SOLUTIONS.

Teacher's Name: Students Circle

Algreen-Ussing (Mr) Itter (Mrs) James (Mr)

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SEMESTER 2 EXAMINATIONS - NOVEMBER 2017

Year 11 Mathematical Methods Examination

Reading time: 10 Minutes
Writing time: 60 Minutes

Marks Allocated:

Sections within Booklet	Number of Questions	Number of Marks
EXAM 1 Short Answer	8	40

Specific Instructions

No calculator and no summary sheet allowed.
In all questions where a numerical answer is required, an exact value must be given unless otherwise specified.
In questions where more than one mark is available, appropriate working must be shown.
Unless otherwise indicated, the diagrams in this book are not drawn to scale.

Supplies and Equipment

Supplies: Please ensure you have the correct supplies/instruments for taking the examination before you enter the examination venue (e.g. pencils, pens, calculator, ruler, etc). There will be no sharing allowed. No other paper, etc. will be allowed to come in with you unless instructed as Specific Instructions. A clear bottle containing only water is permissible.

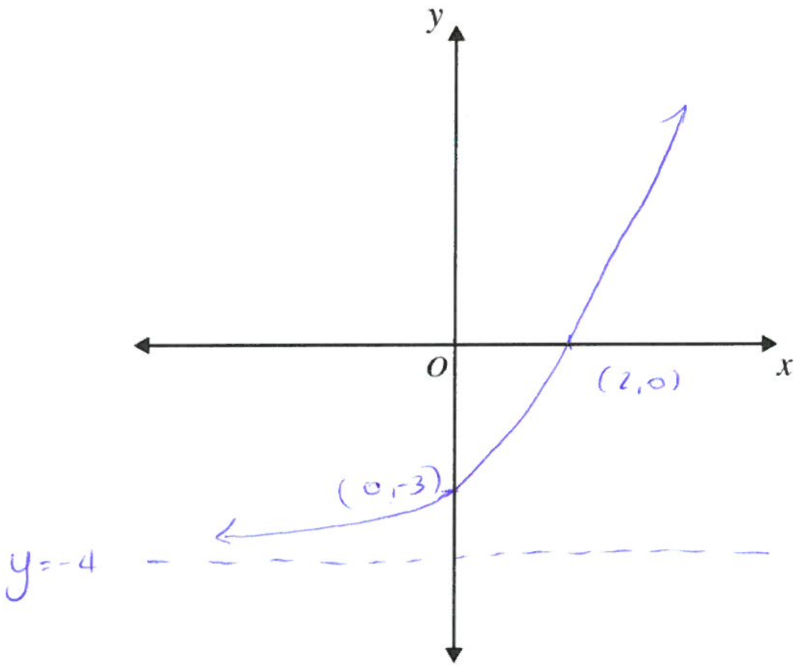
At the Conclusion: Please wait quietly for specific instruction as to how you will be dismissed. Leave your examination paper on your table. Pick up unwanted papers around you, push your chair under the table, and put your rubbish in the bin on your way out of the examination room.

Short Answer
Working required where appropriate:

Question 1 (7 marks)
Let $f: R \rightarrow R, f(x) = 2^x - 4$

a. On the set of axes below, sketch the graph of $y = f(x)$. Indicate clearly the coordinates of any axes intercepts as well as the equation of any asymptotes.

3 marks



y-axis int $(x=0)$

$f(0) = 2^0 - 4$

$= 1 - 4$

$= -3$

$(0,-3)$

x-axis int $(f(x)=0)$

$0 = 2^x - 4$

$4 = 2^x$

$2^2 = 2^2$

$\therefore x = 2$

$(2,0)$

b. Write down

i. the domain of *f*. 1 mark

$(-\infty, \infty)$

ii. the range of *f*. 1 mark

$(-4, \infty)$

a. Find the rule for f^{-1}. 2 marks

$x \Leftrightarrow y$ let $y = f(x)$

$y = 2^x - 4$

$x = 2^y - 4$

$x + 4 = 2^y$

$y = \log_2(x+4)$ $f^{-1}(x) = \log_2(x+4)$.

Question 2 (8 marks)

Solve the following for x.

a. $2^{3x+2} = \frac{1}{4}$ 2 marks

$2^{3x+2} = 2^{-2}$

$3x + 2 = -2$

$3x = -4$

$x = -4/3$

b. $3^{2x} - 10 \times 3^{x} + 9 = 0$ 2 marks

let $a = 3^x$ $(3^x)^2 - 10 \times 3^x + 9 = 0$ $3^x = 9$ $3^x = 1$

$a^2 - 10a + 9 = 0$ $3^x = 3^2$ $x = 0$

$(a-9)(a-1) = 0$ $\therefore x = 2$

$a = 9$ and 1

c. $\log_3(x^2 - 3x + 1) = 0$ 2 marks

$3^0 = x^2 - 3x + 1$ $0 = x(x-3)$

$1 = x^2 - 3x + 1$ $x = 0$ or 3

$0 = x^2 - 3x$

d. $\log_{10}(x) - 2\log_{10}(5) + \log_{10}(2) = 0$ 2 marks

$\log_{10} x - \log_{10} 5^2 + \log_{10} 2 = 0$

$\log_{10} \frac{x}{25} + \log_{10} 2 = 0$ $1 = 2x/25$

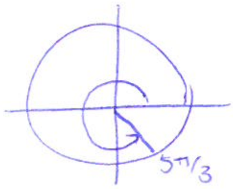
$\log_{10} 2x/25 = 0$ $25 = 2x$

$10^0 = 2x/25$ $x = 12\frac{1}{2}$

Question 3 (3 marks)

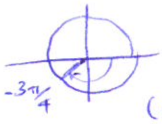
a. Evaluate

i. $\cos\left(\frac{5\pi}{3}\right)$ $= 1/2$ 1 mark



$5\pi/3$ (4th Quad)

ii. $\sin\left(-\frac{3\pi}{4}\right)$ 1 mark



$-3\pi/4$ (Quad 3)

$= -1/\sqrt{2}$

b. Given $\sin(\theta) = 0.8$ and $\frac{\pi}{2} < \theta < \pi$, evaluate $\cos(\theta)$ 1 mark

Quadrant 2 (cos -ve)

$\cos^2\theta + \sin^2\theta = 1$

$\cos^2\theta + 0.8^2 = 1$

$\cos^2\theta + 0.64 = 1$

$\cos^2\theta = 0.36$

$\cos\theta = \pm 0.6$

2nd Quadrant

$\cos\theta = -0.6$.

Question 4 (5 marks)

$0 \leq x \leq 2\pi$

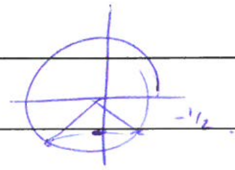
$0 \leq 2x \leq 4\pi$

a. Solve $2\sin(2x)+1=0$ for $x \in [0, 2\pi]$. 2 marks

$\sin 2x = -1/2$

$2x = 7\pi/6, \ 11\pi/6, \ 19\pi/6, \ 23\pi/6$

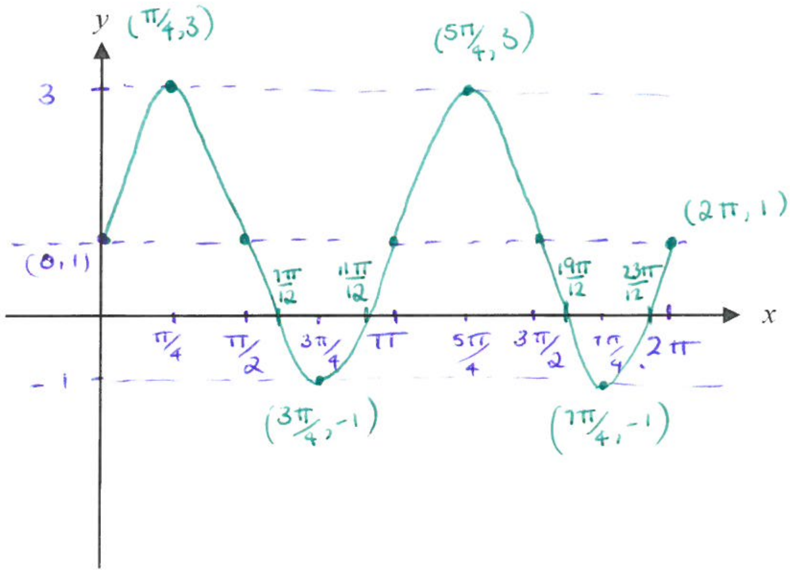
$\rightarrow x = 7\pi/12, \ 11\pi/12, \ 19\pi/12, \ 23\pi/12$



x-int

b. On the set of axes below, sketch the graph of $y = 2\sin(2x)+1$ for $x \in [0, 2\pi]$.
Label endpoints, x-intercepts, and turning points clearly.

3 marks



Question 5 (3 marks)

a. Find the derivative of $2x^2 - 3x + 1$ with respect to x. 1 mark

$dy/dx = 4x - 3$

b. Let $f(x) = \frac{8}{x^3} + 4\sqrt{x}$, $x \neq 0$. 1 mark

i. Show that $f'(x) = -\frac{24}{x^4} + \frac{2}{\sqrt{x}}$

$f(x) = 8x^{-3} + 4x^{1/2}$

$f'(x) = -24x^{-4} + 2x^{-1/2}$

$= \frac{-24}{x^4} + \frac{2}{\sqrt{x}}$

ii. Evaluate $f'(1)$. 1 mark

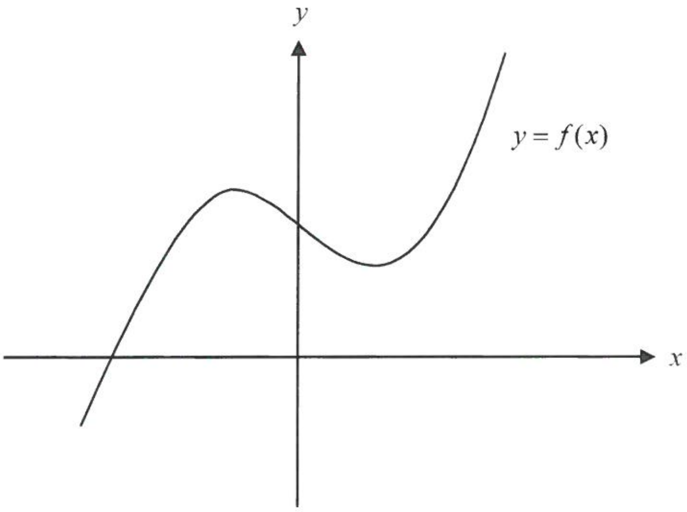
$f'(1) = \frac{-24}{1^4} + \frac{2}{\sqrt{1}}$

$= -24 + 2$

$= -22$

Question 6 (5 marks)

The graph of the function $f(x) = ax^3 + bx + 1$, where a and b are constants is shown below.



The graph passes through the point $(1, 2)$ and has a gradient of 5 at this point.

a. Show that $a = 2$ and $b = -1$.

$f(x) = ax^3 + bx + 1$ $\qquad$ $f'(x) = 3ax^2 + b$

$f(1) = 2$ $\qquad$ from $f'(1) = 5$

$2 = a + b + 1$ $\qquad$ $5 = 3a + b.$ ②

$1 = a + b.$ ①

② - ① $\quad 5 = 3a + b$

$1 = a + b$

$4 = 2a$

$a = 2$

when $a = 2$, $1 = 2 + b$

$b = -1$

3 marks

Question 6 (cont'd)

$f(x) = 2x^3 - x + 1$

b. Find the average rate of change of the function between $x = -1$ and $x = 1$. 1 mark

$f(-1) = 2(-1)^3 - (-1) + 1$ $\quad$ $f(1) = 2(1)^3 - 1 + 1$

$= -2 + 1 + 1$ $\quad$ $= 2 - 1 + 1$

$= 0$ $\quad$ $= 2$

Average R.O.C: $\frac{f(1) - f(-1)}{1 - -1} \Rightarrow \frac{2 - 0}{2}$

$= 1$

c. Find the x-coordinates of the turning point of $f(x)$. 1 mark

$f'(x) = 0$ $\quad$ $f(x) = 2x^3 - x + 1$

$f'(x) = 6x^2 - 1$

$0 = 6x^2 - 1$

$1 = 6x^2$

$x^2 = \frac{1}{6}$

$x = \pm\sqrt{\frac{1}{6}}$

Question 7 (3 marks)

The curve $y = \frac{1}{x}, x > 0$ undergoes a sequence of transformations defined by T such that

$$T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} -3 & 0 \\ 0 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}.$$

a. Find the equation of the image of the curve after it has undergone the sequence of transformations. 2 marks

$x' = -3x \qquad y' = 2y$

$x = -\frac{x'}{3} \qquad y = \frac{y'}{2}$

$\Rightarrow \quad y = \frac{1}{x}$

$\frac{y'}{2} = \frac{1}{-x/3}$

$\frac{y'}{2} = -\frac{3}{x}$

$y = \frac{-6}{x}$

b. Describe **one** of the transformations that the curve undergoes. 1 mark

Reflection in y-axis

Question 8 (6 marks)

a. The graph of $y = 2x^3 + 3x^2 - 12x + 5$ has two stationary points. Find the x-coordinate of each stationary point.

2 marks

☐ $dy/dx = 0$ (stationary point).

$0 = 6x^2 + 6x - 12$

$0 = 6(x^2 + x - 2)$

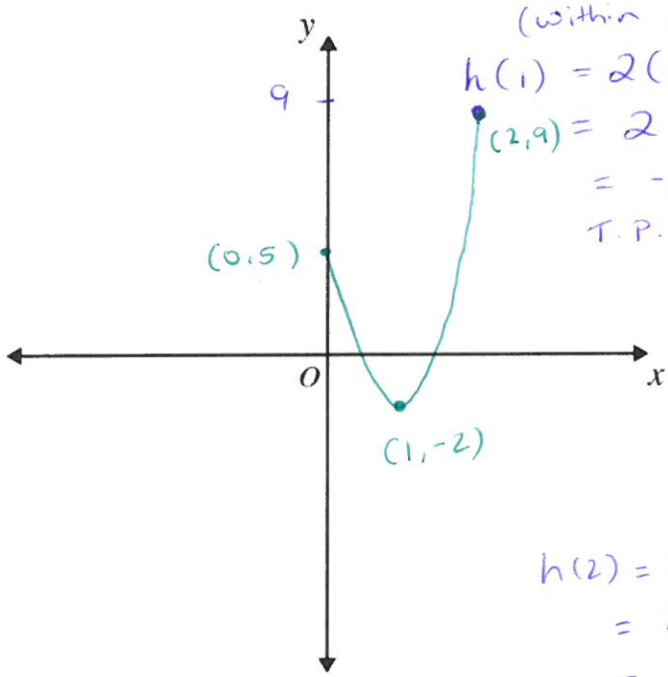
$0 = 6(x+2)(x-1)$

$x = -2$ and $\underline{1}$

b. Sketch the graph of $h : [0,2] \to R, h(x) = 2x^3 + 3x^2 - 12x + 5$. Label the y intercept, the turning point and endpoints with coordinates.

2 marks

☐



T.P. when $x = 1$
(within restricted domain)

$h(1) = 2(1)^3 + 3(1)^2 - 12(1) + 5$

$= 2 + 3 - 12 + 5$

$= -2$

T.P. $(1, -2)$

End points.

$x = 0, \ h(0) = 5$

$x = 2, \ h(2) \Rightarrow$

$h(2) = 2 \times (2)^3 + 3(2)^2 - 12(2) + 5$

$= 2 \times 8 + 3 \times 4 - 24 + 5$

$= 16 + 12 - 24 + 5$

$= 9$ Endpoint $(2, 9)$.

c. Find the values of x for which the gradient of the graph $h : [0,2] \to R, h(x) = 2x^3 + 3x^2 - 12x + 5$ is negative.

2 marks

$h'(x) < 0, \ 0 < x < 1$

☐ $[0, 1)$

END OF EXAMINATION