The Mathematical Association of Victoria

Trial Examination 2017

MATHEMATICAL METHODS

Written Examination 2

STUDENT NAME ____

Reading time: 15 minutes
Writing time: 2 hours

QUESTION AND ANSWER BOOK

Structure of examination

Section	*Number of questions*	*Number of questions to be answered*	*Number of marks*
A	20	20	20
B	4	4	60
			Total 80

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers, a protractor, set-squares, aids for curve sketching, one bound reference, one approved technology (calculator or software) and, if desired, one scientific calculator. Calculator memory DOES NOT need to be cleared. For approved computer-based CAS, full functionality may be used.
- Students are NOT permitted to bring into the examination room: blank sheets of paper and/or white out liquid/tape.

Materials supplied

- Question and answer book of 23 pages.
- Formula sheet.
- Answer sheet for multiple-choice questions.

Instructions

- Write your **name** in the space provided above on this page.
- Write your **name** on the multiple-choice answer sheet.
  Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.
- All written responses must be in English.

At the end of the examination

- Place the answer sheet for multiple-choice questions inside the front cover of this book.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

SECTION A- Muliple-choice questions

Instructions for Section A

Answer **all** questions in pencil on the answer sheet provided for multiple – choice questions.
Choose the response that is **correct** for the question.
A correct answer scores1, an incorrect answer scores 0.
Marks will **not** be deducted for incorrect answers.
No marks will be given if more than one answer is completed for any question.
Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Question 1

The range and period of the graph of $f:\left[0,\frac{\pi}{2}\right]\to R,\ f(x)=-3\cos\left(4x+\pi\right)+1$ are respectively

A. $[-3,3]$ and $\frac{3\pi}{2}$

B. $[-3,3]$ and $\frac{\pi}{2}$

C. $[-2,4]$ and $\frac{\pi}{2}$

D. $[-2,4]$ and 4π

E. $[-2,4]$ and 2π

Question 2

If $\tan(x)=\frac{3}{5}$ and $\pi\le x\le\frac{3\pi}{2}$ then $\sin\left(x\right)$ equals

A. $-\frac{3}{4}$

B. $-\frac{3}{\sqrt{34}}$

C. $\frac{4}{\sqrt{34}}$

D. $\frac{3}{5}$

E. $-\frac{3}{5}$

Question 3

Let $h(x) = x^4$.

Which one of the following is **false**?

A. $h(x) = h(-x)$

B. $-h(x) = -h(-x)$

C. $h(xy) = h(x) \times h(y)$

D. $h(x + y) = h(x) + h(y)$

E. $h\left(\frac{x}{y}\right) = \frac{h(x)}{h(y)}$

Question 4

Which one of the following is the inverse function of $h:(-\infty, 4) \to R, h(x) = 2(4-x)^2 - 1$?

A. $h^{-1}:(-\infty, 4) \to R, h^{-1}(x) = 4 \pm \frac{\sqrt{2(1+x)}}{2}$

B. $h^{-1}:(-\infty, 4) \to R, h^{-1}(x) = 4 - \frac{\sqrt{2(1+x)}}{2}$

C. $h^{-1}:(-1, \infty) \to R, h^{-1}(x) = 4 + \frac{\sqrt{2(1+x)}}{2}$

D. $h^{-1}:[-1, \infty) \to R, h^{-1}(x) = 4 - \frac{\sqrt{2x+2}}{2}$

E. $h^{-1}:(-1, \infty) \to R, h^{-1}(x) = 4 - \frac{\sqrt{2x+2}}{2}$

Question 5

Consider the functions *f* with rule $f(x) = 3e^{2x}$ and *g* with rule $g(x) = \log_e(x+2)$ over their maximal domains.
The function $h = f(g(x))$ can be defined as

A. $h:(0, \infty) \to R, h(x) = x^2$

B. $h: R \to R, h(x) = 3(x+2)^2$

C. $h:(-2, \infty) \to R, h(x) = 3(x+2)^2$

D. $h: R \to R, h(x) = \log_e\left(2 + 3e^{2x}\right)$

E. $h:(0, \infty) \to R, h(x) = \log_e\left(2 + 3e^{2x}\right)$

Question 6

Consider the following equations.

$mx + y = 2$

$2x - 3y = k$

The values of the real numbers *m* and *k* that will give no real solutions are

A. $m = -\frac{2}{3}, k = -6$

B. $m = -\frac{2}{3}, k \in R \setminus \{-6\}$

C. $m \in R \setminus \left\{-\frac{2}{3}\right\}, k \in R$

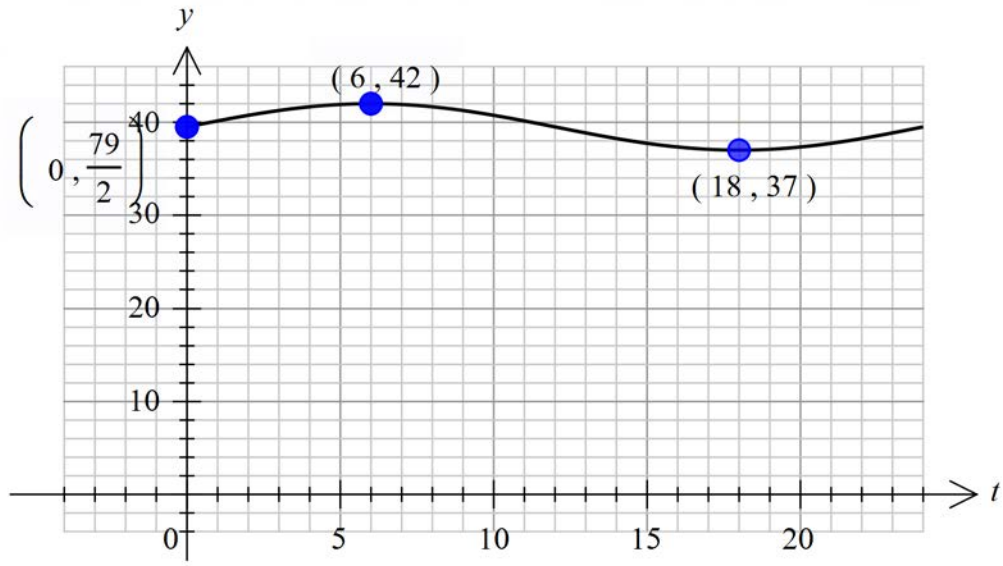
D. $m \in R \setminus \left\{-\frac{2}{3}\right\}, k \in R \setminus \{-6\}$

E. $m \in R \setminus \left\{-\frac{2}{3}\right\}, k = -6$

SECTION A - continued

Question 7

The temperature, T° C of a sick child in a hospital in Melbourne is illustrated in the graph below, where t is the number of hours after the 8 a.m. and $y = T$.



The graph is most likely to have the equation

A. $y = 38.5\sin(12t)$

B. $y = 2.5\sin\left(\frac{\pi t}{12}\right) + 42$

C. $y = 2.5\sin(\pi t) + 39.5$

D. $y = -2.5\sin\left(\frac{\pi t}{12}\right) + 39.5$

E. $y = 2.5\sin\left(\frac{\pi t}{12}\right) + 39.5$

Question 8

Let $f(x) = \sqrt{2x-1} + 3$ and $g(x) = \frac{1}{(x-2)^2} + 4$. The maximal domain of $f + g$ is

A. R

B. $[1,2) \cup (2,\infty)$

C. $(2,\infty)$

D. $\left[\frac{1}{2},2\right) \cup (2,\infty)$

E. $R \setminus \{2\}$

SECTION A - continued
TURN OVER

Question 9

If $f'(x) = e^{x+1} + x + 1$ and $f(0) = 2$ then $f(x)$ equals

A. $2 + e^{x+1} - e$

B. $1 + e^{x+1}$

C. $\frac{x^2}{2} + x + e^{x+1}$

D. $\frac{x^2}{2} + x + 2 - e\left(e^x - 1\right)$

E. $\frac{x^2}{2} + x + 2 + e\left(e^x - 1\right)$

Question 10

If $\int_1^3 g(x)dx = 2$ then $2\int_3^1 \left(g(x)+1\right)dx$ equals

A. 6

B. 5

C. –8

D. –6

E. –4

Question 11

The average value of the function *f* with rule $f(x) = -\sin(\pi - 2x) + 3$ between $x = 0$ and $x = \pi$ is

A. 2

B. 3

C. 4

D. 3π

E. 0

SECTION A - continued

Question 12

The maximum **horizontal** distance between the curves with equations $f(x)=e^x$ and $g(x)=\log_e(x)+7$, within the bounded region, is closest to

A. 0.57
B. 4.67
C. 5.33
D. 1.49
E. 4.66

Question 13

The equation of a straight line which is **both** perpendicular to the line passing through the two axial intercepts of the graph with equation $f(x)=\log_e(3-x)$ and is **also** a tangent to the curve with equation $g(x)=x^2$ is

A. $y=-\frac{\log_e(3)}{2}x$

B. $y=\frac{2}{\log_e(3)}x$

C. $y=\frac{2}{\log_e(3)}x-\frac{1}{\left(\log_e(3)\right)^2}$

D. $y=-\frac{2}{\log_e(3)}x-\frac{1}{\left(\log_e(3)\right)^2}$

E. $y=\frac{\log_e(3)}{2}x-\frac{\left(\log_e(3)\right)^2}{16}$

Question 14

A particle, starting from rest, moves along a straight line. Its acceleration, a ms^{-2}, at time t seconds is given by $a=2t^2-1$. The distance, in metres, travelled by the particle in the first four seconds is

A. 4
B. 16
C. $\frac{104}{3}$
D. $\frac{425}{12}$
E. 31

SECTION A - continued
TURN OVER

Question 15

An approximation is being used to find the area under the graph of $y = \log_e(2x+1)$ between the *x*-axis, the *x*-intercept and the line $x = 1$. Which one of the following methods will give the best estimate?

A. right-endpoint rectangles of width 0.5
B. left-endpoint rectangles of width 0.5
C. right-endpoint rectangles of width 0.25
D. right-endpoint rectangles of width 0.25 minus left-endpoint rectangles of width 0.25
E. $\dfrac{\left(\text{left-endpoint rectangles of width 0.25 + right-endpoint rectangles of width 0.25}\right)}{2}$

Question 16

Consider the probability distribution which is shown in the table below, where *a* and *b* are real constants.

x	0	1	a	3
$\Pr(X = x)$	0.1	b	0.2	0.3

If the Var(*X*) = 4 then *a* could equal

A. $\dfrac{-\sqrt{1205}+13}{8}$ only

B. $\dfrac{3\sqrt{5}+1}{2}$ only

C. $\dfrac{-\sqrt{1205}+13}{8}$ or $\dfrac{\sqrt{1205}+13}{8}$

D. $\dfrac{\sqrt{1205}+13}{8}$ only

E. $\dfrac{3\sqrt{5}+1}{2}$ or $\dfrac{-3\sqrt{5}+1}{2}$

Question 17

There are three white chocolates and seven dark chocolates in box A and six white chocolates and four dark chocolates in Box B. Sara chose a box at random and withdrew a chocolate at random. It was found to be white. The probability it was from Box A is

A. $\frac{1}{3}$

B. $\frac{3}{10}$

C. $\frac{9}{20}$

D. $\frac{3}{5}$

E. $\frac{3}{20}$

Question 18

A binomial random variable X has mean 20 and standard deviation 4. The values of the parameters n and p are respectively

A. 100 and $\frac{1}{5}$

B. 100 and $\frac{4}{5}$

C. 25 and $\frac{4}{5}$

D. 25 and $\frac{1}{5}$

E. 200 and $\frac{1}{10}$

Question 19

The probability of winning a game of chance is 0.65. The least number of games, n, that must be played to ensure that the probability of winning at least twice is more than 0.95 can be found by evaluating

A. $0.35^n + 0.65n(0.35)^{n-1} > 0.05$

B. $0.35^n > 0.05$

C. $0.65^n + 0.35n(0.65)^{n-1} < 0.05$

D. $0.35^n < 0.05$

E. $0.35^n + 0.65n(0.35)^{n-1} < 0.05$

Question 20

The weights of a particular species of fish are normally distributed with a standard deviation of 40 g. If 20% weigh more than 300 g then the mean, in g, of the distribution is closest to

A. 150
B. 266
C. 267
D. 333
E. 334

END OF SECTION A

SECTION B

Instructions for Section B

Answer **all** questions in the spaces provided.
In all questions where a numerical answer is required an exact value must be given unless otherwise specified.
In questions where more than one mark is available, appropriate working **must** be shown.
Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Question 1 (15 marks)

Let $f:\left[-\pi,\frac{7\pi}{2}\right]\to R,\ f(x)=-a\sin\left(\frac{x}{3}+\frac{\pi}{3}\right)-\pi$, where a is a real constant.

a. Find the period of *f*. 1 mark

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The range of *f* is $\left[-\frac{3\pi}{2},-\frac{\pi}{2}\right]$.

b. Show that the value of a is $\frac{\pi}{2}$. 1 mark

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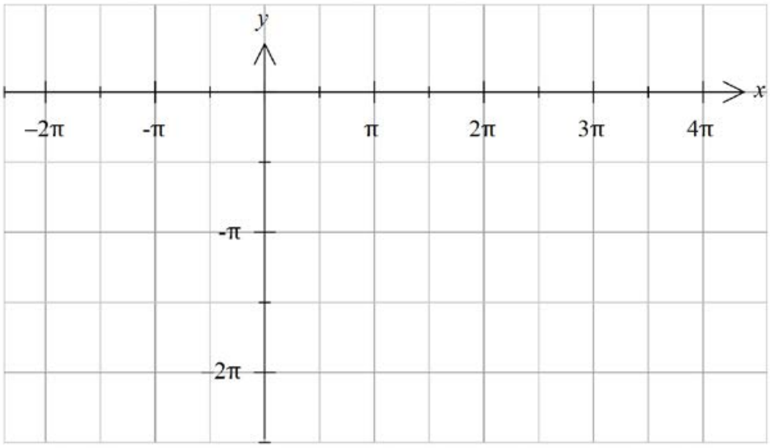
SECTION B - Question 1 - continued
TURN OVER

c. Sketch the graph of *f*, labelling the axial intercepts and endpoints with their coordinates. 2 marks



For the inverse function f_1^{-1} to exist, the domain of f is restricted to $[-\pi, b]$ to form a new function f_1, where b is a real constant.

d. Find the maximum possible value of *b*. 1 mark

e. For this value of *b*, state the domain of f_1^{-1}. 1 mark

Two tangents are drawn to the graph of f.

f. Find the equation of each tangent to the graph of f at $x = 0$ and $x = \pi$. 2 marks

g. Find the coordinates of the point of intersection of the tangents found in **part f.** 1 mark

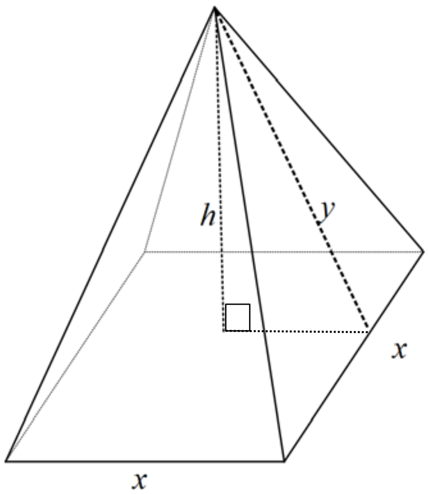
h. Write down an integration statement that when evaluated will give the area enclosed between *f* and the tangents found in **part f.** 2 marks

i. Hence find the area enclosed between *f* and the tangents found in **part f**, correct to 2 decimal places. 1 mark

j. The rule of $f_2(x) = \sin(x)$ can be obtained from the rule of f. State the sequence of transformations required to get from f to f_2. 3 marks

Question 2 (15 marks)

A group of architects, the *Pyramid Group*, are planning a cast iron installation in Federation Square in Melbourne. The first design is a simple right square based pyramid with vertical height, h metres, and length of the sides of the base, x metres.



The *Pyramid Group* know that the total cast iron required for the four sloping sides as well as the square base is 60 m^2.

a. Show that the slant height, y m, of the pyramid can be expressed as $y = \sqrt{h^2 + \frac{x^2}{4}}$. 1 mark

b. Find an expression for the Total Surface Area (TSA) of the pyramid in terms of both x and h. 2 marks

SECTION B - Question 2 - continued

c. Hence show that the volume, V m^3, of the square based pyramid can be expressed in terms of x only as

$$V = \frac{1}{3}x\sqrt{900 - 30x^2}\ .$$

2 marks

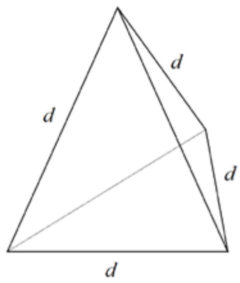
d. State the implied domain for V.

1 mark

e. Find the maximum volume, in m^3, that the *Pyramid Group* can have for this square based pyramid. State the value of x, in m, that gives this maximum volume.

2 marks

The second design is a tetrahedron triangular pyramid, with each of the 4 faces an equilateral triangle of side length, d metres. The *Pyramid Group* like this design because a tetrahedron is the only ordinary convex polyhedron that has fewer than 5 faces.



f. **i.** Find an expression for the TSA of the 4 faces of the tetrahedron in terms of d. Write your answer in the form $\sqrt{a}d^b$ where a and b are real constants. 2 marks

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The volume, V_T m^3, of this regular tetrahedron can be found using the formula $V_T = \frac{d^3}{6\sqrt{2}}$.

ii. If the TSA of the 4 faces of the tetrahedron equals 60 m^2, find the volume of the tetrahedron, in m^3 correct to 3 decimal places. 2 marks

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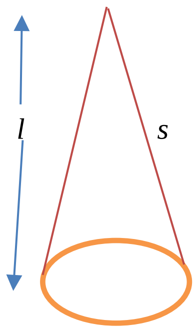
The third design that the *Pyramid Group* chooses is a right circular cone with radius r metres and height l metres.



The formula for finding the TSA of a right circular cone is TSA = $\pi r(r+s)$ where r is the radius of the circular base and s is the slant height of the cone. Both are measured in metres.

The *Pyramid Group* again wants to use a TSA of 60 m^2.

g. If the TSA of the right circular cone equals 60 m^2, find its maximum volume, correct to two decimal places. 2 marks

h. The *Pyramid Group* wants the design to have the maximum possible volume. Which of the three installations will they select? 1 mark

Question 3 (15 marks)

A manufacturer, Glowglen, claims that 92% of their light globes last longer than 100 hours. Customers start to complain, claiming that this is not true. Inspectors decide to take a random sample of 500 light globes and find that 426 of them last longer than 100 hours.

a. Find a 99% confidence interval for the proportion of light globes lasting more than 100 hours. Give your answers correct to three decimal places. 1 mark

b. If the inspectors took another 200 random samples of 500 light globes, how many of the 99% confidence intervals would be expected to contain the population proportion? 1 mark

c. According to the survey results, do the customers have a right to complain? Briefly explain your answer. 1 mark

d. What is the largest sample size Glowglen could have agreed to so that their claim is within a 99% confidence interval, assuming the sample proportion $\hat{p}$ is 0.852? Round your answer to the nearest integer. 2 marks

It was found that 85.2% of the Glowglen light globes last longer than 100 hours. Five light globes were selected at random.

e. What is the probability that more than three of them last longer than 100 hours? Give your answer correct to three decimal places. 2 marks

f. Given only three of them last longer than 100 hours, what is the probability it was the first three. 2 marks

The length of time a Glowglen light globe lasts is also normally distributed. The inspectors also found that 2% of the light globes last longer than 150 hours.

g. Find the mean and standard deviation of the distribution. Give your answers correct to two decimal places. 3 marks

Jessica and David both bought Glowglen light globes so that they could do their homework. The probability that Jessica does her homework on Monday night is 0.2, and the probability that David does his on a Monday night is q^2, where q is a positive real constant. These events are independent.

h. Find the probability that neither of them will do their homework on Monday night in terms of q. 2 marks

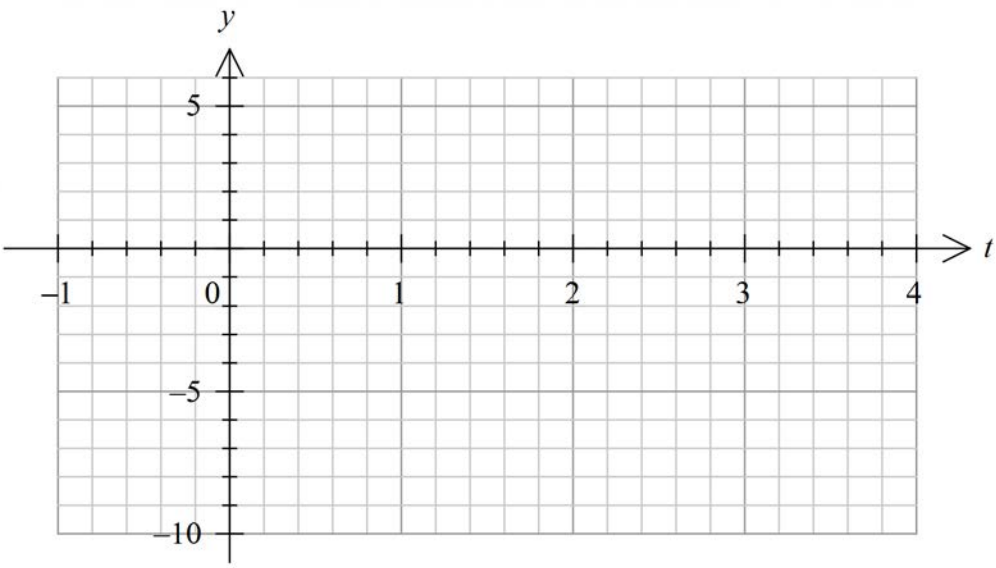
i. If the probability that David does not do his homework is 0.4, find the value of q. Write your answer in the form $\frac{\sqrt{a}}{b}$ where a and b are real constants. 1 mark

Question 4 (15 marks)

Due to a disease, the rate at which a rabbit population is changing in a particular area is given by $g'(t) = -3(1-t)^2 e^{(1-t)^3}$, where t is the time in months, $t > 0$, and g' the number of rabbits **in hundreds** per month.

a. Sketch the graph of $y = g'(t)$ on the set of axes below. Label any asymptotes with their equations and the stationary points and endpoint(s) with their coordinates correct to the nearest hundred rabbits.

3 marks



b. By how many rabbits did the population decline in the first two months? Give your answer to the nearest integer.

2 marks

SECTION B - Question 4 - continued
TURN OVER

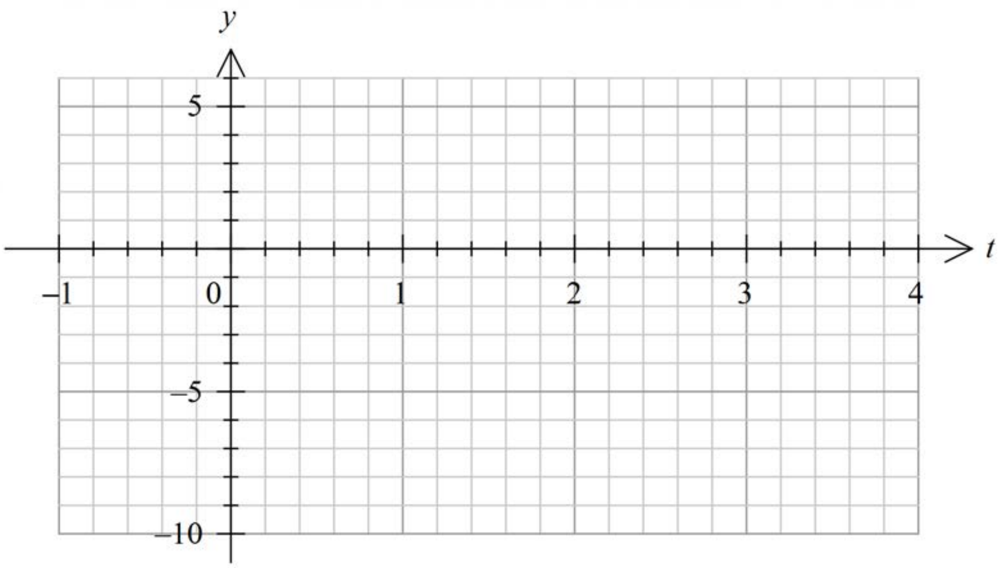
Assume $g(1) = 2$.

c. Find a rule for g. 2 marks

d. How many rabbits were there initially? Give your answer to the nearest integer. 1 mark

e. If conditions do not change, how many rabbits will there be in the future? Give your answer to the nearest integer. 1 mark

f. Sketch the graph of $y = g(t)$ on the set of axes below. Label any asymptotes with their equations and the stationary points and endpoints with their coordinates correct to the nearest hundred rabbits. 2 marks



SECTION B - Question 4 - continued

Rabbits run across paddocks at fast speeds to escape from predators. The velocity, v m/s, of a particular rabbit, which was travelling in a straight line, was recorded and is modelled by the rule $v(t) = -\frac{3}{2}t(t-1)(t-b)(t-4)$, where t is the time in seconds and b is a real constant, $1 < b < 4$.

g. Find the value of b if the rabbit first stopped after running 2.2 m. 2 marks

h. Find the maximum speed of the rabbit in m/s correct to one decimal place, and the total distance the rabbit ran, in m, during the first four seconds. 2 marks

END OF QUESTION AND ANSWER BOOK