The Mathematical Association of Victoria

Trial Examination 2017

MATHEMATICAL METHODS

WRITTEN EXAMINATION 1

STUDENT NAME ____

Reading time: 15 minutes
Writing time: 1 hour

QUESTION AND ANSWER BOOK

Structure of book

Number of questions	*Number of questions to be answered*	*Number of marks*
8	8	40

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers.
- Students are NOT permitted to bring into the examination room: any technology (calculators or software), notes of any kind, blank sheets of paper and/or correction fluid/tape.

Materials supplied
- Question and answer book of 10 pages,
- Formula Sheet
- Working space is provided throughout the book.

Instructions
- Write your **name** in the space provided above on this page.
- Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.
- All written responses must be in English.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

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Question 1 (5 marks)

a. Find $\frac{dy}{dx}$ when $y = \tan\left(x^2 + 2\right)$. 2 marks

b. Find $f'(-2)$ when $f(x) = \frac{\log_e\left(x^2 - 1\right)}{x^2 - 1}$. 3 marks

Question 2 (4 marks)

The depth of water in a wave pool during a particular hour is given by $d(t) = -5\cos\left(\frac{\pi t}{8}\right) + 5$ metres, where t is the time in minutes and $0 \le t \le 60$.

a. Find the value of t for which the depth of water is first at its maximum. 1 mark

b. Find the fraction of time for the particular hour when the depth of water is more than 2.5 metres. 3 marks

TURN OVER

Question 3 (7 marks)

Consider the polynomial $P(x) = 5x^3 - x^2 + x + 7$.

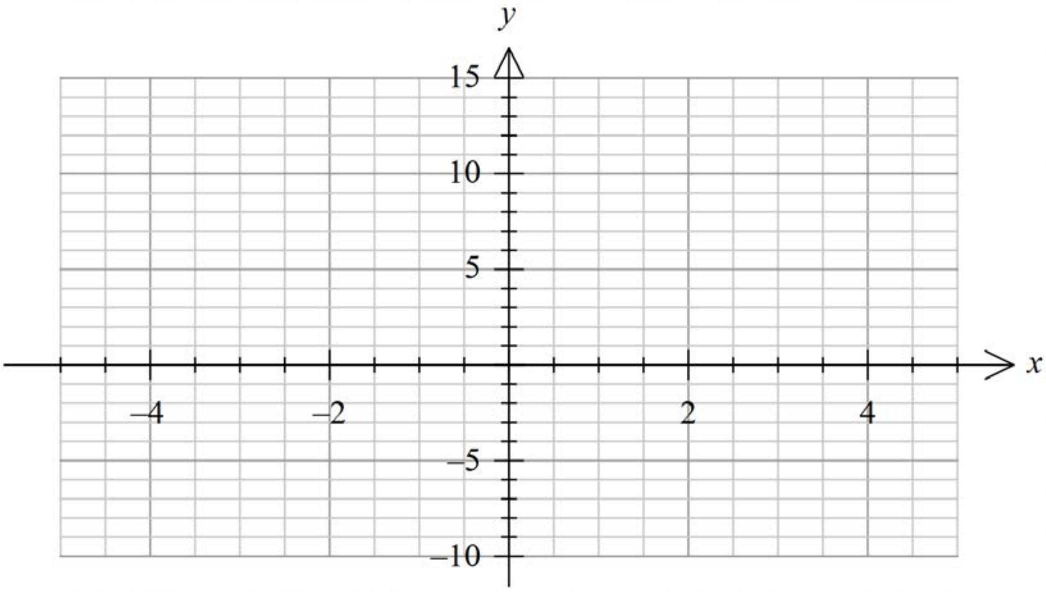
a. Find a linear factor of $P(x)$. 1 mark

b. Find $Q(x)$, the quadratic factor of $P(x)$. 1 mark

c. Show that there are no linear factors for $Q(x)$. 1 mark

d. The graph of $y = P(x)$ has no stationary points. Find the *x* coordinate of the point where the gradient of the graph of *P* is a minimum. 2 marks

e. Hence sketch the graph of $y = P(x)$ labelling any axial intercepts with coordinates. 2 marks



TURN OVER

Question 4 (6 marks)

Let $f(x) = x\sin(x)$.

a. Find the average rate of change of f over the interval $\left[\frac{\pi}{2}, \pi\right]$. 2 marks

b. Find $f'(x)$. 1 mark

c. Hence find $\int_0^{\frac{\pi}{2}} x\cos(x)\,dx$. 3 marks

Question 5 (3 marks)

Solve $\log_2\left((2x-2)^2\right)-4\log_2(1-x)=1$ for x.

Question 6 (3 marks)

The government is concerned about the number of teenagers in Australia who smoke on a regular basis. They randomly selected 10 000 teenagers and found 2000 of them smoke regularly. Find an approximate 95% confidence interval for the proportion of teenagers in Australia who smoke on a regular basis. Use an integer multiple of the standard deviation in your calculation.

TURN OVER

Question 7 (5 marks)
A random variable X has a probability density function given by

$$g(x) = \begin{cases} x & 0 \le x \le 1 \\ \dfrac{1}{x^2} & 1 < x \le a \\ 0 & \text{elsewhere} \end{cases}$$, where a is a real constant.

a. Show that $a = 2$. 2 marks

b. Find $E(X)$. 3 marks

Question 8 (7 marks)

Consider the functions *f* and *g* with rules $f(x) = 3\sqrt{4-2x}+1$ and $g(x) = \sqrt{x}+4$.

a. If the graph of *f* is mapped to the graph of *g* using the transformation $T: R^2 \to R^2, T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}\left(\begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} c \\ d \end{bmatrix}\right)$, find *a*, *b*, *c* and *d*, where *a*, *b*, *c* and *d* are real constants. 3 marks

b. Find the rule for f^{-1} and state the domain. 2 marks

TURN OVER

c. Find the rule for $g\left(f(x)\right)$ and state the range. 2 marks

END OF QUESTION AND ANSWER BOOK