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Year 12 *Trial Exam Paper*

2018
MATHEMATICAL METHODS
Written examination 1

Reading time: 15 minutes
Writing time: 1 hour

STUDENT NAME:

QUESTION AND ANSWER BOOK
Structure of book

Number of questions	Number of questions to be answered	Number of marks
9	9	40

- Students are permitted to bring the following items into the examination: pens, pencils, highlighters, erasers, sharpeners and rulers.
- Students are NOT permitted to bring blank sheets of paper, notes of any kind or correction fluid/tape into the examination.
- Calculators are NOT permitted in this examination.

Materials provided

- Question and answer book of 11 pages with a separate sheet of miscellaneous formulas.
- Working space is provided throughout this book.

Instructions

- Write your **name** in the box provided.
- Remove the formula sheet during reading time.
- Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.
- You must answer the questions in English.

Students are NOT permitted to bring mobile phones or any other unauthorised electronic devices into the examination.

Instructions

Answer **all** questions in the spaces provided.

In all questions where a numerical answer is required, an exact value must be given, unless otherwise specified.

In questions where more than one mark is available, appropriate working **must** be shown.

Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Question 1 (4 marks)

a. Let $y = \cos(x)\cos(4x)$.

Find $\dfrac{dy}{dx}$.

2 marks

b. Let $f(x) = 2e^{2x}$.

Evaluate $f'(\log_e(2))$.

2 marks

Question 2 (5 marks)

Let $y = \dfrac{\log_e(x)}{x}$.

a. Find $\dfrac{dy}{dx}$.

2 marks

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b. Hence, calculate $\displaystyle\int_1^2 \frac{\log_e(x)}{x^2}\, dx$.

3 marks

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TURN OVER

Question 3 (3 marks)

Let $f: D \rightarrow R$, $f(x) = \log_e(x) + \log_e(5 - 2x)$.

a. State the maximal domain of f.

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b. Solve the equation $f(x) = 0$.

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Question 4 (5 marks)

Lily enjoys seeing birds in a tree outside her house. The random variable *X* represents the number of birds in the tree when she looks outside each morning.

The distribution of *X* is given in the table below.

X	0	1	2	3
Pr(X = x)	0.45	0.3	0.15	0.1

a. Find $\Pr(X = 0 \mid X < 2)$.

1 mark

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b. Find $E(X)$.

1 mark

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c. Lily records how many birds she sees each morning before school for a week. Let $\hat{P}$ represent the sample proportion of days that Lily sees at least two birds in a particular five day week.

    i. Find $E(\hat{P})$.

1 mark
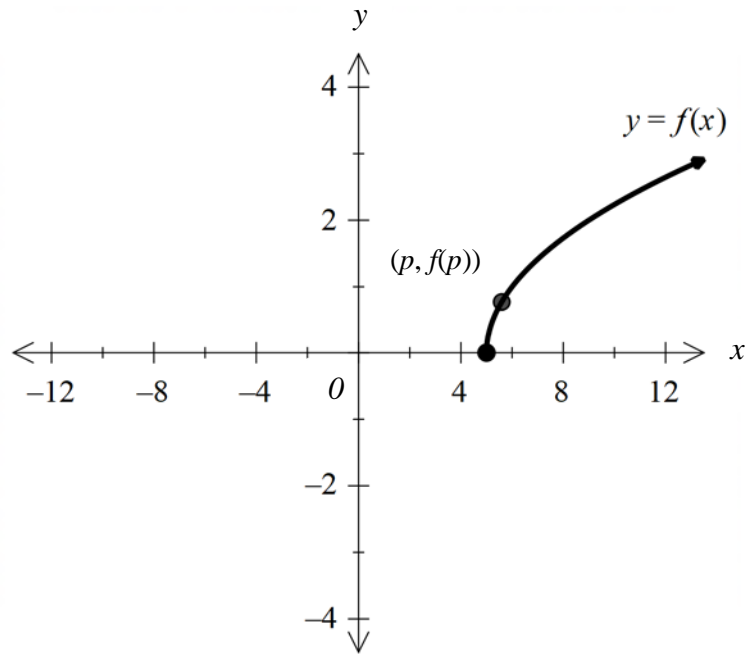
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    ii. Find $\Pr\left(\hat{P} = 0.2\right)$.

Express your answer in the form $\dfrac{a}{b^c}$, where *a*, *b* and *c* are positive integers.

2 marks

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Question 5 (4 marks)

Let $f:[5,\infty) \to R$, where $f(x) = \sqrt{x-5}$.

The point, P, with coordinates $(p, f(p))$, is on the curve defined by $y = f(x)$.

a. Find the gradient of the tangent to the curve at the point P in terms of p.

1 mark

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b. Find the value of p such that the tangent to the curve at P passes through the point $(0,0)$.

3 marks

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Question 6 (2 marks)

The events A and B from a sample space are independent. If $\Pr(A' \cap B) = 0.2$ and $\Pr(A) = 0.4$, find $\Pr(A \cap B)$.

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Question 7 (3 marks)

Let X be a continuous random variable with probability density function

$$f(x) = \begin{cases} kx^3(1-x)^2 & 0 < x < 1 \\ 0 & \text{elsewhere} \end{cases}$$
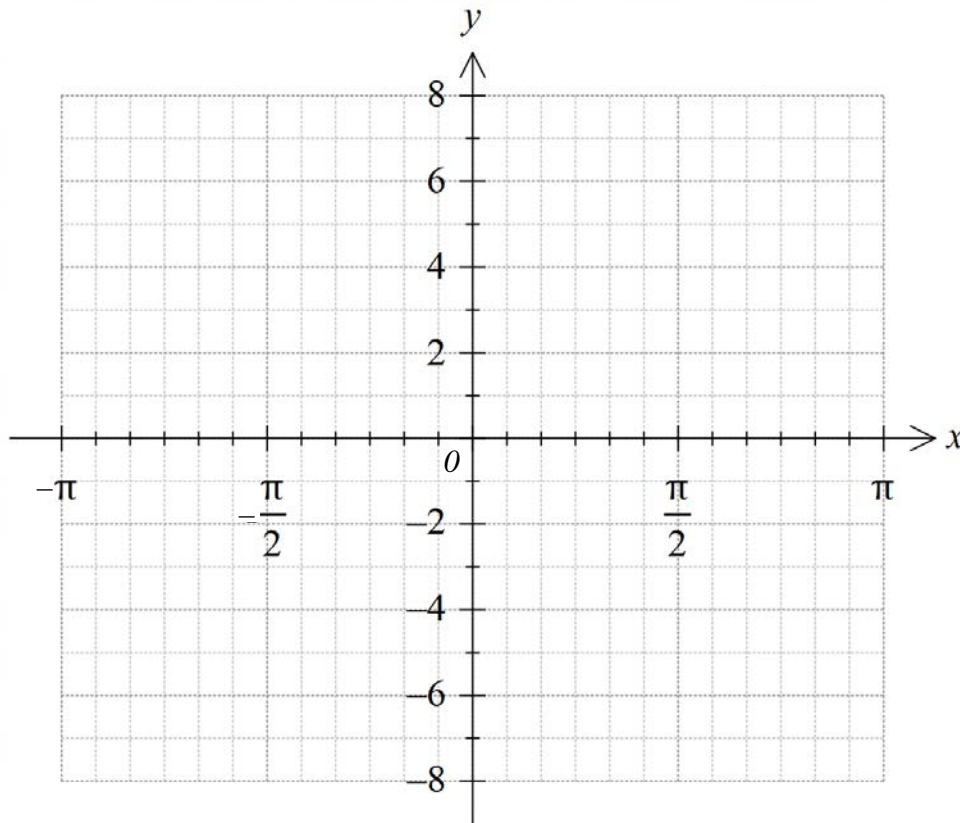
where k is a real number.

Find the value of k.

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Question 8 (7 marks)

Let $f: \left[-\dfrac{7\pi}{12}, \dfrac{\pi}{4}\right] \to R$, where $f(x) = 2 - 4\sin(2x)$.

a.   Sketch the graph of $y = f(x)$ on the axes below. Label all end points and axes intercepts with their coordinates.

4 marks

b.   Find the area enclosed by the graph of $y = f(x)$ and the x-axis over the interval $x \in \left[-\dfrac{7\pi}{12}, \dfrac{\pi}{12}\right]$.

3 marks

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Question 9 (7 marks)

Let $f: R \backslash \{b\} \to R$, $f(x) = \dfrac{a}{x-b}$, where a and b are real numbers.

a. Find the domain and rule of the inverse function f^{-1}.

2 marks

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The transformation $T: R^2 \to R^2$ with rule $T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} 1 & 0 \\ 0 & g \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} h \\ k \end{bmatrix}$, where g, h

and k are integers, maps the graph of $y = f(x)$ onto the graph of $y = f^{-1}(x)$.

b. Find the values g, h and k in terms of a and b.

2 marks

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c. Find the values of a, in terms of b, for which the equation $f(x) = f^{-1}(x)$ has no real solutions.

3 marks

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END OF QUESTION AND ANSWER BOOK