Year 2017 VCE Mathematical Methods Trial Examination 1



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• While every care has been taken, no guarantee is given that these questions are free from error. Please contact us if you believe you have found an error.

Victorian Certificate of Education 2017

STUDENT NUMBER

						Letter
Figures						
Words						

MATHEMATICAL METHODS

Trial Written Examination 1

Reading time: 15 minutes Total writing time: 1 hour

QUESTION AND ANSWER BOOK

Structure of book

Number of	Number of questions	Number of
questions	to be answered	marks
8	8	40

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers.
- Students are NOT permitted to bring into the examination room: any technology (calculators or software) notes of any kind, blank sheets of paper, and/or correction fluid/tape.

Materials supplied

- Question and answer book of 18 pages.
- Detachable sheet of miscellaneous formulas at the end of this booklet.
- Working space is provided throughout the booklet.

Instructions

- Detach the formula sheet from the end of this book during reading time.
- Write your **student number** in the space provided above on this page.
- Unless otherwise indicated, the diagrams in this booklet are **not** drawn to scale.
- All written responses must be in English.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

Instructions

Answer all questions in the spaces provided.

In all questions where a numerical answer is required an exact value must be given unless otherwise specified.

In questions where more than one mark is available, appropriate working **must** be shown. Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Question 1 (4 marks)

a. Let $f(x) = e^{\tan(3x)}$, find $f'(\frac{\pi}{9})$

2 marks

b. Consider the function $g:(0,\pi) \to R$, $g(x) = \cos(2x)$. If the tangent to the graph of g at the point where x = p makes an angle of 120° with the positive x-axis, find the possible value(s) of p.

Ouestion	2.	(3	marks)
Oucsuon	4	U	marks

Sam is a frequent flyer and occasionally gets a free upgrade. In a sample of ten flights, he found that once he got a free upgrade.

Let \hat{P} represent the distribution of the sample proportion, on flights when he gets a free upgrade.

i.	Sam wants to find the smallest value of the sample size n , such that the standard				
	deviation of \hat{P} is less than or equal to $\frac{1}{50}$. Determine the value of n .				
		2 marks			
ii.	Find the probability that when Sam takes five flights, he gets three upgrades.				
		1 mark			

Question 3 (4 marks)

a. State the amplitude, period and range of the function $h: R \to R$, $h(x) = 3 - 6\cos\left(\frac{\pi x}{3}\right)$

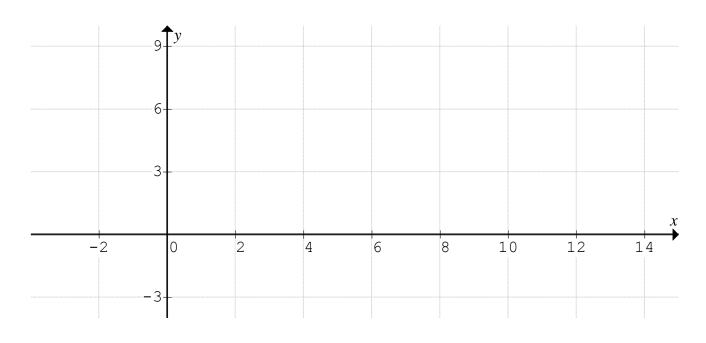
1 mark

b. Find the general solution of $3-6\cos\left(\frac{\pi x}{3}\right) = 0$

1 mark

c. Sketch the graph of the function $f:[0,12] \to R$, $f(x) = 3 - 6\cos\left(\frac{\pi x}{3}\right)$, on the axes

below, stating coordinates of all axial intercepts and endpoints.

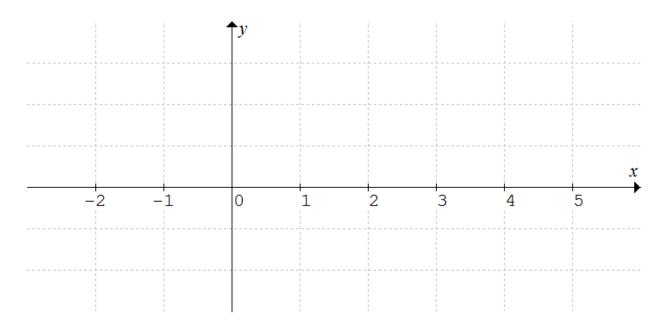


Question 4 (9 marks)

Consider the function $f: D \to R$, $f(x) = 3\log_e(x+1)$

a. Sketch the graph of the function on the axes below, clearly showing axial intercepts, and stating the equations of any asymptotes. State the maximal domain *D* and range of the function.

2 marks



Mia wants to calculate the area bounded by the graph of $y = 3\log_e(x+1)$, the x-axis and the ordinates x = 0 and x = 4.

b.i. As a first approximation, she decides to use four equally spaced upper rectangles. Draw these rectangles on the diagram above.

1 mark

ii.	The area using these four upper rectangles can be expressed as $b \log_e(c)$, find the values of b and c .				
	That the values of b and c.	1 mark			
c.	Find the rule for the inverse function f^{-1} .	2 marks			

		$\int_{0}^{\log_e(125)} (x)$	
d.i.	Evaluate	$\left e^{\overline{3}} - 1 \right $	dх
		J_0	

2 marks

ii. Hence find the area bounded by the graph of $y = 3\log_e(x+1)$, the x-axis and the ordinates x = 0 and x = 4.

1 mark

Ouestion	5	(4	marks`	١
Oucsuon	9	(+	marks	J

Consider the function $f:[0,\infty) \to R$, $f(x) = k\sqrt{3x+4}$

a. If the average rate of change of f between x = 0 and x = 4 is equal to 9, find the value of k.

2 marks

b. Find the average value of f between x = 0 and x = 4.

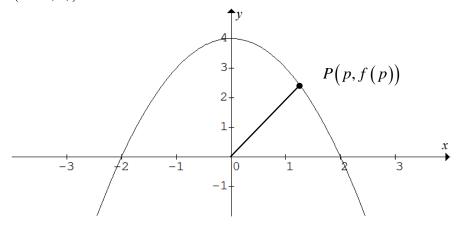
Ouestion	6	(3 marks)

A transformation $T: \mathbb{R}^2 \to \mathbb{R}^2$ is defined by $T \begin{pmatrix} x \\ y \end{pmatrix} = \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} h \\ k \end{bmatrix}$

The image of the curve $y = \sqrt{16 - x^2}$ under the transformation T, has the equation $y = 5 - 2\sqrt{55 - 6x - x^2}$. Find the values of a, b, h, and k.

Question 7 (4 marks)

The diagram shows part of the function $f: R \to R$, $f(x) = 4 - x^2$. A point P with coordinates (p, f(p)) where $0 \le p \le 2$ is shown.



Determine the value of p, for which the distance from the origin O to the point P is a minimum.

Question 8 (9 marks)

a. Determine $\frac{d}{dx} \left[x \cos(2x) \right]$

1 mark

Let X be a continuous random variable with probability density function

$$f(x) = \begin{cases} k x \sin(2x) & 0 \le x \le \frac{\pi}{2} \\ 0 & \text{elsewhere} \end{cases}$$

b.i Show that $k = \frac{4}{\pi}$

ii. The median of X is m, and satisfies the equation $\sin(2m) - 2m\cos(2m) = p$. Find the value of p.

2 marks

- The mode of X is M, and satisfies the equation $\frac{\tan(2M)}{2M} = q$. Find the value of q. 2 marks
- iv. Sketch the graph of f, on the axis below, clearly showing the relative positions of the median m, the mode M.



END OF QUESTION AND ANSWER BOOKLET END OF EXAMINATION

EXTRA WORKING PAGE						

MATHEMATICAL METHODS

Written examination 1

FORMULA SHEET

Directions to students

Detach this formula sheet during reading time.

This formula sheet is provided for your reference.

Mathematical Methods formulas

Mensuration

area of a trapezium	$\frac{1}{2}(a+b)h$	volume of a pyramid	$\frac{1}{3}Ah$
curved surface area of a cylinder	$2\pi rh$	volume of a sphere	$\frac{4}{3}\pi r^3$
volume of a cylinder	$\pi r^2 h$	area of triangle	$\frac{1}{2}bc\sin(A)$
volume of a cone	$\frac{1}{3}\pi r^2 h$		

Calculus

$\frac{d}{dx}\left(x^{n}\right) = nx^{n-1}$		$\int x^n dx = \frac{1}{n+1} x^{n+1} + c , n \neq -1$		
$\frac{d}{dx}\left(\left(ax+b\right)^{n}\right) = na\left(ax+b\right)^{n-1}$		$\int (ax+b)^n dx = \frac{1}{a(n+1)} (ax+b)^{n+1} + c, \ n \neq -1$		
$\frac{d}{dx}(e^{ax}) = ae^{ax}$		$\int e^{ax} dx = \frac{1}{a} e^{ax} + c$		
$\frac{d}{dx}(\log_{e}(x)) = \frac{1}{x}$		$\int \frac{1}{x} dx = \log_e(x) + c, \ x > 0$		
$\frac{d}{dx}(\sin(ax)) = a\cos(ax)$		$\int \sin(ax) dx = -\frac{1}{a}\cos(ax) + c$		
$\frac{d}{dx}(\cos(ax)) = -a\sin(ax)$		$\int \cos(ax) dx = \frac{1}{a} \sin(ax) + c$		
$\frac{d}{dx}(\tan(ax)) = \frac{a}{\cos^2(ax)} = a\sec^2(ax)$				
product rule	$\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$	quotient rule	$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$	
chain rule	$\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx}$			

Probability

Pr(A) = 1 - Pr(A')		$\Pr(A \cup B) = \Pr(A) + \Pr(B) - \Pr(A \cap B)$		
$\Pr(A \mid B) = \frac{\Pr(A \cap B)}{\Pr(B)}$				
mean	$\mu = E(X)$	variance	$\operatorname{var}(X) = \sigma^{2} = E((X - \mu)^{2}) = E(X^{2}) - \mu^{2}$	

Probability distribution		Mean	Variance
discrete	$\Pr(X=x) = p(x)$	$\mu = \sum x p(x)$	$\sigma^2 = \sum (x - \mu)^2 p(x)$
continuous	$\Pr(a < X < b) = \int_{a}^{b} f(x) dx$	$\mu = \int_{-\infty}^{\infty} x f(x) dx$	$\sigma^2 = \int_{-\infty}^{\infty} (x - \mu)^2 f(x) dx$

Sample proportions

$\hat{P} = \frac{X}{n}$		mean	$E(\hat{P}) = p$
standard deviation	$\operatorname{sd}(\hat{P}) = \sqrt{\frac{p(1-p)}{n}}$	approximate confidence interval	$\left(\hat{p}-z\sqrt{\frac{\hat{p}(1-\hat{p})}{n}},\hat{p}+z\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}\right)$

END OF FORMULA SHEET