

**MATHEMATICAL METHODS UNIT 4**

**SAC 2 ANALYSIS TASK Tech Free**

**STUDENT NAME = \_\_\_\_\_**

**Time Allowed: 75 Minutes**

**Total Marks = 38**

General instruction regarding the assessment of questions worth more than 1 mark:-

- Show all working clearly.
- Justify your answers when required.

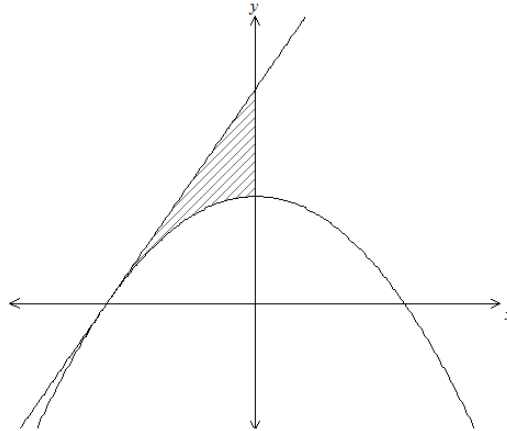
**All answers need to be given in exact form.**

**Technology Free with a Bound Book.**



**Question 1**

The graph of  $f : \mathbb{R} \rightarrow \mathbb{R}$ ,  $f(x) = 4 - x^2$  and the tangent to the graph of  $f$ , where it crosses the negative  $x$ -axis, are shown.



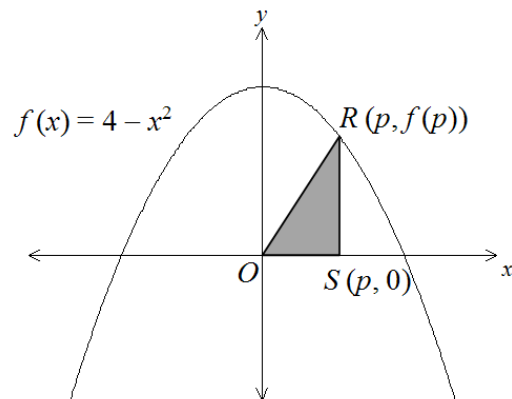
- a) Find the equation of the tangent to the graph of  $f$  where it crosses the negative  $x$ -axis.

2 marks

- b) Find the area of the shaded region.

3 marks

- c) Consider the triangle  $ORS$ , where vertex  $R$  is on the graph of  $f$ , with coordinates  $(p, f(p))$ , and vertices  $O$  and  $S$  are on the  $x$ -axis, with coordinates  $(0,0)$  and  $(p,0)$ , respectively.

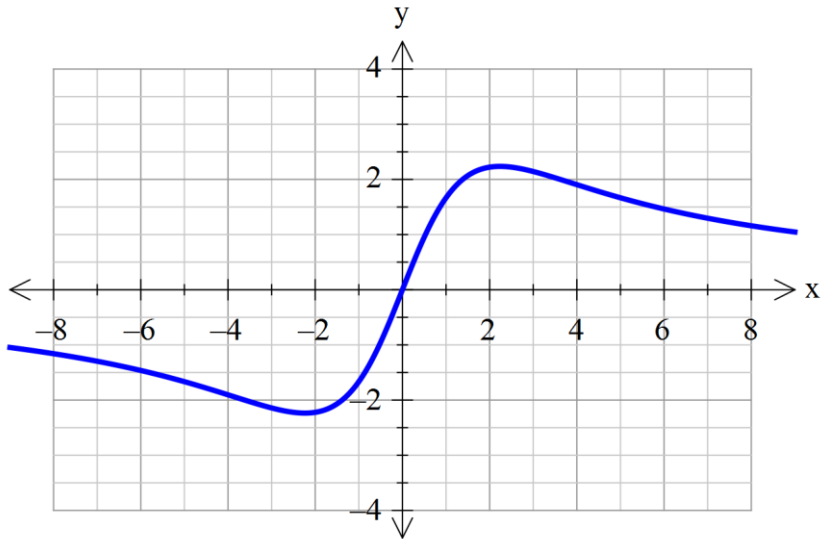


If  $p \in [-2, 2]$ , find the value(s) of  $p$  for which the area of triangle  $ORS$  is a maximum.

4 marks

## Question 2

The graph of  $h(x) = \frac{10x}{5+x^2}$  is shown.



a) Find  $h'(x)$ .

2 marks

b) Find the exact coordinates of the minimum and maximum turning points.

2 marks

c) Find the derivative of  $\log_e(5+x^2)$  and hence find an antiderivative for  $\frac{10x}{5+x^2}$ .

3 marks

d) Find the area enclosed between the curve, the  $x$ -axis, the line where the  $x$  equals the  $x$ -coordinate of the maximum turning point, and the line  $x=6$ .

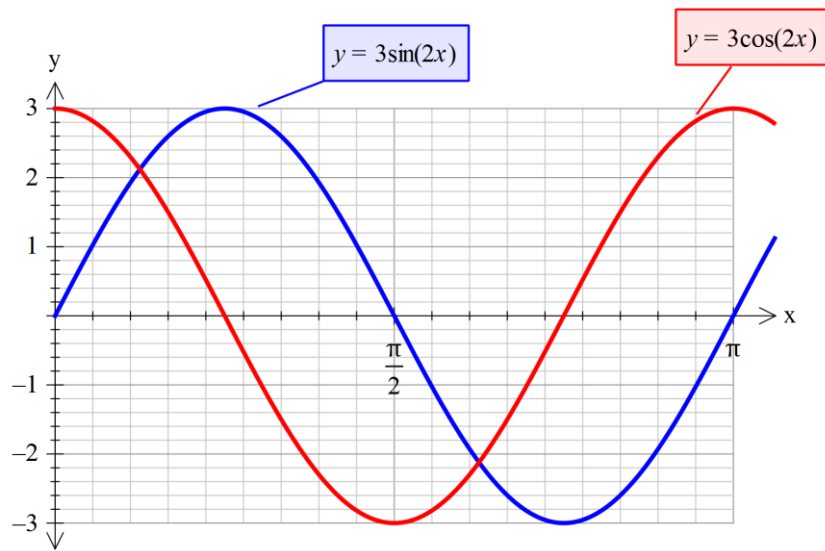
3 marks

e) Find the average value of the function  $h(x)$  over the interval  $[0,6]$ .

2 marks

### Question 3

The graphs of  $f(x) = 3\sin(2x)$  and  $g(x) = 3\cos(2x)$  are shown for  $x \in [0, \pi]$ .



- a) Find the coordinates of the points of intersection of  $f$  and  $g$  for the interval  $\left[0, \frac{\pi}{2}\right]$ .

3 marks



b) Determine the area enclosed between the two curves on the interval  $\left[0, \frac{\pi}{2}\right]$ .

4 marks

#### Question 4

Consider the function  $f(x) = e^x$  and the point on the graph of  $f(x)$  with the coordinates  $(a, e^a)$ ,  $a > 0$ .

a) Sketch the graph of  $f$  and mark the point  $(a, e^a)$ .

1 mark

b) Draw the tangent to the graph at the point  $(a, e^a)$ .

1 mark

c) Find the equation of the tangent above, in terms of  $a$ .

2 marks

- d) Write down the definite integral which determines the area of the region bound by the graph of  $f$ , the tangent at  $(a, e^a)$  and the coordinate axes.

2 marks

- e) Find the value of  $a$  if the area described above is equal to  $e^a - 1$ .

4 marks

**END OF SAC**

**Additional Working Space.**

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