MATHEMATICAL METHODS

Written examination 1

TSSM™
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2017 Trial Examination

SOLUTIONS

Question 1

a. $y = (x^2 + 1)e^{4x} \rightarrow \frac{dy}{dx} = e^{4x}(4x^2 + 4 + 2x)$ 1 mark

$\rightarrow \frac{dy}{dx} = 2e^{4x}(2x^2 + x + 2)$

b.

i. $f'(x) = \frac{x^2 \times \frac{2}{x} - 2log_e(x) \times 2x}{x^4}$ 1 mark

$= \frac{x(2 - 4log_e(x))}{x^4}$ 1 mark

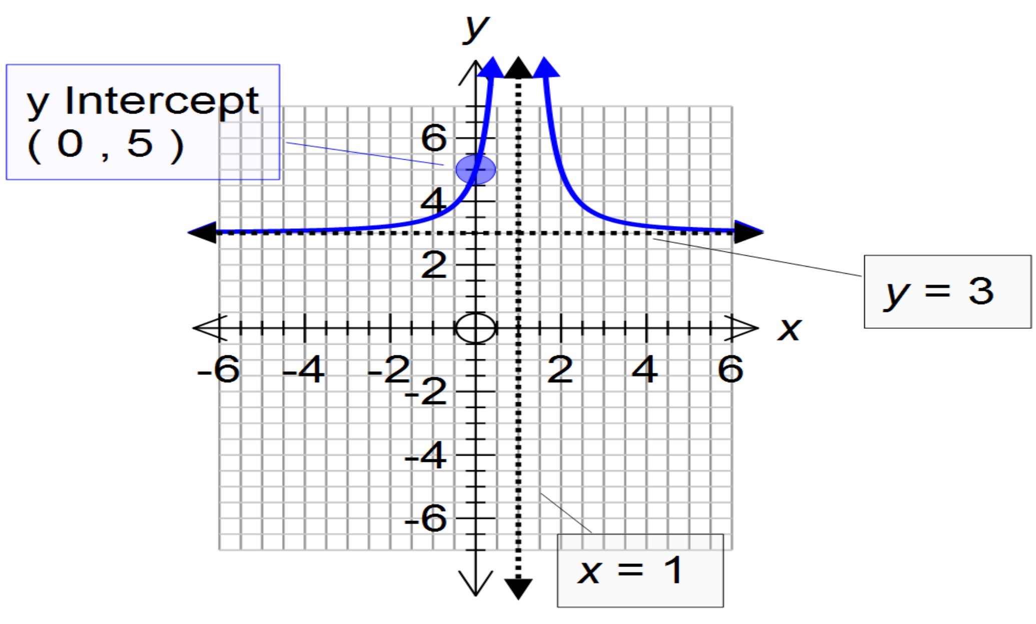
$= \frac{2 - 4log_e(x)}{x^3}$

ii. $f'(e) = \frac{2-4}{e^3} = -\frac{2}{e^3}$

1 mark

Question 2

a.



1 mark for equations of asymptotes, 1 mark for y-intercept, 1 mark for shape

b. $Area = \int_2^5 (3 + 2(x-1)^{-2})\, dx$ 1 mark

$Area = \left[3x - \frac{2}{x-1}\right]_2^5 . Area = \left(15 - \frac{1}{2}\right) - (6 - 2) = 10.5 \;\; sq \;\; units$

1 mark

Question 3

a. $\frac{dy}{dx} = -\tan(x) \to m_T = -1$ 1 mark

$y - \ln\left(\frac{\sqrt{2}}{2}\right) = -1\left(x - \frac{\pi}{4}\right)$

$y = -x + \left(\frac{\pi}{4} + \ln\left(\frac{\sqrt{2}}{2}\right)\right)$ 1 mark

b. $tan\theta = -1$

$\theta = \frac{3\pi}{4}$

1 mark

c. $-tanx = 0 \to x = 0,\ \pi,\ 2\pi$ 1 mark

Question 4

a. $\left(\frac{2}{3}\right)^4 = \frac{16}{81}$

1 mark

b. $\Pr(B \geq 1) = 1 - \Pr(B = 0) = 1 - \frac{16}{81} = \frac{65}{81}$

1 mark

c. $\mathrm{C}(5, 3) \times \left(\frac{16}{81}\right)^3 \times \left(\frac{65}{81}\right)^2$ 1 mark

$= 10\left(\frac{16}{81}\right)^3\left(\frac{65}{81}\right)^2$

1 mark

Question 5

a.

i. $1 - x^2 > 0 \to x^2 < 1 \to -1 < x < 1$

Domain: $(-1,\ 1)$

1 mark

ii. $x = log_e(1 - y^2)$

$e^x = 1 - y^2 \to y^2 = 1 - e^x \to y = \pm\sqrt{1 - e^x}$ 1 mark

$g^{-1}(x) = -\sqrt{1 - e^x}$ 1 mark

iii. $Domain: (-\infty, 0]$

$Range: (-1, 0]$

1 mark each

b.

i. $h(k(x)) = \sqrt{1 - e^{-1-x^2}}$

1 mark

ii. $Domain = Domain\ \ of\ \ k(x) = R$

1 mark

iii. $\frac{d(h(k(x)))}{dx} = \frac{1}{2\sqrt{1-e^{-1-x^2}}}(2xe^{-1-x^2})$ 1 mark

For stationary point, numerator must equal zero

$e^{-1-x^2} \times 2x = 0 \to x = 0$ 1 mark

$\left(0,\ \sqrt{1 - e^{-1}}\right)$ 1 mark

Question 6

a. $\sin(2x) + 1 = 0 \to 2x = \frac{3\pi}{2}, \frac{7\pi}{2}$ 1 mark

$\left(\frac{3\pi}{4},\ 0\right)\ \ and\ \ \left(\frac{7\pi}{4},\ 0\right)$ 1 mark

b. $Average\ \ ROC = \frac{f(2\pi) - f(0)}{2\pi} = 0$ 1 mark

c. $Average\ \ value = \frac{1}{\frac{7\pi}{4} - \frac{3\pi}{4}} \times \int_{3\pi/4}^{7\pi/4} (\sin(2x) + 1)\ dx$ 1 mark

$Average\ \ value = \frac{1}{\pi} \times \left(-\frac{\cos(2x)}{2} + x\right)_{\frac{3\pi}{4}}^{\frac{7\pi}{4}}$ 1 mark

$Average\ \ value = \frac{1}{\pi} \times \left(\frac{7\pi}{4} - \frac{3\pi}{4}\right) = 1$ 1 mark

Question 7

a. $\Pr(\text{faulty}) = \frac{50\text{x}0.04 + 80\text{x}0.05}{130} = \frac{3}{65}$

1 mark

b. $\Pr(B|Faulty) = \frac{\Pr(B \cap faulty)}{\Pr(faulty)}$ 1 mark

$= \frac{\frac{4}{80} \times \frac{8}{13}}{\frac{4}{80} \times \frac{8}{13} + \frac{2}{50} \times \frac{5}{13}} = \frac{2}{3}$ 1 mark

Question 8

a. $\frac{d}{dx}\left(e^{2x}(2+bx)\right) = 2e^{2x} \times (2+bx) + be^{2x} = e^{2x}(4+b) + 2bxe^{2x}$

1 mark

b. $\Pr\left(X < \frac{1}{4}\right) = \int_0^{1/4} 4xe^{2x}\,dx$

Use $b = 2$

$\Pr\left(X > \frac{1}{4}\right) = \left(e^{2x}(2+2x)\right)_0^{1/4} - 6\int_0^{1/4} e^{2x}\,dx$

$\Pr\left(X > \frac{1}{4}\right) = \left(e^{2x}(2+2x)\right)_0^{1/4} - (3e^{2x})_0^{1/4}$ 1 mark

$\Pr\left(X > \frac{1}{4}\right) = \frac{5}{2}e^{\frac{1}{2}} - 2 - \left(3e^{\frac{1}{2}} - 3\right) = 1 - \frac{e^{\frac{1}{2}}}{2}$

1 mark

c. $\Pr(X < m) = \frac{1}{2}$

$\int_0^m 4xe^{2x}\,dx = \frac{1}{2}$

$\left(e^{2x}(2+2x)\right)_0^m - (3e^{2x})_0^m = \frac{1}{2}$

$e^{2m}(2+2m) - 2 - (3e^{2m} - 3) = \frac{1}{2}$ 1 mark

$-e^{2m} + 2me^{2m} + 1 = \frac{1}{2}$

$e^{2m} - 2me^{2m} - \frac{1}{2} = 0 \rightarrow 2e^{2m} - 4me^{2m} - 1 = 0$

$2e^{2m} - 4me^{2m} - 1 = 0$

1 mark