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'2016 Examination Package' - Trial Examination 1 of 5

STUDENT NUMBER

Figures									
Words									

Letter

MATHEMATICAL METHODS

Units 3 & 4 – Written examination 2

(TSSM's *2011 trial exam updated for the current study design)*

Reading time: 15 minutes
Writing time: 2 hours

QUESTION & ANSWER BOOK

Structure of book

Section	*Number of questions*	*Number of questions to be answered*	*Number of marks*
1	22	22	22
2	5	5	58
			Total 80

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers, a protractor, set-squares, aids for curve sketching, one bound reference, one approved CAS calculator (memory DOES NOT need to be cleared) and, if desired, one scientific calculator
- Students are NOT permitted to bring into the examination room: blank sheets of paper and/or white out liquid/tape.

Materials supplied

- Question and answer book of 21 pages including answer sheet for multiple-choice questions.

Instructions

- Print your name in the space provided on the top of this page and the multiple-choice answer sheet.
- All written responses must be in English.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic communication devices into the examination room.

SECTION 1 – Multiple-choice questions

Instructions for Section 1
Answer all questions on the answer sheet provided for multiple choice questions.
Choose the response that is **correct** for the question.
A correct answer scores 1, an incorrect answer scores 0.
Marks will **not** be deducted for incorrect answers.
No marks will be given if more than one answer is completed for any question.

Question 1

The simultaneous linear equations $(k-1)x+4y=8$ and $3x-(-k+2)y=k+1$ where k is a real constant, have a unique solution for:

A. $k \in R\backslash\{2,\ 5\}$

B. $k=-2$

C. $k=5$

D. $k \in R\backslash\{0\}$

E. $k=2$

Question 2

Let $f: R \to R,\ f(x)=x^3$. Which one of the following is **not** true?

A. $f(2x)=8f(x)$

B. $-f(xy)=-f(x)f(y)$

C. $f(x-y)=f(x)-f(y)$

D. $f(-x)+f(x)=0$

E. $f(x-y)+f(x+y)=2x\left(f\left(\frac{x}{\sqrt[3]{x}}\right)+3f\left(\frac{y}{\sqrt[3]{y}}\right)\right)$

Question 3

At the point $(2,\ 0)$ on the graph of the function with rule $y=-2(x-2)^3(x+2)$,

A. The graph is not continuous.

B. There is a stationary point of inflection.

C. There is a local maximum.

D. The graph is not differentiable.

E. There is a local minimum.

SECTION 1 - continued

Question 4

$\int_1^4 (2f(x)+6)dx$ can be written as:

A. $2\int_1^4 f(x)dx + 6$

B. $2\int_1^4 (f(x)+6)dx$

C. $2\int_1^4 f(x)dx + 18$

D. $2\int_1^4 f(x)dx + \int_1^4 3\,dx$

E. $2\int_1^4 f(x)dx + 6\,x$

Question 5

The tangent at the point $(-2, 10)$ on the graph of the curve $y = f(x)$ has equation $y = 4 - 3x$. The tangent of $f(x)$ at the point $(1, 4)$ on the curve $y = f(x-3) - 6$ has equation:

A. $y = -3x + 1$

B. $y = -3x + 7$

C. $y = -2x + 4$

D. $y = -3x + 6$

E. $y = \frac{1}{3}x - 1$

Question 6

If $\Pr(Z < -c) = 0.75$, where Z is a standard normal random variable, then c is closest to:

A. 0.6745

B. -0.6745

C. 0.5987

D. 0.7734

E. -0.7734

SECTION 1 - continued

TURN OVER

Question 7
A cubic function has the rule $y = f(x)$. The graph of the derivative function $f'(x)$ crosses the x-axis at (2, 0) and (–3, 0). The maximum value of the derivative function is 10. The value of x for which the graph of $y = f(x)$ has a local maximum is:

A. -2
B. 2
C. -3
D. 3
E. $-\frac{1}{2}$

Question 8
Let $f(x) = \frac{x+3}{\sqrt{x-1}}$, $x > 1$ and $g(x) = 2(x-1)^2 - 7$, $x \in D$. Then the largest domain D such that $f \circ g$ exists is:

A. $x < -1$ or $x > 3$

B. R^+

C. $x > -7$

D. $R\backslash[1, -7]$

E. $x < -7$

Question 9
If $f'(x) = (2-x)(2x+1)^2$, then $f(x)$ could be:

A. $\frac{x(6x^3-8x^2-21x-12)}{6}$

B. $\frac{-x(6x^3-8x^2-12x)}{6}$

C. $-x^4 + \frac{4}{3}x^3 + \frac{7}{2}x^2 + 2x$

D. $-x^4 + 4x^3 + \frac{21}{4}x^2 + 2x$

E. $-x^4 + 2x^3 + 7x^2 + 2x$

SECTION 1 - continued

Question 10

The continuous random variable X has a probability density function given by

$$f(x) = \begin{cases} \frac{2}{\pi}\cos^2(x)dx, & 0 \leq x \leq \pi \\ 0 & \text{elsewhere} \end{cases}$$

The value of k such that $Pr(X > k) = 0.3$ is closest to:

A. $\frac{\pi}{2}$

B. 2.63

C. 3.45

D. $\frac{2}{\pi}$

E. 4.06

Question 11

The graph of a function lies above the $x-$axis for $0 \leq x \leq 6$ and $\int_0^6 f\left(\frac{x}{3}\right)\, dx = 12$. The graph of $f(x)$ is dilated by a factor of 3 from the $y-$axis, then translated 1 unit up. The resulting function is $g(x)$. Find $\int_0^6 g(x)dx$.

A. 6

B. 18

C. 12

D. 42

E. 0

Question 12

The continuous random variable X has a normal distribution with mean 4.3 and variance 0.09. The continuous random variable Z has the standard normal distribution. The probability that Z is between -1 and 2 is equal to:

A. $Pr(4.21 < X < 4.18)$

B. $Pr(4.21 < X < 4.39)$

C. $Pr(4 < X < 4.9)$

D. $Pr(3.7 < X < 4.39)$

E. $Pr(4 < X < 4.6)$

SECTION 1 - continued
TURN OVER

Question 13

For $y = e^{-4x}\sin(x-2)$ the rate of change of y with respect to x when $x = 2$ is

A. 1

B. $-4e^{-8}$

C. 0

D. e^{-8}

E. -4

Question 14

The general solution to the equation $\cos(2x) = -\frac{\sqrt{3}}{2}$ is

A. $\frac{5\pi}{12} + k\pi,\ \ k \in Z$

B. $\frac{(12k+5)\pi}{12},\ \ k \in Z$

C. $\pm\frac{5\pi}{12} + k\pi,\ \ k \in Z$

D. $\frac{(6k-1)\pi}{6},\ \ k \in Z$

E. $\frac{(3k+2)\pi}{3},\ \ k \in Z$

Question 15

For the function $f: R \to R,\ f(x) = (x+6)^2(x-3)^2$, the subset of R for which the gradient of f is positive is closest to

A. $(-\infty, -6) \cup (-1.5, \infty)$

B. $(-\infty, 3)$

C. $(-6, \infty)$

D. $(-\infty, -2.5) \cup (3, \infty)$

E. $(-6, -1.5) \cup (3, \infty)$

SECTION 1 - continued

Question 16

A transformation $T: R^2 \rightarrow R^2$ that maps the curve with equation $y = -e^{2x}$ onto the curve with equation $y = 2 + 5(-e^{2x+4})$ is given by

A. $T\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & 5 \end{bmatrix} + \begin{bmatrix} 2 \\ 2 \end{bmatrix}$

B. $T\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 & -5 \\ 2 & 0 \end{bmatrix} + \begin{bmatrix} 4 \\ 2 \end{bmatrix}$

C. $T\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2 & 0 \\ 0 & -5 \end{bmatrix} + \begin{bmatrix} 4 \\ 2 \end{bmatrix}$

D. $T\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & 0 \\ 0 & 5 \end{bmatrix} + \begin{bmatrix} -2 \\ 2 \end{bmatrix}$

E. $T\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 5 \end{bmatrix} + \begin{bmatrix} -2 \\ 2 \end{bmatrix}$

Question 17

A certain drug is known to be successful in 45% of all people using the drug. Twenty sufferers at a company are using the drug. The probability that exactly five people will be cured is closest to

A. 0.0049

B. 0.0365

C. 0.0503

D. 0.0185

E. 0.2059

Question 18

The function with rule $f(x) = -3\tan\left(\frac{x}{5}\right)$ has a period and an asymptote respectively

A. 5π and $x = \frac{5\pi}{2}$

B. $\frac{\pi}{5}$ and $x = \frac{\pi}{2}$

C. 10π and $x = 5\pi$

D. 5 and $x = \frac{5}{2}$

E. 10 and $x = 5$

Question 19

The right rectangle approximation using rectangles of width 1 to the area of the region enclosed by the curve with equation $y = 3 - e^{x-1}$, the x −axis, the line $x = 2$ and y −axis is

A. 5

B. $5 - e^{-1}$

C. $5 - e$

D. $-e^{-1}$

E. $-2e$

Question 20

The function $f\colon (-\infty,\ m] \to R$, with rule $f(x) = -2\log_e(x^2)$, will have an inverse function if

A. $m \leq -1$

B. $m > 0$

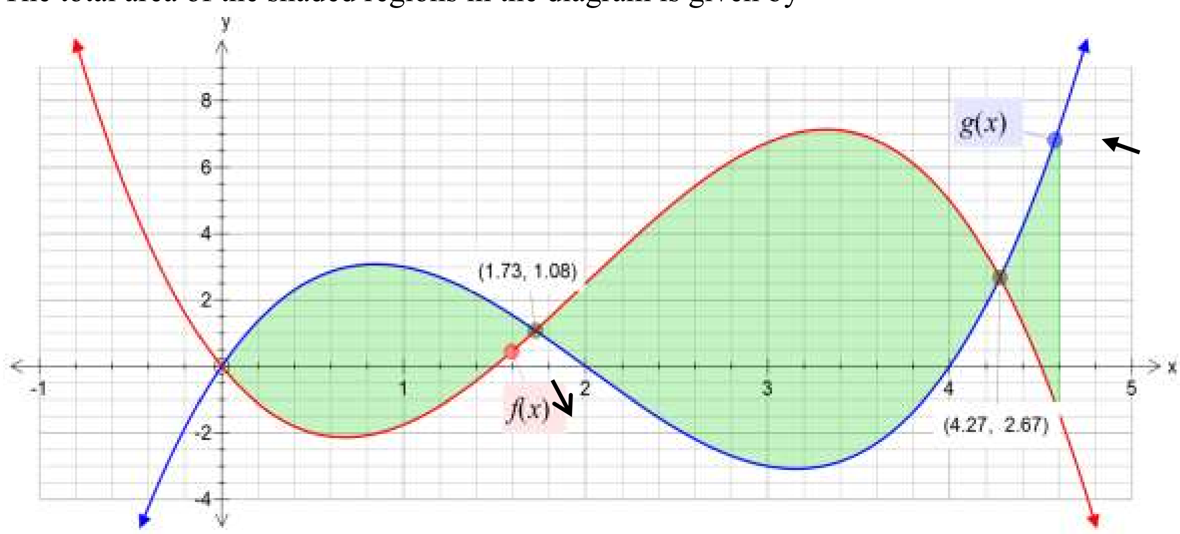
C. $m \geq -1$

D. $m \leq 0$

E. $m \geq 1$

Question 21

The total area of the shaded regions in the diagram is given by



A. $\int_0^{4.27} f(x) - g(x)\,dx$

B. $-\int_0^{4.27} g(x) - f(x)dx$

C. $\int_0^{1.73} g(x) - f(x)dx + \int_{1.73}^{4.27} f(x) - g(x)dx + \int_{4.27}^{4.6} g(x) - f(x)dx$

D. $\int_0^{1.73} f(x) - g(x)dx + \int_{1.73}^{4.27} g(x) - f(x)dx + \int_{4.27}^{4.6} f(x) - g(x)dx$

E. $-\left(\int_0^{1.73} g(x) - f(x)dx + \int_{1.73}^{4.27} f(x) - g(x)dx + \int_{4.27}^{4.6} g(x) - f(x)dx\right)$

Question 22

The function g has rule $g(x) = 3\log_e(x - a) + b$ where a and b are real constants. The maximal domain of g is

A. $R\backslash\{a\}$

B. $R\backslash\{b\}$

C. R^+

D. (a, b)

E. (a, ∞)

END OF SECTION 1

TURN OVER

SECTION 2

Instructions for Section 2

Answer **all** questions in the spaces provided.

A decimal approximation will not be accepted if an **exact** answer is required to a question.

In questions where more than one mark is available, appropriate working **must** be shown.

Where an instruction to **use calculus** is stated for a question, you must show an appropriate derivative or anti-derivative.

Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Question 1

For the function $f: R\backslash\{3\} \to R, f(x) = \frac{2}{x-3} - 4$.

a. Use calculus to show that the gradient of the normal to the function $f(x)$ at any point, is $\frac{(x-3)^2}{2}$.

2 marks

b. Find the equation of $g(x)$, the normal to the function $f(x)$, at $x = 1$.

2 marks

SECTION 2 – Question 1 - continued

c. Find the equation of the composite function $g o f$ and state its domain.

3 marks

d. Sketch the graph of $g\big(f(x)\big)$, showing all key features.

2 marks

e. Sketch the graph of $h(x) = g\big(f(x)\big) + g(x)$ on the axes above, showing all key features.

2 marks

f. If the graph of $y = f(x)$ is transformed to give $y = -2f(2x - 4) + 1$, describe the transformations applied to $f(x)$ and find the equation of the transformed graph.

4 marks

Total 15 marks

SECTION 2 - continued

TURN OVER

Question 2

Jealeen and Chirs have clients who buy coloured light globes. The lifespan of these globes is binomially distributed. A sample of n lightglobes was tested and the expected number of lightglobes that last over 200 hours is 212.5 with a variance of 31.875.

a. Find n the number of lightglobes tested and p, the probability that a lightglobe will last over 200 hours.

3 marks

Jealeen and Chris trial a new brand of coloured lightglobe that has a probability of lasting over 200 hours of 0.9.

b. Find the probability that if 5 of the new brands lightglobes are tested, at least 3 will last longer than 200 hours. Answer to 3 decimal places.

1 mark

c. How many lightglobes would need to be tested to ensure that the probability that at least one of them lasts less than 200 hours is more than 0.95?

3 marks

A third brand of coloured lightglobes claims the lifespan of their globes is normally distributed with a mean of 196 hours and standard deviation of 4.2 hours.

d. Find the probability that one of the new lightglobes randomly selected will last at least 200 hours writing your answer correct to 2 decimal places.

1 mark

e. If the probability that one of these lightglobes lasts more than *m* hours is 0.42, find the value of *m* to 2 decimal places.

2 marks

f. Chris realized that some of his customers prefer the coloured light globes that he bought from company B. The choices that a customer has to make between coloured light globes and candles are independent of previous choices that they have made. The probability that his customers will buy candles from company B is 0.6.
Chris wants to ensure that the probability that his customers will buy coloured light globes on at least two days is at least 0.8.f.Calculate the minimum number of coloured light globes Chris' customers have to buy in a day to achieve his aim.

3 marks

Jealeen and Chris sell speciality light globes that change colour. The life span of these light globes in hours, *t*, is given by the probability density function:

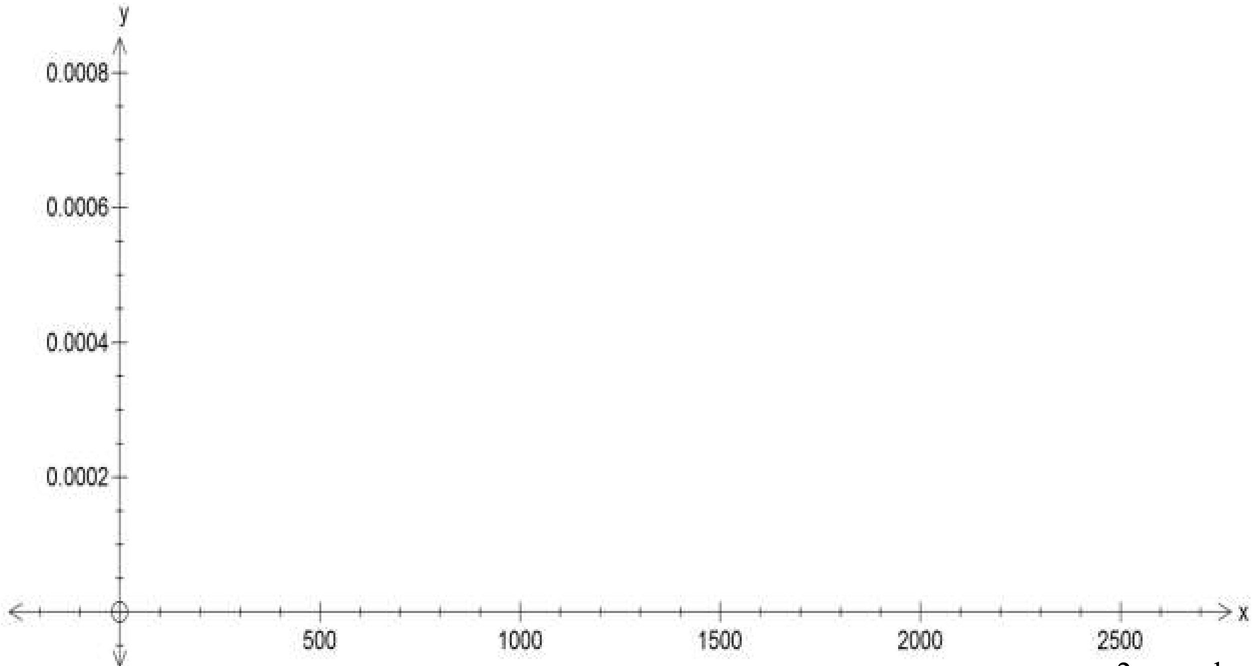
$f(t) = \frac{1}{9.765625\times 10^{12}}(6.25 \times 10^{6}t - t^{3}),\ 0 \leq t \leq 2500$ hours and zero elsewhere.

g. Prove that this is a probability distribution.

2 marks

SECTION 2 – Question 2 - continued

h. Sketch a graph of the probability distribution and label the maximum with its coordinates.



2 marks

i. Calculate the exact value for the average life of a light globe.

2 marks

j. Find the probability that the life of a light globe will last longer than average, given that it has already lasted 1000 hours. Give your answer correct to four decimal places.

2 marks

Total 21 marks

SECTION 2 – continued

TURN OVER

Question 3

The gradient of a curve at any point is given by $f'(x) = (x+1)^2(5-4x)$ for $x \in R$.

a. What is the average value of the gradient function for $-1.5 \leq x \leq 1.5$?

2 marks

b. Show that $f(x) = -x^4 - x^3 + 3x^2 + 5x$ if $f(2) = -2$.

2 marks

c. Find the x −coordinate of the stationary points of f and state the nature of each of these stationary points. Give reasons for your answer.

4 marks

SECTION 2 – Question 3 - continued

d. Find the area between f and the x −axis from $-1 \leq x \leq 2$, correct to two decimal places.

2 marks

Total 10 marks

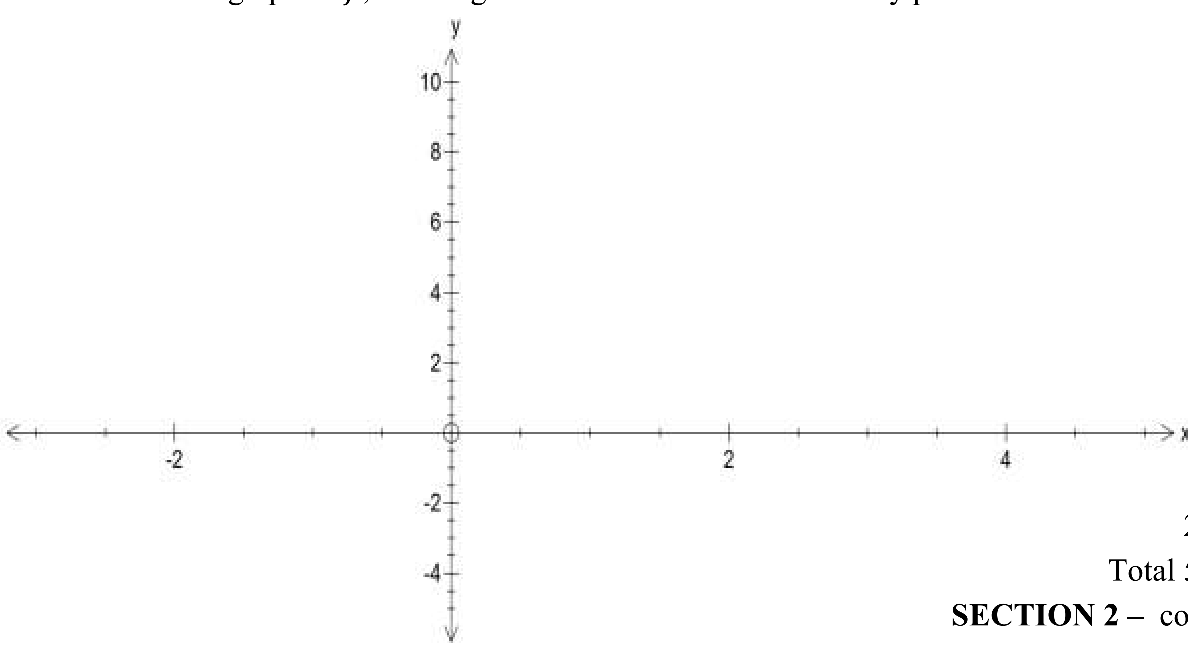
Question 4

In the following f is the function $f: R \rightarrow R,\ f(x) = \begin{cases} 2x^2 - 3x - 2\,, & x \leq 0 \ \cup \ x \geq \frac{3}{2} \\ -2x^2 + 3x - 2, & 0 < x < \frac{3}{2} \end{cases}.$

a. Show that the x −intercepts of $f(x)$ are $x = -\frac{1}{2}$ or $x = 2$.

3 marks

b. Sketch the graph of f, showing the coordinates of the stationary point.



2 marks

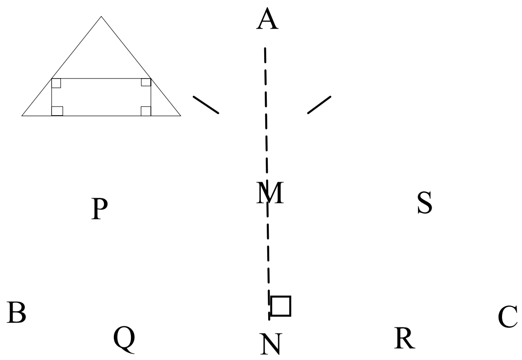
Total 5 marks

SECTION 2 – continued

TURN OVER

Question 5

An isosceles triangle ABC has a height of 200 mm and its base is 200 mm. Rectangle PQRS is an inscribed rectangle with P on AB and S on AC as shown in the diagram below.



a. Show that the area of PQRS is $400x - 4x^2$ if $\text{NR} = x$.

4 marks

b. Find the value for x that will maximise the area.

2 marks

SECTION 2 – Question 5 - continued

c. Find the maximum area of the rectangle PQRS.

1 mark

Total 7 marks

END OF QUESTION AND ANSWER BOOK

MULTIPLE CHOICE ANSWER SHEET

Student Name:___________________________________

Circle the letter that corresponds to each correct answer.

Question					
1	A	B	C	D	E
2	A	B	C	D	E
3	A	B	C	D	E
4	A	B	C	D	E
5	A	B	C	D	E
6	A	B	C	D	E
7	A	B	C	D	E
8	A	B	C	D	E
9	A	B	C	D	E
10	A	B	C	D	E
11	A	B	C	D	E
12	A	B	C	D	E
13	A	B	C	D	E
14	A	B	C	D	E
15	A	B	C	D	E
16	A	B	C	D	E
17	A	B	C	D	E
18	A	B	C	D	E
19	A	B	C	D	E
20	A	B	C	D	E
21	A	B	C	D	E
22	A	B	C	D	E