MATHEMATICAL METHODS

Units 3 & 4 – Written examination 1

(**TSSM's** ***2012 trial exam updated for the current study design)***

SOLUTIONS

Question 1

a. $f(x^4+2) = log_e(x^4+2+3) = log_e(x^4+5)$

M1+A1
2 marks

b. $f'(g(x)) = \frac{1}{(x^4+5)} \times 4x^3 = \frac{4x^3}{(x^4+5)}$

M1+A1
2 marks

c. $f'\left(g(-2)\right) = \frac{4(-2)^3}{((-2)^4+5)} = \frac{-32}{21}$

A1
1 mark

Question 2

a. $\left(\frac{5x^2}{2} - 10x\right)_0^a = 0$

$$\frac{5a^2}{2} - 10a = 0$$
$$5a(a-4) = 0$$
$$a = 0 \; or \; a = 4$$

Since $a \neq 0\,,\; a = 4$

M1+A1
2 marks

b. $\int_0^{\pi} \cos\left(\frac{x}{2}\right) dx = 2sin\left(\frac{x}{2}\right)_0^{\pi} = 2$

M1+A1
2 marks

Question 3

a. $x = -4e^{\frac{y}{2}} + 1$

$\frac{x-1}{-4} = e^{\frac{y}{2}}$

$\frac{y}{2} = log_e(\frac{1-x}{4})$

$f^{-1}(x) = 2log_e(\frac{1-x}{4})$

M2+A1
3 marks

b. $1 - x > 0$

$Domain\ of\ f^{-1}(x)\ is\ (-\infty, 1))$

M1
1 mark

c. $2log_e(\frac{1-x}{4}) = 0$

$(\frac{1-x}{4}) = 1$

$\frac{1-x}{4} = 1$

x = -3

M1+A1
2 marks

Question 4

a. $3\cos(2x) = -\frac{3\sqrt{3}}{2}$

$\cos(2x) = -\frac{\sqrt{3}}{2} \quad for \quad -\pi \le 2x \le \pi$

$2x = \frac{5\pi}{6}, -\frac{5\pi}{6}$

$x = \frac{5\pi}{12}, -\frac{5\pi}{12}$

M1+A1
2 marks

b. $-3\sin(2x) \times 2 = 0$

$2x = 0, \pi, -\pi$

$x = 0, \frac{\pi}{2}, -\frac{\pi}{2}$

M1+A1
2 marks

c. $(0, \frac{\pi}{2})$

M1
1 mark

Question5

a. $Pr\left(\hat{p} = \frac{1}{4}\right) = Pr(1\ blue\ ball)$

$Pr(1\ blue) = \frac{3}{10} \times \frac{7}{9} \times \frac{6}{8} \times \frac{5}{7} + \frac{7}{10} \times \frac{3}{9} \times \frac{6}{8} \times \frac{5}{7} + \frac{7}{10} \times \frac{6}{9} \times \frac{3}{8} \times \frac{5}{7} + \frac{7}{10} \times \frac{6}{9} \times \frac{5}{8} \times \frac{3}{7}$
$= \frac{1}{8} + \frac{1}{8} + \frac{1}{8} + \frac{1}{8}$
$= \frac{1}{2}$

M1+A1
2 marks

b. $\Pr\left(\hat{p} \geq \frac{1}{4}\right) = 1 - \Pr\left(\hat{p} < \frac{1}{4}\right) = 1 - \Pr\ (no\ blue\ balls)$
$\Pr(no\ blue) = \frac{7}{10} \times \frac{6}{9} \times \frac{5}{8} \times \frac{4}{7} = \frac{1}{6}$
$\Pr\left(\hat{p} \geq \frac{1}{4}\right) = 1 - \frac{1}{6} = \frac{5}{6}$

M1+A1
2 marks

Question 6

$f(1) = e^2$
$(1, e^2)$
$f'(x) = 2e^{2x}$
$f'(1) = 2e^2$
$y - e^2 = 2e^2(x - 1)$
$y = 2e^2x - e^2$

M2+A1
3 marks

Question 7

$\sin(2x) = \cos(2x)$
$\tan(2x) = 1$
$2x = \frac{\pi}{4}, \frac{5\pi}{4} \ldots..$
$2x = k\pi + \frac{\pi}{4}, k \in Z$
$x = \frac{1}{2}(k\pi + \frac{\pi}{4})$
$x = \frac{\pi}{8}(4k + 1), k \in Z$

M2+A1
3 marks

Question 8

a. $\int_1^4 k(-x^2 + 5x - 4)dx = 1$

$$k\left(-\frac{x^3}{3} + \frac{5x^2}{2} - 4x\right)_1^4 = 1$$

which gives $k = \frac{2}{9}$

M1+A2
3 marks

b. $\Pr(X > 3) = \frac{2}{9}\int_3^4(-x^2 + 5x - 4)dx$

$$= \frac{2}{9}\left(\frac{-64}{3} + 40 - 16 + \frac{27}{3} - \frac{45}{2} + 12\right)$$

$$= \frac{7}{27}$$

A2
2 marks

Question 9

a. $m = 200$ (95% means 2 standard deviations from the mean)

M1
1 mark

b. $\Pr(X < 210) = \Pr\left(Z < \frac{210-220}{10}\right)$

$= \Pr(Z < -1) = \Pr(Z > 1) = 0.16$

M1+A1
2 marks

c. $\Pr(X > 230|X > 220) = \frac{\Pr(X>230)}{\Pr(X>220)} = \frac{\Pr(Z>1)}{\Pr(Z>0)} = \frac{0.16}{0.5} = \frac{16}{50} = \frac{8}{25}$ or 0.32

M1+A1
2 marks