The Mathematical Association of Victoria

Trial Examination 2017

MATHEMATICAL METHODS

Trial Written Examination 1 - SOLUTIONS

Question 1

a. $y=\tan\left(x^2+2\right)$

$\frac{dy}{dx}=\sec^2\left(x^2+2\right)\times 2x$ **1M**

$=2x\sec^2\left(x^2+2\right)$ **1A**

OR

$\frac{2x}{\cos^2(x^2+2)}$ **1A**

b. $f(x)=\frac{\log_e\left(x^2-1\right)}{x^2-1}$

$f'(x)=\frac{\left(x^2-1\right)\times\frac{2x}{x^2-1}-\log_e\left(x^2-1\right)\times 2x}{\left(x^2-1\right)^2}$ **1M**

$=\frac{2x-2x\log_e\left(x^2-1\right)}{\left(x^2-1\right)^2}$ **1M**

$\therefore\ f'(-2)=\frac{-4-(-4)\log_e(3)}{(3)^2}$

1A

$=\frac{-4+4\log_e(3)}{9}$

Question 2

$d(t)=-5\cos\left(\frac{\pi t}{8}\right)+5$

a. $d'(t)=\frac{5\pi}{8}\sin\left(\frac{\pi t}{8}\right)$

$d'(t)=0\Rightarrow\sin\left(\frac{\pi t}{8}\right)=0$

$\frac{\pi t}{8}=0,\pi,\ldots$

$t=0,8,\ldots$

Understanding that the graph starts at (0, 0).

First at its maximum at $t=8$. **1A**

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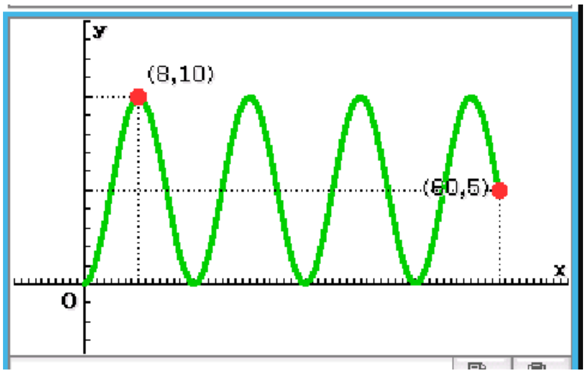
OR

Consider the graph of $d(t) = -5\cos\left(\frac{\pi t}{8}\right) + 5$, with amp = 5 and period = 16.



First at its maximum at $t = 8$ **1A**

b. Solve $d(t) > 2.5$

$$-5\cos\left(\frac{\pi t}{8}\right) + 5 = 2.5$$

$$\cos\left(\frac{\pi t}{8}\right) = \frac{1}{2}$$

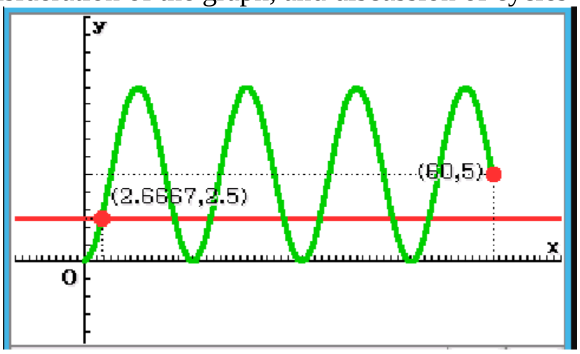
For one cycle solutions are

$$\frac{\pi t}{8} = \frac{\pi}{3}, 2\pi - \frac{\pi}{3}$$

$$\frac{\pi t}{8} = \frac{\pi}{3}, \frac{5\pi}{3}$$ **1M**

$$\therefore t = \frac{8}{3}, \frac{40}{3}$$

Consideration of the graph, and discussion of cycles



We have 3 complete cycles with $\frac{40}{3} - \frac{8}{3} = \frac{32}{3}$ giving a time fraction each of $\frac{\frac{32}{3}}{16} = \frac{2}{3}$.

We have an additional 12 minutes with $\frac{180}{3} - \frac{152}{3} = \frac{28}{3}$ giving a time fraction of $\frac{\frac{28}{3}}{12} = \frac{7}{9}$. **1M**

Time for $0 \le t \le 60$ minutes when the depth of water is more than 2.5 metres equals

$\frac{2}{3} \times 48 + \frac{7}{9} \times 12 = \frac{124}{3}$ minutes.

Giving the fraction of time $= \frac{\frac{124}{3}}{60} = \frac{124}{180} = \frac{31}{45}$ **1A**

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Question 3

$P(x) = 5x^3 - x^2 + x + 7$.

a. Using the factor theorem.

$P(1) = 5 - 1 + 1 + 7 \neq 0$

$P(-1) = -5 - 1 - 1 + 7 = 0$.

$\therefore (x+1)$ is a factor **1A**

b. Using a form of division

	5	−1	1	7
−1		−5	6	−7
	5	−6	7	0

$Q(x) = 5x^2 - 6x + 7$ **1A**

c. $Q(x) = 5x^2 - 6x + 7$

$\Delta = (-6)^2 - 4 \times 5 \times 7 = 36 - 140 = -104$

$\Delta < 0$ giving no linear factors for $Q(x)$. **1A**

d. $y = P(x)$ has no stationary points, giving $\frac{dy}{dx} \neq 0$ over the domain R

$\frac{dy}{dx} = 15x^2 - 2x + 1$ **1A**

Minimum is at the turning point

$x = \frac{-b}{2a} = \frac{1}{15}$ **1A**

OR

$\frac{d^2y}{dx^2} = 30x - 2 = 0$

$x = \frac{1}{15}$ **1A**

e. Shape **1A**

Intercepts **1A**

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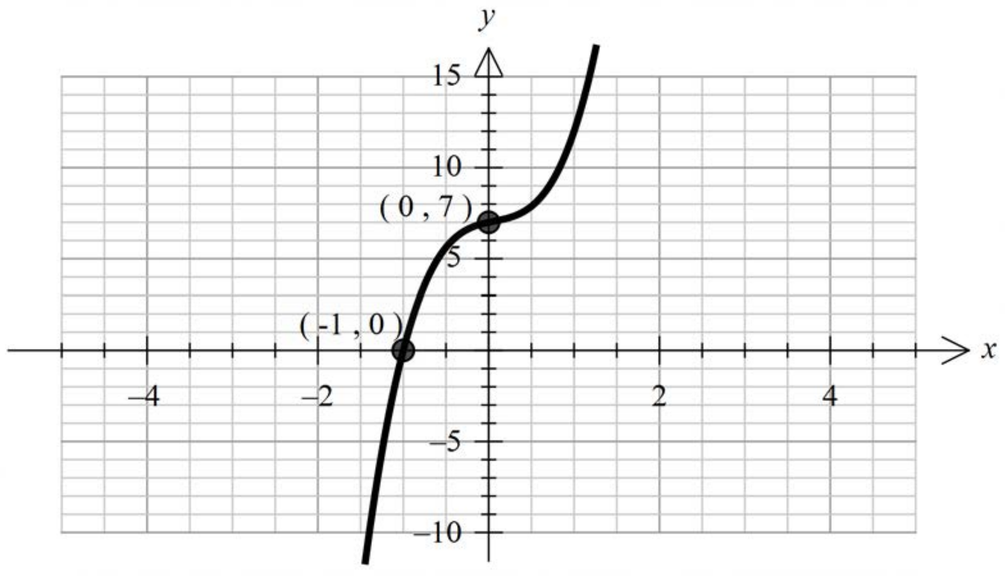

Question 4

$f(x) = x\sin(x)$.

a. Average rate of change for $\left[\frac{\pi}{2}, \pi\right]$ equals $\dfrac{f(\pi) - f\left(\frac{\pi}{2}\right)}{\pi - \frac{\pi}{2}}$

$$= \frac{\pi\sin(\pi) - \frac{\pi}{2}\sin\left(\frac{\pi}{2}\right)}{\pi - \frac{\pi}{2}}$$ **1M**

$$= \frac{0 - \frac{\pi}{2}}{\frac{\pi}{2}}$$

Average rate of change $= -1$ **1A**

b. $f'(x) = \sin(x)\times 1 + x\cos(x) = \sin(x) + x\cos(x)$ **1A**

c. If $f'(x) = \sin(x) + x\cos(x)$ then $\int\left(\sin(x) + x\cos(x)\right)dx = x\sin(x) + c$ **1M**

Rearrange to get

$$\int\left(\sin(x)\right)dx + \int\left(x\cos(x)\right)dx = x\sin(x) + c$$

$$\int\left(x\cos(x)\right)dx = x\sin(x) - \int\left(\sin(x)\right)dx + c$$

We need $\int_0^{\frac{\pi}{2}} x\cos(x)dx$

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$$\int_0^{\frac{\pi}{2}}\left(x\cos(x)\right)dx=\left[x\sin(x)\right]_0^{\frac{\pi}{2}}-\int_0^{\frac{\pi}{2}}\left(\sin(x)\right)dx$$ **1M**

$$=\frac{\pi}{2}\sin\left(\frac{\pi}{2}\right)-0-\left[-\cos(x)\right]_0^{\frac{\pi}{2}}$$

$$=\frac{\pi}{2}+\cos\left(\frac{\pi}{2}\right)-\cos\left(0\right)$$

Giving $\int_0^{\frac{\pi}{2}}x\cos(x)dx=\frac{\pi}{2}-1$ **1A**

Question 5

$\log_2\left((2x-2)^2\right)-4\log_2(1-x)=1$

$\log_2\left(\frac{(2x-2)^2}{(1-x)^4}\right)=1$ **1A**

$\frac{(2x-2)^2}{(1-x)^4}=2$

$\frac{4(x-1)^2}{(x-1)^4}=2$

$\frac{4}{(x-1)^2}=2$

$2x^2-4x-2=0$ **1A**

$x=\frac{2\pm\sqrt{8}}{2}$

$x=1\pm\sqrt{2}$

As $x<1$

$x=1-\sqrt{2}$ **1A**

Question 6

$$\left(\hat{p}-2\sqrt{\frac{\hat{p}\left(1-\hat{p}\right)}{n}},\hat{p}+2\sqrt{\frac{\hat{p}\left(1-\hat{p}\right)}{n}}\right)$$

$$=\left(\frac{1}{5}-2\sqrt{\frac{\frac{1}{5}\times\frac{4}{5}}{10\ 000}},\frac{1}{5}+2\sqrt{\frac{\frac{1}{5}\times\frac{4}{5}}{10\ 000}}\right)$$ **1M**

$$=\left(\frac{1}{5}-2\sqrt{\frac{4}{250\ 000}},\frac{1}{5}+2\sqrt{\frac{4}{250\ 000}}\right)$$

$$=\left(\frac{1}{5}-\frac{1}{125},\frac{1}{5}+\frac{1}{125}\right)$$ **1A**

$$=\left(\frac{24}{125},\frac{26}{125}\right)$$ **1A**

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Question 7

a. $\int_0^1 (x)dx$

$=\left[\frac{x^2}{2}\right]_0^1$

$=\frac{1}{2}$

$\int_1^a \left(\frac{1}{x^2}\right)dx=\frac{1}{2}$ **1A**

$\left[-\frac{1}{x}\right]_1^a=\frac{1}{2}$

$-\frac{1}{a}+1=\frac{1}{2}$

$-\frac{1}{a}=-\frac{1}{2}$

$a=2$ as required **1M**

b. $E(X)=\int_0^1 (x^2)dx+\int_1^2 \left(\frac{1}{x}\right)dx$ **1M**

$=\left[\frac{x^3}{3}\right]_0^1+\left[\log_e(x)\right]_1^2$ **1M**

$=\frac{1}{3}+\log_e(2)-\log_e(1)$

$=\frac{1}{3}+\log_e(2)$ **1A**

Question 8

a. $f(x)=3\sqrt{4-2x}+1$

$f(x)=3\sqrt{-2(x-2)}+1$

Translate 2 units left and 11 units up.

As the graph has to be dilated by a factor of $\frac{1}{3}$ from the *x*-axis,

the vertical translation can be worked by solving $\frac{1+d}{3}=4, d=11$.

$f_1(x)=3\sqrt{-2x}+12$

$c=-2, d=11$ **1A**

Dilate by a factor of $\frac{1}{3}$ from the *x*-axis.

$f_2(x)=\sqrt{-2x}+4$

Dilate by a factor of 2 from the *y*-axis.

$f_3(x)=\sqrt{-x}+4$

Reflect in the *y*-axis.

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$g(x)=\sqrt{x}+4$

$a=-2, b=\frac{1}{3}$ **2A**

OR

$x'=a(x+c)=-2(x-2)$

$a=-2, c=-2$ **1A**

$y'=b(y+d)=\frac{y-1}{3}+4=\frac{1}{3}(y+11)$ **1M**

$b=\frac{1}{3}, d=11$ **1A**

OR

$x'=a(x+c), x=\frac{x'}{a}-c$

$y'=b(y+d), y=\frac{y'}{b}-d$ **1M**

$\frac{y'}{b}-d=3\sqrt{4-2\left(\frac{x'}{a}-c\right)}+1$

$y'=3b\sqrt{-\frac{2x'}{a}+4+2c}+(1+d)b$

$3b=1, b=\frac{1}{3}$

$(1+d)b=4, d=11$

$-\frac{2}{a}=1, a=-2$

$4+2c=0, c=-2$ **1A 2 correct**
2A all correct

b. $y=3\sqrt{4-2x}+1$

Inverse swap x and y

$x=3\sqrt{4-2y}+1$

$\frac{x-1}{3}=\sqrt{4-2y}$

$f^{-1}(x)=2-\frac{(x-1)^2}{18}$ **1A**

Domain $[1,\infty)$ **1A**

$f^{-1}:[1,\infty)\to R, f^{-1}(x)=-\frac{(x-1)^2}{18}+2$

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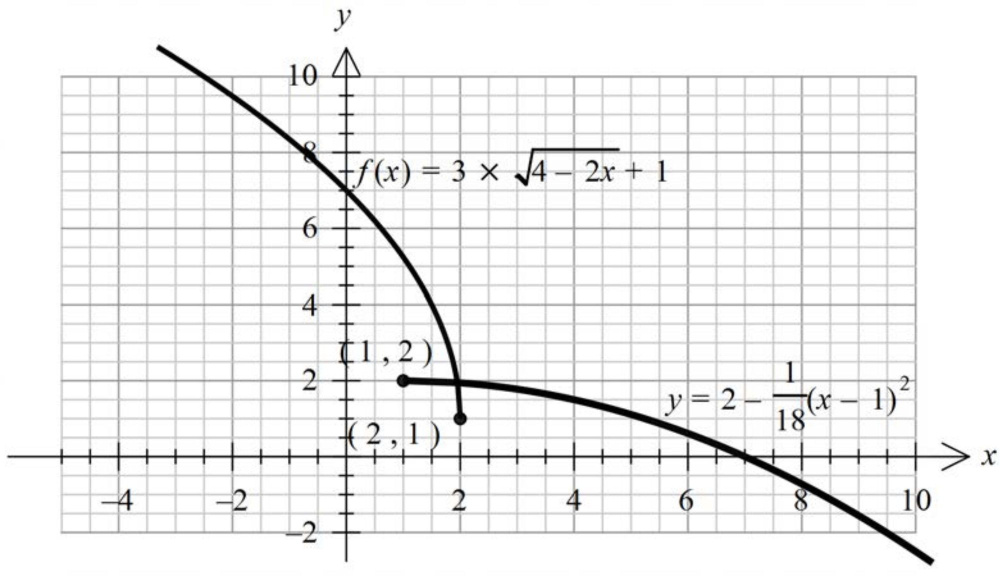
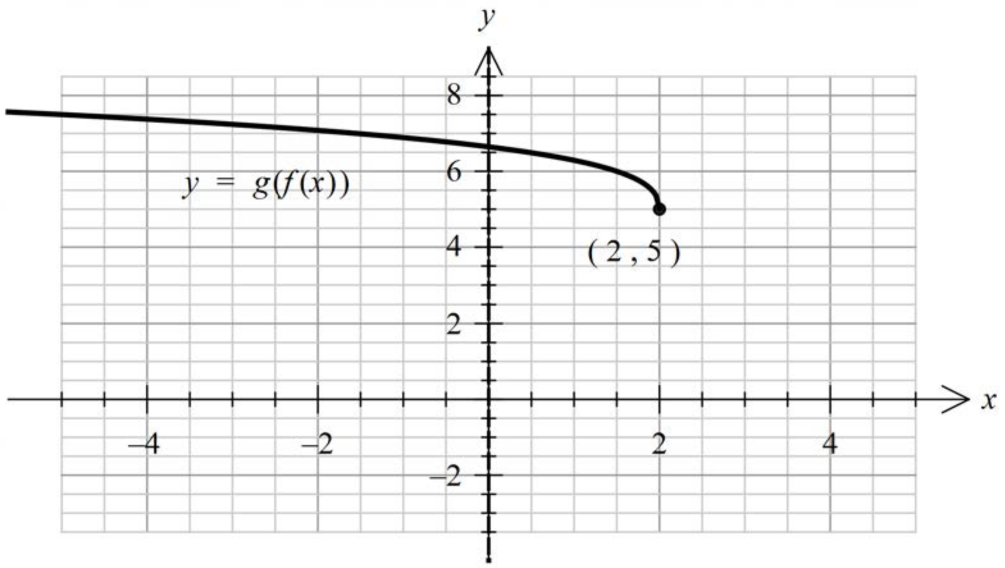


c. $g\left(f(x)\right) = \sqrt{3\sqrt{4-2x}+1}+4$ **1A**

Range $\left[5, \infty\right)$ **1A**



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