MATHEMATICAL METHODS UNIT 4

SAC 2 ANALYSIS TASK Tech Active

STUDENT NAME = _____

Time Allowed: 75 Minutes

Total Marks = 35

General instruction regarding the assessment of questions worth more than 1 mark:-

Whether your solution is based upon algebraic analysis &/or calculator analysis, be sure that it shows:

- the mathematical concepts upon which you based your solution and
- the processes by which you applied those concepts in order to obtain your solution.

'Show that…' questions require arriving at the given answer. Meaning you cannot take the answer given and substitute it back; this would be verification. All steps in 'Show that' questions need to be shown algebraically and very clearly.

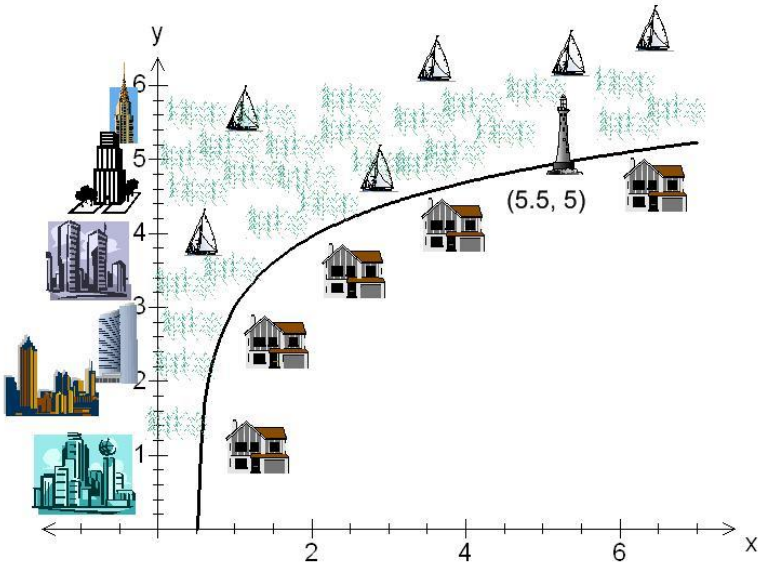
Follow the required accuracy of answers on the calculator part!

Technology Active with a Bound Book.

A local council of an outer suburb out of the City of San Log decides to build a new road along the coast. To make it safe it was decided to design the road so that it followed the curve represented by the following logarithmic function:

$f(x) = 2\log_{10}(2x - a) + 3,\ x \in (m, 7]$, where $a > 0$. **Road 1.**

The diagram below shows the road. In each direction, 1 unit represents 1 kilometre.



Question 1

a) If the new section of the road passes by the lighthouse marked on the graph, show algebraically that $a = 1$.

2 marks

b) Find the value of m, the x-coordinate of the point where the road begins. Give your answer to four decimal places.

1 mark

c) Find the equation of the asymptote for this function.

1 mark

d) Hence find the shortest distance from the beginning of the road to the asymptote, to the nearest metre.

2 marks

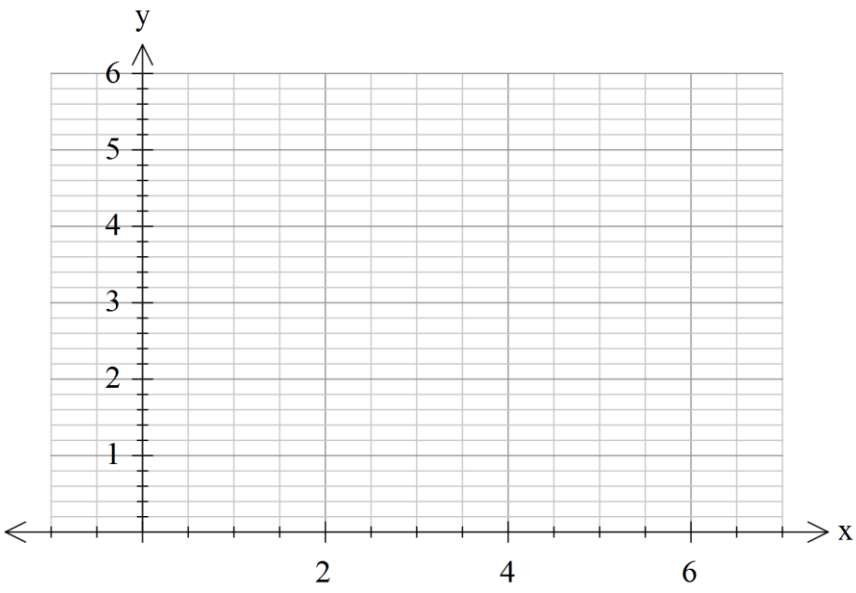
The population in this coastline increased rapidly and there was a need for an alternative road, **Road 2**. The engineers thought that the inverse function of the original road would be suitable.

Question 2

a) Use algebra to find the equation for the inverse function $f^{-1}(x)$.

3 marks

b) Sketch both the graph of the original function $f(x)$ and the inverse function $f^{-1}(x)$ on the grid provided. Show the asymptotes and end points and label them clearly.



4 marks

Due to the traffic, the local council decided to build a bypass bridge. The bridge is to be modelled by a straight line tangent to Road 1 from the lighthouse at (5.5, 5) to the City.

Question 3

a) Find the gradient of the curve at the point where the lighthouse is located. Express your answer to four decimal places.

2 marks

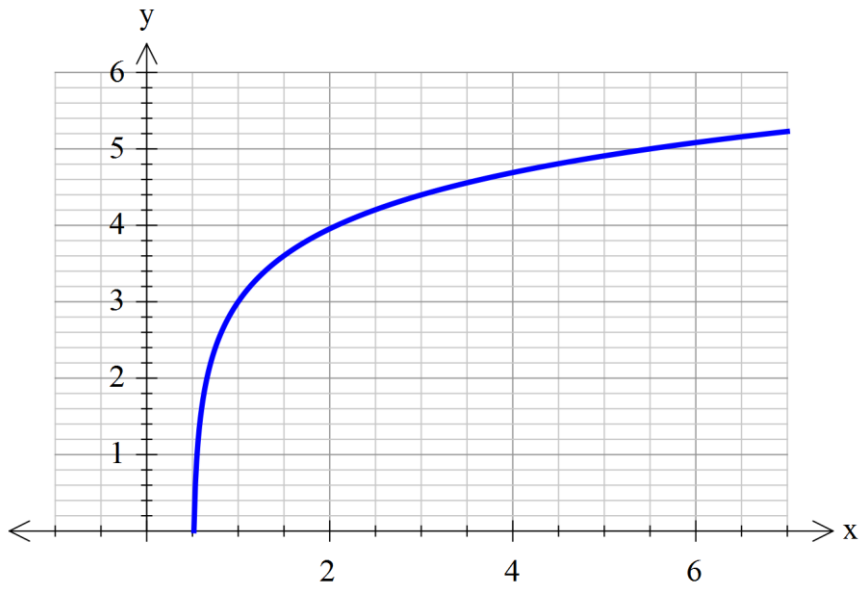
b) Find the equation of the tangent as described above. Give the coefficients to four decimal places.

2 marks

The Planning Department of the local council needs to estimate the area between the Road 1 and the *x*-axis using the approximation method with left and right rectangles.

Question 4

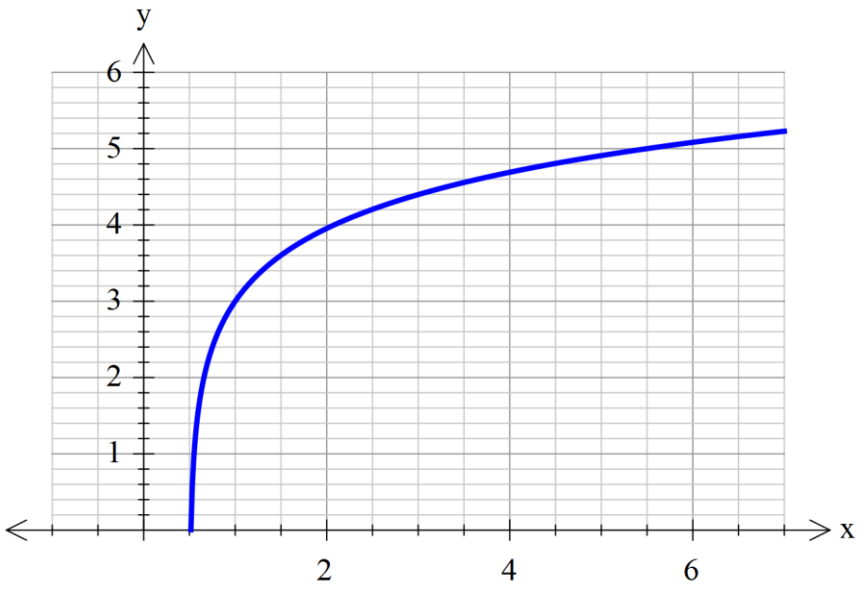
a) i) Draw the specified rectangles on the diagram below when the right-end point estimate method between $x = 1$ and $x = 5$ with four strips of width 1 is applied.



ii) Approximate the area under the curve $y = 2\log_{10}(2x-1)+3$ between $x = 1$ and $x = 5$ using the right-endpoint estimate with four strips of width 1, correct to 2 decimal places.

1 + 2 = 3 marks

b) i) Draw the specified rectangles on the diagram below when the left-end point estimate method between $x = 1$ and $x = 5$ with four strips of width 1 is applied.



ii) Approximate the area under the curve $y = 2\log_{10}(2x-1)+3$ between $x = 1$ and $x = 5$ using the left-endpoint estimate with four strips of width 1, correct to 2 decimal places.

1 + 2 = 3 marks

c) Use your CAS calculator to calculate the exact value of the area under the curve $y = 2\log_{10}(2x-1)+3$ between $x = 1$ and $x = 5$. Give your answer to 2 decimal places.

1 mark

d) Comment on how the approximate method can be improved.

1 mark

In addition the local council also needs to include in the plans the exact area between Road 1 and Road 2.

Question 5

a) Find the coordinates of the points of intersection between the above two roads, correct to two decimal places.

2 marks

b) Write down the definite integral which denotes the exact area between the two roads.

2 marks

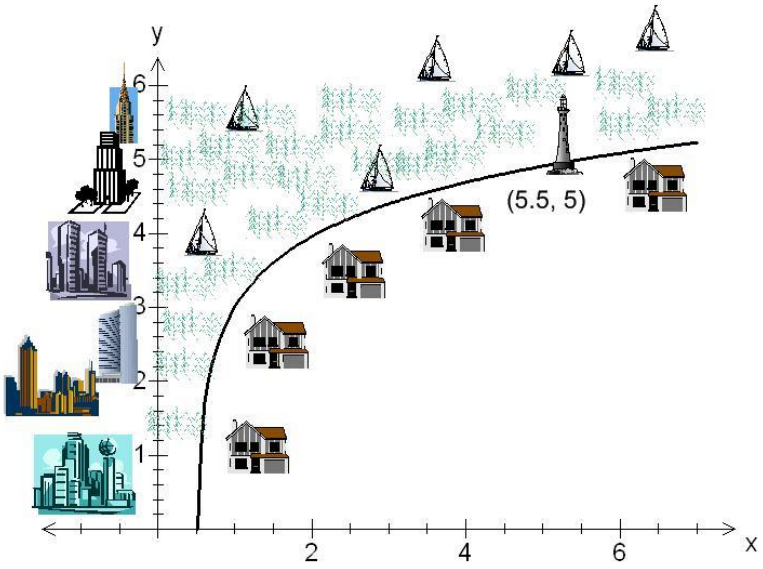
c) Hence find the area enclosed by the two roads to the nearest square kilometre.

2 marks

The water authorities decided to declare the area enclosed by the Road 1, the bypass bridge and the line with equation $x = m$ as the **no motor-boat zone**.

Question 6

a) Shade the **no motor-boat zone** on the diagram below.



1 mark

b) Calculate the area of the **no motor-boat zone** to 2 decimal places.

3 marks

END OF SAC 2 Tech Active

Additional Working Space.

Additional Working Space.