Neap:

Trial Examination 2017

VCE Mathematical Methods Units 3&4

Written Examination 1

Suggested Solutions

Question 1 (4 marks)

a. $\frac{dy}{dx} = 6x(3x^2 - 4)^{-\frac{1}{2}}$

$= \frac{3x}{\sqrt{3x^2 - 4}}$ A1

b. $f'(x) = 2x^2\sec^2(2x) + 2x\tan(2x)$ M1

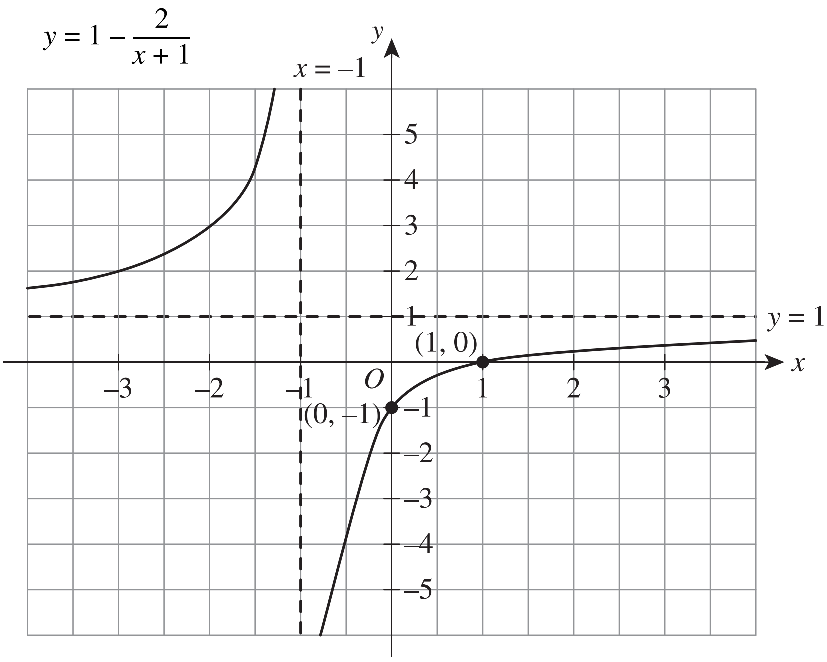
$f'\left(\frac{\pi}{2}\right) = \frac{\left(\frac{\pi}{2}\right)^2 \times 2}{(\cos(\pi))^2} + 2\left(\frac{\pi}{2}\right)\tan(\pi)$ M1

$= \frac{\pi^2}{2} + 0$

$= \frac{\pi^2}{2}$ A1

Question 2 (5 marks)

a.



correct shape A1
correct asymptotes labelled at x = –1, y = 1 A1
correct intercepts labelled at (0, –1) and (1, 0) A1

b. $\text{area} = -\int_0^1 1 - \frac{2}{x+1}dx$ M1

$= -[x - 2\log_e|x+1|]_0^1$

$= -[1 - 2\log_e(1+1)] - [0 - 2\log_e(0+1)]$

$= -[1 - \log_e(4)] - [0]$

$= \log_e(4) - 1$ A1

Question 3 (3 marks)

$2(2^{4b}) + 7(2^{2b}) = 4 \quad \text{let } u = 2^{2b}$

$2u^2 + 7u - 4 = 0$ M1

$(2u-1)(u+4) = 0$

$u = \frac{1}{2}, -4$

$\therefore 2^{2b} = \frac{1}{2} \text{ since } 2^{2b} \neq -4$ A1

$2^{2b} = 2^{-1},\ b = -\frac{1}{2}$ A1

Note: Students must disregard the incorrect solution to get full marks.

Question 4 (2 marks)

Let $y = e^{3x} + 4$ (for inverse swap x and y).

$x = e^{3y} + 4$ M1

$x - 4 = e^{3y}$

$\log_e(x-4) = 3y$

$f^{-1}(x) = \frac{\log_e(x-4)}{3}$ A1

Alternatively:

$f^{-1}(x) = \frac{1}{3}\log_e(x-4)$

$= \log_e(x-4)^{\frac{1}{3}}$ A1

Question 5 (3 marks)

$-2x - 1 = x'$

$x = \frac{x' + 1}{-2}$

$3y - 2 = y'$

$y = \frac{y' + 2}{3}$ M1

substitute into original equation $y = \frac{1}{x} + 3$

$\frac{y' + 2}{3} = \frac{1}{\frac{x' + 1}{-2}} + 3$

$y' + 2 = \frac{-6}{x' + 1} + 9$

$y' = \frac{-6}{x' + 1} + 7$

$y_{\text{new}} = \frac{-6}{x + 1} + 7$ A1

$a = -6,\ b = 1,\ c = 7$ *must state values for all of a, b, and c* A1

Question 6 (3 marks)

$\mathrm{E}(X) = np$

$= 3$

$\mathrm{var}(X) = np(1 - p)$

$= \frac{12}{5}$

$\frac{\mathrm{var}(X)}{\mathrm{E}(X)} = \frac{np(1 - p)}{np}$

$= \frac{\frac{12}{5}}{3}$ M1

$1 - p = \frac{12}{15}$

$p = \frac{3}{15}$

$= \frac{1}{5}$ A1

$n \times \frac{1}{5} = 3$

$n = 15$ A1

Question 7 (4 marks)

a. $f'(x) = \cos(2x) - 2x\sin(2x)$ A1

b. $$\int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \cos(2x) - 2x\sin(x)dx = [x\cos(2x)]_{\frac{\pi}{6}}^{\frac{\pi}{2}}$$ M1

$$\int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \cos(2x)dx - [x\cos(2x)]_{\frac{\pi}{6}}^{\frac{\pi}{2}} = 2\int_{\frac{\pi}{6}}^{\frac{\pi}{2}} x\sin(2x)dx$$ M1

$$\left[\frac{\sin(2x)}{2}\right]_{\frac{\pi}{6}}^{\frac{\pi}{2}} - [x\cos(2x)]_{\frac{\pi}{6}}^{\frac{\pi}{2}} = 2\int_{\frac{\pi}{6}}^{\frac{\pi}{2}} x\sin(2x)dx$$

$$\left[\frac{\sin(\pi)}{2} - \frac{\sin\left(\frac{\pi}{3}\right)}{2}\right] - \left[\frac{\pi}{2}\cos(\pi) - \frac{\pi}{6}\cos\left(\frac{\pi}{3}\right)\right] = 2\int_{\frac{\pi}{6}}^{\frac{\pi}{2}} x\sin(2x)dx$$

$$\left[0 - \frac{\sqrt{3}}{4}\right] - \left[-\frac{\pi}{2} - \frac{\pi}{12}\right] = 2\int_{\frac{\pi}{6}}^{\frac{\pi}{2}} x\sin(2x)dx$$

$$\frac{7\pi}{24} - \frac{\sqrt{3}}{8} = \int_{\frac{\pi}{6}}^{\frac{\pi}{2}} x\sin(2x)dx$$ A1

Question 8 (4 marks)

a.
$$\Pr(Z > c) = \Pr(X < 2.3)$$
$$= \Pr(Z < -2)$$
$$\Pr(Z < -2) = \Pr(Z > 2)$$
$$c = 2$$ A1

b.
$$\Pr(X < 2.7 | X < 3.5) = \frac{\Pr(X < 2.7 \cap X < 3.5)}{\Pr(X < 3.5)}$$
$$= \frac{\Pr(X < 2.7)}{\Pr(X < 3.5)}$$
$$= \frac{\Pr(Z < -1)}{\Pr(Z < 1)}$$ *equating Z values to X values* M1
$$= \frac{\frac{16}{100}}{\frac{84}{100}}$$ *calculating Pr(Z < 1) = 1 – $\frac{16}{100}$* M1
$$= \frac{16}{84}$$
$$= \frac{4}{21}$$ A1

Question 9 (12 marks)

a. **i.** $A = 2\left[(9 \times 9) - (3 \times 1) - \int_3^9 \frac{1}{9}x^2 dx\right]$ M1

$= 2\left[81 - 3 - \left[\frac{x^3}{27}\right]_3^9\right]$ M1

$= 2[78 - (27 - 1)]$

$= 2(78 - 26)$

$= 104 \text{ cm}^3$ A1

ii. volume $= 104 \times 18$

$= 1872 \text{ cm}^3$ A1

b. maximum rate when $R'(t) = 0$ M1

$R'(t) = 6t - 4$

$= 0$

$t = \frac{2}{3} \Rightarrow t = 40$ minutes A1

c. $V(t) = \int R(t)dt$ M1

$= \int (3t^2 - 4t + 1)dt$

$= t^3 - 2t^2 + t$ *as V(t) = 0 at t = 0 as no water emptied* M1

$t^3 - 2t^2 + t = 1872$ M1

$(t - 13)(t^2 + 11t + 144) = 0$

$\therefore t = 13$ hours A1

d. At $t = 10$, $V(10) = 10^3 - 2(10)^2 + 10$

$= 810 \text{ cm}^3$ M1

$\therefore$ yes, overflows as only 810 cm^3 removed A1