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2017 Mathematical Methods Trial Exam 1 Solutions

Q1a The mean of $q = E(\hat{P}) = p = \frac{500}{1500} = \frac{1}{3}$

Q1b $\text{sd}(\hat{P}) = \sqrt{\frac{p(1-p)}{n}} = \sqrt{\frac{\frac{1}{3}\times\frac{2}{3}}{50}} = \frac{1}{15}$

$\frac{2}{5} - \frac{1}{3} = \frac{1}{15}$, .: $\frac{2}{5}$ is a standard deviation higher than the mean.

$\Pr\left(q > \frac{2}{5}\right) \approx \frac{1-0.68}{2} = 0.16$, .: 16%

Q1c $n = 25$, $\hat{p} = \frac{5}{25} = \frac{1}{5}$

95% confidence interval for p

$\approx \left(\hat{p} - 2\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}, \hat{p} + 2\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}\right) = (0.04, 0.36)$

95% confidence interval for X

$\approx (1500\times 0.04, 1500\times 0.36) = (60, 540)$

Q2a $f(x) = \frac{x^2 - 3x + 2}{x^2 - 1}$, $x > -2$

$f(x) = \frac{(x-2)(x-1)}{(x+1)(x-1)} = \frac{x-2}{x+1}$ for $x > -2$ and $x \neq \pm 1$

.: $f(x) = 0$ when $x = 2$

Q2b $f(x) = \frac{(x-2)(x-1)}{(x+1)(x-1)} = \frac{x-2}{x+1}$ for $x > -2$ and $x \neq \pm 1$

$f'(x) = \frac{(x+1)-(x-2)}{(x+1)^2} = \frac{3}{(x+1)^2}$, for $x > -2$ and $x \neq \pm 1$

Q2c Domain: $\{x : x > -2 \text{ and } x \neq \pm 1\}$

If $f'(x)$ is continuous at $x = 1$, $f'(1) = \frac{3}{4}$, .: range: $(0, \infty)\setminus\left\{\frac{3}{4}\right\}$

Q3a $f(x) = \tan\left(\frac{2x}{3}\right)$, period $= \frac{\pi}{\frac{2}{3}} = \frac{3\pi}{2}$

$g(x) = f\left(x - \frac{3\pi}{2}\right) = \tan\left(\frac{2}{3}\left(x - \frac{3\pi}{2}\right)\right) = \tan\left(\frac{2x}{3} - \pi\right)$

Q3b $f'(x) = \frac{2}{3}\sec^2\left(\frac{2x}{3}\right)$, $f'\left(\frac{11\pi}{4}\right) = \frac{2}{3}\sec^2\left(\frac{11\pi}{6}\right)$

$= \frac{2}{3}\times\frac{1}{\left(\cos\left(\frac{11\pi}{6}\right)\right)^2} = \frac{2}{3}\times\frac{1}{\left(\frac{\sqrt{3}}{2}\right)^2} = \frac{8}{9}$

Q4 Let $e^{2x} = 2(e^x + 1)$, .: $(e^x)^2 - 2e^x = 2$, $(e^x)^2 - 2e^x + 1 = 3$,

$(e^x - 1)^2 = 3$, $e^x - 1 = \sqrt{3}$, $e^x = \sqrt{3} + 1$

.: $y = 2(e^x + 1) = 4 + 2\sqrt{3}$

Q5 Let $y = \log_{10}\left(\frac{x}{x+1}\right)$, inverse: $x = \log_{10}\left(\frac{y}{y+1}\right)$

.: $\frac{y}{y+1} = 10^x$, $\frac{1}{1+\frac{1}{y}} = 10^x$, $1 + \frac{1}{y} = 10^{-x}$, $\frac{1}{y} = 10^{-x} - 1$

.: $y = \frac{1}{10^{-x} - 1}$, .: $a = 1$, $b = 1$ and $c = -1$

Q6a Average value of $f(x)$

$= \frac{1}{3-0}\int_0^3 \frac{1}{4}(x-2)^4\,dx = \frac{1}{12}\left[\frac{(x-2)^5}{5}\right]_0^3 = \frac{11}{20}$

Q6b $f(0) = 4$, $f(3) = \frac{1}{4}$

Average rate of change of $f(x) = \frac{\frac{1}{4} - 4}{3 - 0} = \frac{-\frac{15}{4}}{3} = -\frac{5}{4}$

Q7a $\frac{d}{dx}(x\cos x) = \cos x - x\sin x$

Q7b $x\sin x = \cos x - \frac{d}{dx}(x\cos x)$

$\int_0^\pi (x\sin x)\,dx = \int_0^\pi \cos x\,dx - \int_0^\pi \left(\frac{d}{dx}(x\cos x)\right)dx$

$= [\sin x]_0^\pi - [x\cos x]_0^\pi = \pi$

Q8a y-intercept: Let $x = 0$, $\frac{m+1}{m} = -\frac{m+1}{m-1}$

.: $(m+1)(m-1) = -m(m+1)$, $(m+1)(m-1) + m(m+1) = 0$

$(m+1)((m-1)+m) = 0$, $(m+1)(2m-1) = 0$, .: $m = -1$, $\frac{1}{2}$

Q8b $\begin{bmatrix} m & -(m-1) \\ (m-1) & m \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} m+1 \\ m+1 \end{bmatrix}$

$\Delta = m^2 + (m-1)^2 > 0$ for any $m \in R$

.: the two simultaneous equations always have a unique solution.

Q9 $\frac{1}{2} - \log_e\left(\frac{x-1}{2}\right) \geq 0$ and $\frac{x-1}{2} > 0$

.: $\log_e\left(\frac{x-1}{2}\right) \leq \frac{1}{2}$, $\frac{x-1}{2} \leq \sqrt{e}$, $x \leq 1 + 2\sqrt{e}$ and $x > 1$

Maximal domain: $\left(1, 1 + 2\sqrt{e}\right]$

Q10a $\frac{1}{2}\times 5\times c = 1$, $c = 0.4$

Q10b For $x \in [0, 3]$, $y = \frac{2}{15}x$. Let $\int_0^a \frac{2}{15}x\,dx = 0.5$, .: $a = \frac{\sqrt{30}}{2}$

Please inform mathline@itute.com re conceptual and/or mathematical errors