Year 2017

VCE

Mathematical Methods

Trial Examination 2

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Victorian Certificate of Education
2017

STUDENT NUMBER

									Letter
Figures									
Words									

MATHEMATICAL METHODS
Trial Written Examination 2

Reading time: 15 minutes
Total writing time: 2 hours

QUESTION AND ANSWER BOOK

Structure of book

Section	*Number of questions*	*Number of questions to be answered*	*Number of marks*
A	20	20	20
B	4	4	60
			Total 80

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers, a protractor, set squares, aids for curve sketching, one bound reference, one approved technology (calculator or software) and, if desired, one scientific calculator. Calculator memory DOES NOT need to be cleared. For approved computer-based CAS, their full functionality may be used.
- Students are NOT permitted to bring into the examination room: blank sheets of paper and/or white out liquid/tape.

Materials supplied

- Question and answer booklet of 34 pages.
- Detachable sheet of miscellaneous formulas at the end of this booklet.
- Answer sheet for multiple-choice questions.

Instructions

- Detach the formula sheet from the end of this book during reading time.
- Write your **student number** in the space provided above on this page.
- Write your **name** and **student number** on your answer sheet for multiple-choice questions, and sign your name in the space provided to verify this.
- Unless otherwise indicated, the diagrams in this booklet are **not** drawn to scale.
- All written responses must be in English.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

SECTION A

Instructions for Section A

Answer **all** questions in pencil on the answer sheet provided for multiple-choice questions.
Choose the response that is **correct** for the question.
A correct answer scores 1 mark, an incorrect answer scores 0.
Marks will **not** be deducted for incorrect answers.
No mark will be given if more than one answer is completed for any question.
Unless otherwise indicated, the diagrams in this booklet are **not** drawn to scale.

Question 1

The function $f : D \to R$, $f(x) = -\sqrt{4b^2 - x^2}$ where $b > 0$ has a range of $\left[-\sqrt{3}b, 0\right)$.
The domain D could be equal to

A. $[-b, 0)$

B. $(-b, 0]$

C. $(b, 2b]$

D. $(-2b, -b]$

E. $[-2b, -b)$

Question 2

Consider the polynomial $p(x) = x^3 + (3-k)x^2 - (3k+10)x + 10k$, where $k \in R$.

Which of the following is **false**?

A. If $(x-1)$ is a factor of $p(x)$ then $k = 1$.

B. $(x+k)$ is a factor of $p(x)$.

C. $(x-2)$ is a factor of $p(x)$.

D. $(x+5)$ is a factor of $p(x)$.

E. $p(k) = 0$.

Question 3

The function $f:(-\infty,b)\to R$, $f(x)=-x^4+2x^3$ will have an inverse function, if

A. $b<\frac{3}{2}$

B. $b>\frac{3}{2}$

C. $b>0$

D. $b<2$

E. $b>2$

Question 4

The simultaneous linear equations $2x-ky=5$ and $(k+1)x-6y=3k+1$,
have no solution when

A. $k\in R\setminus\{-4,3\}$

B. $k=-4$ or $k=3$

C. $k\in R\setminus\{3\}$

D. $k=3$

E. $k=-4$

Question 5

The graph of $y=x^5-5x^3$, has a positive gradient, when

A. $x\in\left(-\sqrt{3},0\right)\cup\left(0,\sqrt{3}\right)$

B. $x\in\left(-\sqrt{5},0\right)\cup\left(0,\sqrt{5}\right)$

C. $x\in\left(-\infty,-\sqrt{3}\right)\cup\left(\sqrt{3},\infty\right)$

D. $x\in\left(-\infty,-\sqrt{5}\right)\cup\left(\sqrt{5},\infty\right)$

E. $x\in\left(-\sqrt{3},\sqrt{3}\right)$

Question 6

Let $f(x)=\int_0^{x^2}\sin(t^2)\,dt$. Then $f'(x)$ is equal to

A. $x^2\sin(x^2)$

B. $2x\sin(x^4)$

C. $2x\sin(x^4)-1$

D. $x^2\cos(x^2)-1$

E. $2x\cos(x^4)$

Question 7

The graph of $y=x^3-3x^2+c$, where $c\in R$, crosses the *x*-axis three times if

A. $c>0$

B. $c<0$

C. $0<c<4$

D. $c<4$

E. $c\in(-\infty,4)\cup(0,\infty)$

Question 8

If $f(x)=\dfrac{\log_e(3x)}{g(x)}$ and $g(2)=4$, $g'(2)=3$, then $f'(2)$ is equal to

A. $\dfrac{2-3\log_e(6)}{16}$

B. $\dfrac{6-3\log_e(6)}{16}$

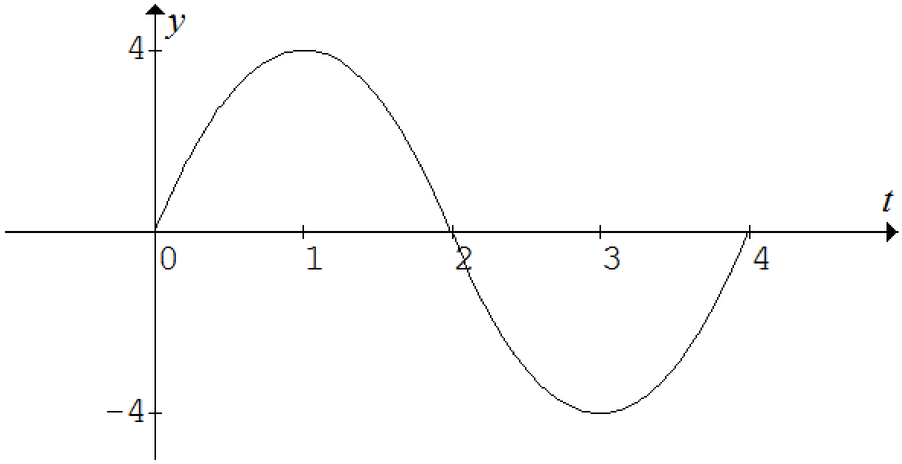
C. $\dfrac{1}{18}$

D. $\dfrac{1}{6}$

E. $\dfrac{1}{3}\log_e(6)$

Question 9

The graph of $y = f(t)$ is shown below.



Let $F(x) = \int_0^x f(t)\,dt$, that is $F(x)$ is an anti-derivative of $f(x)$.
Then which of the following is **false**?

A. $F(0) = 0$

B. $F(4) = 0$

C. $F(2) = 2F(1)$

D. $F(2) = 2F(3)$

E. $F(3) + F(1) = 0$

Question 10

The temperature, in degrees Celsius of water in a pond is modelled by the function of $15 - 5\cos\left(\frac{2\pi}{365}(t+10)\right)$ where $t \geq 0$ is the number of days since January 1st.
The instantaneous rate of change of temperature of the water after 90 days, is closest to

A. 0.002

B. 0.029

C. 0.063

D. 0.085

E. 12.362

Question 11

Consider the function $f(x)=\begin{cases} x \text{ for } & x<2 \\ 4 \text{ for } & x\geq 2 \end{cases}$

Then $\int_{-1}^{3} f(x)\,dx$ is

A. undefined

B. 4

C. 5

D. $5\frac{1}{2}$

E. $6\frac{1}{2}$

Question 12

The table below shows selected values of a differentiable and decreasing function *f*.

x	0	1	2
$f(x)$	4	2	–3
$f'(x)$	–2	–4	–6

If *g* is the inverse of the function *f*, then the value of $g'(2)$ is equal to

A. 4

B. –6

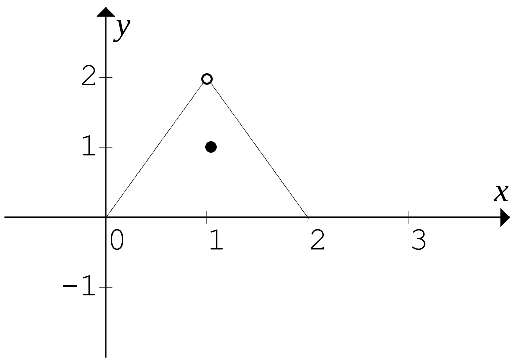
C. $-\frac{1}{4}$

D. $\frac{1}{4}$

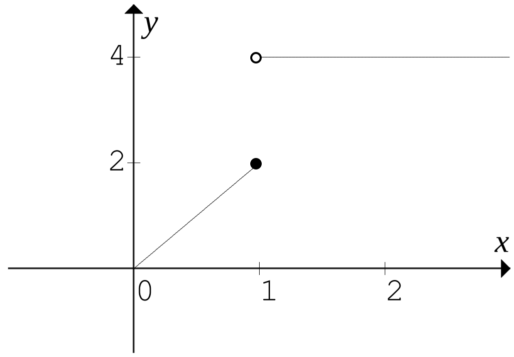
E. $\frac{1}{6}$

Question 13

The graphs of two functions f and g are shown below.



The graph of f



The graph of g

Then

A. $f(g(1)) = 0$ and $g(f(1)) = 2$

B. $f(g(1)) = 1$ and $g(f(1)) = 4$

C. $f(g(1)) = 0$ and $g(f(1)) = 4$

D. $f(g(1))$ does not exist.

E. $g(f(1))$ does not exist.

Question 14

The amount of bacteria grows at a rate of $5e^{0.2t}$ grams per day, where $t \geq 0$, is the time measured in days. Over the time interval from $t = 0$ to $t = 10$ days, the amount in grams has grown by

A. e^2

B. $e^2 - 1$

C. $\frac{1}{2}\left(e^2 - 1\right)$

D. $\frac{5}{2}\left(e^2 - 1\right)$

E. $25\left(e^2 - 1\right)$

Question 15

An urn contains *r* red balls, *b* black balls and *w* white balls. Three balls are drawn at random from the urn, one at a time, without replacement. The probability that the three balls drawn are all of different colours is equal to

A. $\dfrac{r(r-1)(r-2)+b(b-1)(b-2)+w(w-1)(w-2)}{(r+b+w)^3}$

B. $\dfrac{4rbw}{(r+b+w)^3}$

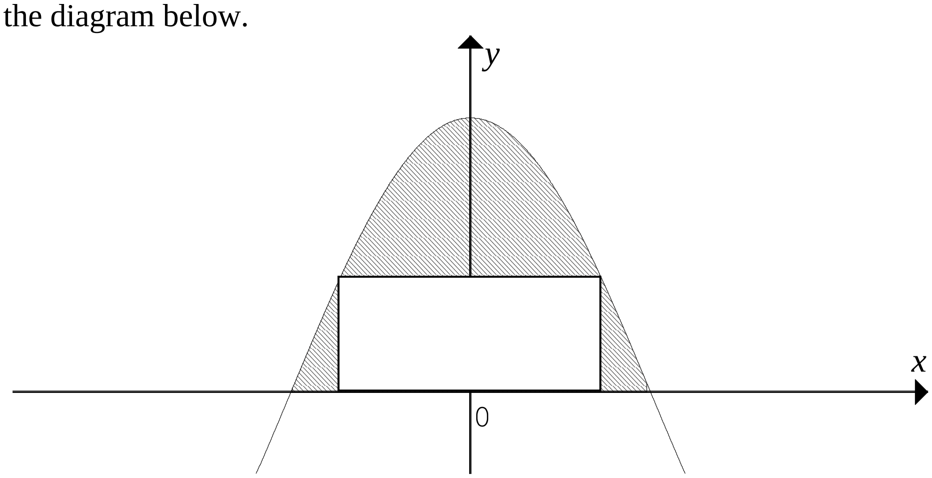
C. $\dfrac{rbw}{(r+b+w)(r+b+w-1)(r+b+w-2)}$

D. $\dfrac{6rbw}{(r+b+w)(r+b+w-1)(r+b+w-2)}$

E. $\dfrac{r(r-1)(r-2)+b(b-1)(b-2)+w(w-1)(w-2)}{(r+b+w)(r+b+w-1)(r+b+w-2)}$

Question 16

A rectangle has two vertices on the curve $y=\cos(x)$ and two on the *x*-axis as shown in the diagram below.



The minimum area of the shaded region is closest to

A. 0.799

B. 0.878

C. 1.140

D. 1.439

E. 2.000

Question 17

In a particular electorate of several thousand voters, 35% favour one political party.
In a random sample of 200 voters from this particular electorate, the width of a 95% confidence interval, for the proportion of voters who favour this political party is closest to

A. 0.0046

B. 0.068

C. 0.132

D. 0.135

E. 0.187

Question 18

The random variable Z has the standard normal distribution, with $\Pr(-c < Z < c) = C$.

Then $\Pr(Z < c \mid Z > 0)$ is equal to

A. $\frac{C}{2}$

B. C

C. $\frac{2-C}{2}$

D. $\frac{1-C}{2}$

E. $1-C$

Question 19

A and B are two events with $\Pr(A) = 3p$, $\Pr(B) = 2\sqrt{p}$ and $\Pr(A \cap B) = p$.
The maximum possible value of p is closest to

A. 0.13398

B. 0.25

C. 0.2956

D. 0.3333

E. 0.4

Question 20

A discrete random variable has a binomial distribution. The expression
$1-\left(0.65^8+8(0.35)(0.65)^7\right)$ represents the probability of

A. exactly one success in eight trials each with probability of success equal to 0.65.

B. at least one success in eight trials each with probability of success equal to 0.65.

C. at least one success in eight trials each with probability of success equal to 0.35.

D. more than one success in eight trials each with probability of success equal to 0.65.

E. more than one success in eight trials each with probability of success equal to 0.35.

END OF SECTION A

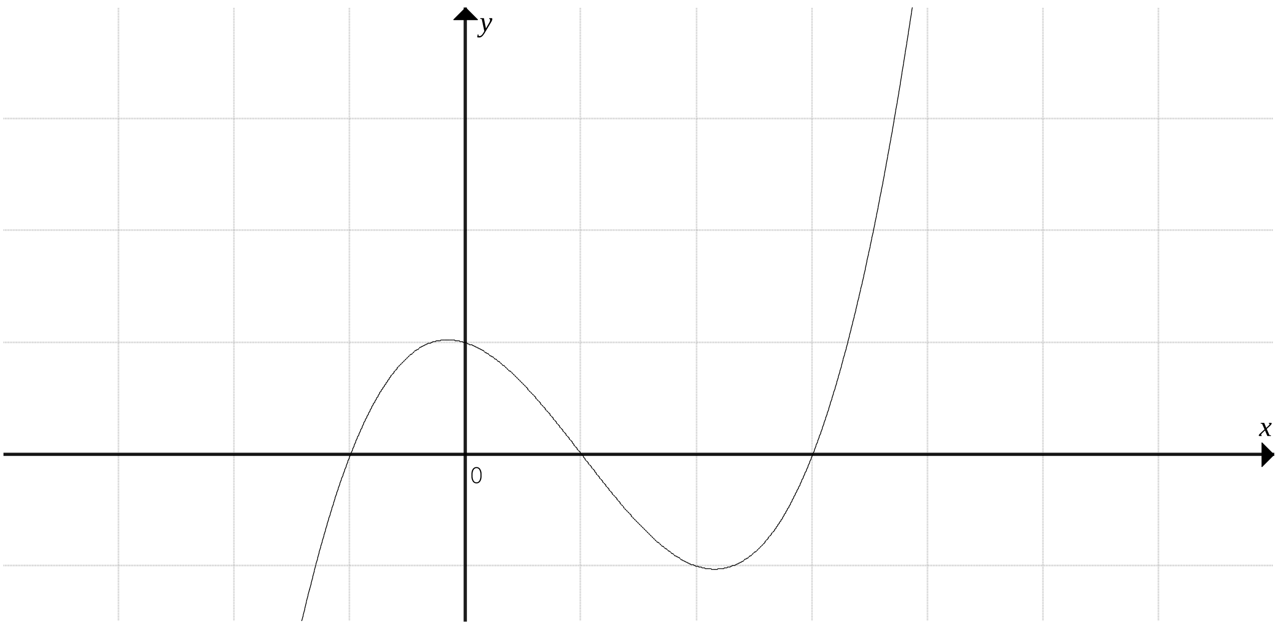
SECTION B

Instructions for Section B

Answer **all** questions in the spaces provided.
In all questions where a numerical answer is required an exact value must be given unless otherwise specified.
In questions where more than one mark is available, appropriate working **must** be shown.
Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Question 1 (13 marks)

Consider the function $f : R \to R$, $f(x) = (x^2 - p^2)(x - 3p)$ where p is a positive real number.



a. Find the equation of the tangent $t1(x)$ to the curve $y = f(x)$ at the point where $x = 2p$.
Give your answer in the form $y = m_1 x + c_1$ and express m_1 and c_1 in terms of p.

2 marks

b. If this tangent $t1(x)$ intersects the curve $y = f(x)$ at the point R, write down the coordinates of the point R, and draw this tangent on the diagram on the previous page.

2 marks

c. Find the equation of the tangent $t2(x)$ to the curve $y = f(x)$ at the point R.
Give your answer in the form $y = m_2 x + c_2$ and express m_2 and c_2 in terms of p.

2 marks

d. If this tangent $t2(x)$ intersects the curve $y = f(x)$ at the point S, determine the coordinates of the point S.

2 marks

e. Let A_1 be the area between the curve $y = f(x)$ and the tangent $t1(x)$.

Write down in terms x and p, a definite integral which gives the area A_1.

2 marks

f. Let A_2 be the area between the tangent $t2(x)$ and the curve $y = f(x)$.

Write down in terms x and p, a definite integral which gives the area A_2.

2 marks

g. Find the value of $\frac{A_2}{A_1}$.

1 mark

Question 2 (21 marks)

a. The weights of eggs produced by a certain breed of hen are normally distributed with a mean of 58 grams with a standard deviation of 6 grams. An egg is classified as large, if its weight exceeds 50 grams.

i. Find the probability correct to four decimal places, that a randomly produced egg by this breed of hen is classified as large.

1 mark

ii. Find the probability correct to four decimal places that a random sample of 12 eggs collected from this breed of hen, contains at least 3 eggs that are not large eggs.

1 mark

iii. Find the minimum number of eggs that we would need to collect from this breed of hen, to ensure that the probability of obtaining at least 5 large eggs, is at least 0.95.

1 mark

b. Australian eggs are classified in increasing weight sizes, as medium, large, extra large and jumbo. It is also known that these weights are normally distributed.

i. It is found that 2.3% of eggs are classified as jumbo, and have weights greater 66.7 grams. 9% of eggs are classified as medium and have weights less than 50 grams. Find the mean and standard deviation weights of Australian eggs, giving your answers in grams correct to one decimal place.

3 marks

ii. 35.2% of eggs are classified as extra large, find the minimum weight of extra large egg, giving your answer in grams correct to one decimal place.

1 mark

iii. Farm A sells eggs. The revenue for jumbo eggs is 25 cents, for extra large eggs the revenue is 20 cents, for large eggs the revenue is 15 cents and for medium eggs the revenue is 5 cents. Complete the following table and find the expected revenue on selling eggs, correct to the nearest cent.

	medium	large	extra large	jumbo
probability				
egg weight, grams				
revenue cents				

2 marks

iv. An egg that is known not to be a jumbo size egg, is selected at random. What is the probability correct to three decimal places that it is an extra large or large egg.

1 mark

c. A random sample of 95 eggs produced from another farm B, has 5 eggs that are classified as jumbo. Let $\hat{P}$ be the random variable of the sample proportion of jumbo eggs produced from farm B.

i. Find the expected value and variance of $\hat{P}$.
Give your answers correct to 3 significant figures.

1 mark

ii. Find the probability that the sample proportion of jumbo eggs lies within two standard deviations of the mean. Give your answer correct to three decimal places. Do not use a normal approximation.

2 marks

d. A supermarket buys eggs from either farm A or farm B. 30% of the eggs purchased are from farm A. It is found that 55% of the jumbo eggs are from farm A, while 25% of the jumbo eggs are from farm B. A particular egg is found not to be a jumbo egg, find the probability correct to three decimal places that it came from farm B?

2 marks

e. Hens lay eggs in the daylight hours. The time taken *t* in hours for a mature hen to lay an egg, is found to be a probability density function $T(t)$ defined by

$$T(t)=\frac{\pi}{3(\pi+4)}\begin{cases}\sin\left(\frac{\pi t}{12}\right) & 0\le t\le 6\\ b\left(1-\frac{t}{12}\right) & 6<t\le 12\\ 0 & \text{otherwise}\end{cases}$$

i. Explain why $b=2$. 1 mark

ii. Sketch the graph of $T(t)$ on the axes below, clearly labelling the scales. 2 marks

iii. Find $\text{Var}(T)$ giving your answer correct to three decimal places. 2 marks

iv. Find the median time correct to three decimal places. 1 mark

Question 3 (13 marks)

Consider the functions $S:R\to R$, $S(x)=\frac{1}{2}\left(e^{x}-e^{-x}\right)$ and $C:R\to R$, $C(x)=\frac{1}{2}\left(e^{x}+e^{-x}\right)$

a. Verify that $\left[C(x)\right]^{2}-\left[S(x)\right]^{2}=1$.

2 marks

b. By replacing *x* with *kx* and using the definitions show that $\frac{d}{dx}\left[S(kx)\right]=kC(kx)$ where *k* is a constant.

2 marks

c. If $T(x) = \dfrac{S(x)}{C(x)}$ then show that $\dfrac{d}{dx}\left[T(kx)\right] = \dfrac{k}{\left[C(kx)\right]^2}$, where k is a constant.

3 marks

d. Find the value of x, for which $S(x) = \frac{3}{4}$ by writing it and solving it as a quadratic in e^x. 2 marks

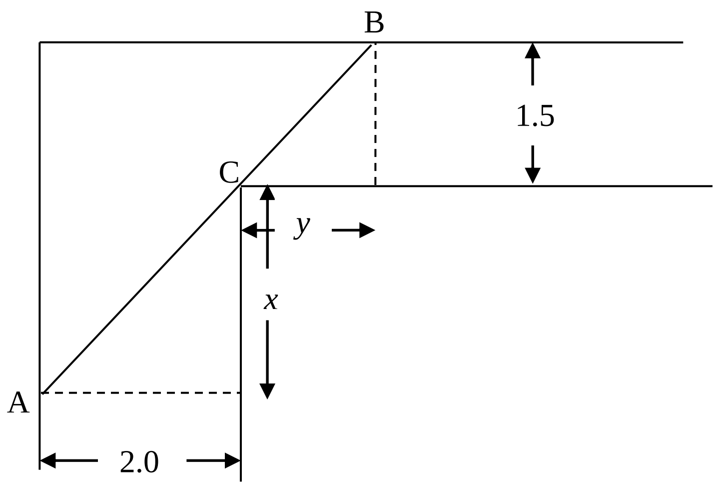
e. Explain why the function $S(x)$ has an inverse function over the set of real numbers. Let $y = S^{-1}(x)$ be the inverse of the function *S*. Express $S^{-1}(x)$ as a natural logarithm. Hence verify your answer to **Question 3d**.

4 marks

Question 4 (13 marks)

A pole AB is to be carried horizontally around a 90^0 corner that has widths of 2.0 metres and 1.5 metres. The length of the longest pole that can be moved around the corner is the pole that touches the inside corner at the point C as shown in the diagram below.

a. Distances of x and y in metres, where $x > 0$ and $y > 0$ are also shown.



i. Express *y* in terms of *x*.

1 mark

ii. Hence, show that the length of the pole AB or L in terms of x can be expressed as

$$L(x)=\left(\frac{3}{2x}+1\right)\sqrt{x^2+4}$$

2 marks

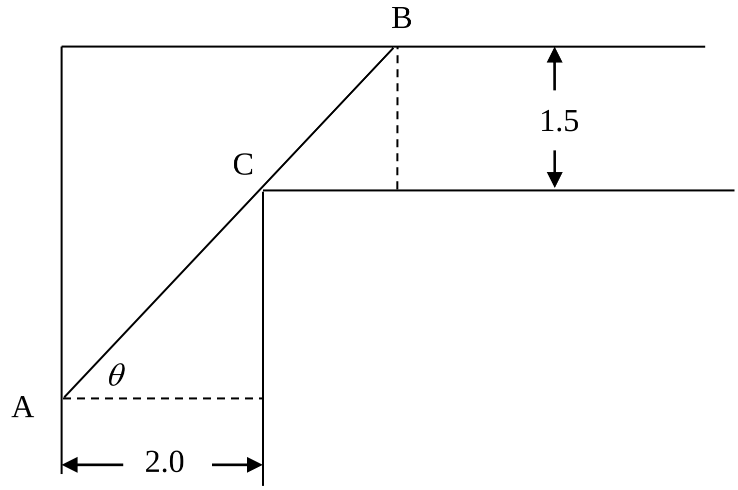
iii. Find $\frac{dL}{dx}$ and hence the value of x which gives the length of the longest pole.

2 marks

iv. Find the length of the longest pole, giving your answer correct to three decimal places.

1 mark

b. Alternatively, consider the angle θ in degrees, that the pole makes with the horizontal.



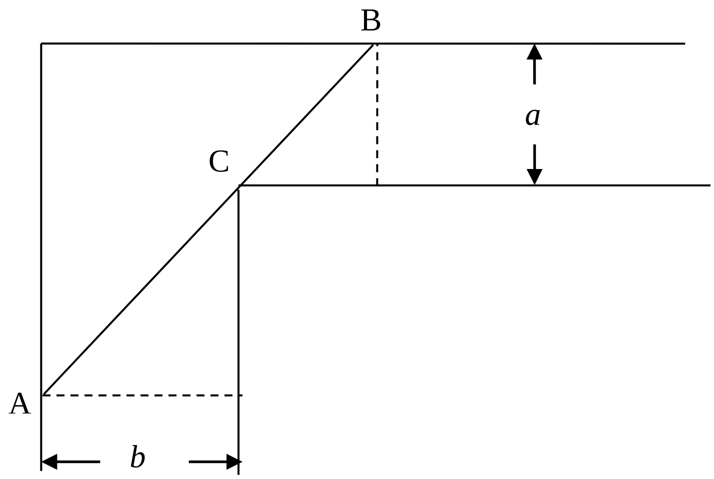
i. Show that the length of the pole AB or L in terms of θ is given by $L(\theta)=\dfrac{2}{\cos(\theta)}+\dfrac{3}{2\sin(\theta)}$, and state the domain of the function.

1 mark

ii. Find $\dfrac{dL}{d\theta}$ and hence the value of θ in degrees correct to two decimal places which gives the length of the longest pole.

2 marks

c. Consider now when the corners have lengths, a and b as shown.



i. Obtain an expression for the length AB in terms of a and b and another variable.

1 mark

ii. The length of the longest pole that can be carried around the corner, can be expressed as $\left(a^m + b^m\right)^n$, find the values of m and n.

3 marks

END OF EXAMINATION

EXTRA WORKING SPACE

MATHEMATICAL METHODS

Written examination 2

FORMULA SHEET

Directions to students

Detach this formula sheet during reading time. This formula sheet is provided for your reference.

Mathematical Methods formulas

Mensuration

area of a trapezium	$\frac{1}{2}(a+b)h$	volume of a pyramid	$\frac{1}{3}Ah$
curved surface area of a cylinder	$2\pi rh$	volume of a sphere	$\frac{4}{3}\pi r^3$
volume of a cylinder	$\pi r^2 h$	area of triangle	$\frac{1}{2}bc\sin(A)$
volume of a cone	$\frac{1}{3}\pi r^2 h$		

Calculus

$\frac{d}{dx}\left(x^n\right)=nx^{n-1}$		$\int x^n dx=\frac{1}{n+1}x^{n+1}+c\ ,\ n\neq -1$	
$\frac{d}{dx}\left((ax+b)^n\right)=na(ax+b)^{n-1}$		$\int (ax+b)^n\,dx=\frac{1}{a(n+1)}(ax+b)^{n+1}+c\ ,\ n\neq -1$	
$\frac{d}{dx}\left(e^{ax}\right)=ae^{ax}$		$\int e^{ax}dx=\frac{1}{a}e^{ax}+c$	
$\frac{d}{dx}\left(\log_e(x)\right)=\frac{1}{x}$		$\int \frac{1}{x}dx=\log_e(x)+c,\ x>0$	
$\frac{d}{dx}\left(\sin(ax)\right)=a\cos(ax)$		$\int \sin(ax)\,dx=-\frac{1}{a}\cos(ax)+c$	
$\frac{d}{dx}\left(\cos(ax)\right)=-a\sin(ax)$		$\int \cos(ax)\,dx=\frac{1}{a}\sin(ax)+c$	
$\frac{d}{dx}\left(\tan(ax)\right)=\frac{a}{\cos^2(ax)}=a\sec^2(ax)$			
product rule	$\frac{d}{dx}(uv)=u\frac{dv}{dx}+v\frac{du}{dx}$	quotient rule	$\frac{d}{dx}\left(\frac{u}{v}\right)=\frac{v\frac{du}{dx}-u\frac{dv}{dx}}{v^2}$
chain rule	$\frac{dy}{dx}=\frac{dy}{du}\frac{du}{dx}$		

Probability

$\Pr(A)=1-\Pr(A')$		$\Pr(A\cup B)=\Pr(A)+\Pr(B)-\Pr(A\cap B)$	
$\Pr(A\mid B)=\dfrac{\Pr(A\cap B)}{\Pr(B)}$			
mean	$\mu=E(X)$	variance	$\operatorname{var}(X)=\sigma^2=E\left((X-\mu)^2\right)=E(X^2)-\mu^2$

Probability distribution		Mean	Variance
discrete	$\Pr(X=x)=p(x)$	$\mu=\sum x\,p(x)$	$\sigma^2=\sum(x-\mu)^2\,p(x)$
continuous	$\Pr(a<X<b)=\int_a^b f(x)\,dx$	$\mu=\int_{-\infty}^{\infty} x\,f(x)\,dx$	$\sigma^2=\int_{-\infty}^{\infty}(x-\mu)^2\,f(x)\,dx$

Sample proportions

$\hat{P}=\dfrac{X}{n}$		mean	$E(\hat{P})=p$
standard deviation	$\operatorname{sd}(\hat{P})=\sqrt{\dfrac{p(1-p)}{n}}$	approximate confidence interval	$\left(\hat{p}-z\sqrt{\dfrac{\hat{p}(1-\hat{p})}{n}}\,,\ \hat{p}+z\sqrt{\dfrac{\hat{p}(1-\hat{p})}{n}}\right)$

END OF FORMULA SHEET

ANSWER SHEET

STUDENT NUMBER

									Letter
Figures									
Words									

SIGNATURE ______________________________

SECTION A

1	**A**	**B**	**C**	**D**	**E**
2	**A**	**B**	**C**	**D**	**E**
3	**A**	**B**	**C**	**D**	**E**
4	**A**	**B**	**C**	**D**	**E**
5	**A**	**B**	**C**	**D**	**E**
6	**A**	**B**	**C**	**D**	**E**
7	**A**	**B**	**C**	**D**	**E**
8	**A**	**B**	**C**	**D**	**E**
9	**A**	**B**	**C**	**D**	**E**
10	**A**	**B**	**C**	**D**	**E**
11	**A**	**B**	**C**	**D**	**E**
12	**A**	**B**	**C**	**D**	**E**
13	**A**	**B**	**C**	**D**	**E**
14	**A**	**B**	**C**	**D**	**E**
15	**A**	**B**	**C**	**D**	**E**
16	**A**	**B**	**C**	**D**	**E**
17	**A**	**B**	**C**	**D**	**E**
18	**A**	**B**	**C**	**D**	**E**
19	**A**	**B**	**C**	**D**	**E**
20	**A**	**B**	**C**	**D**	**E**