Presbyterian Ladies' College

MELBOURNE

VCE
Year 11
2016 Semester 1
Mathematical Methods Unit 1 Examination

PAPER 2

Teacher: Bell, Lewis, Smith, I Taylor, J Taylor (Please circle)

Full Name: ____ **Form:** __________

WRITING TIME: (80 minutes)

Section	Number of Questions	Number of Questions to be answered	Number of Marks
B	15	15	15
C	4	4	57

No. of Pages: 15

Instructions

1. Calculators can be used in both sections of this paper.

2. You are permitted to refer to 8 sides of notes, whilst completing this paper.

3. Whiteout is not permitted.

SECTION B: Multiple Choice: Use the answer sheet provided.

SECTION C: Analysis: Answer the questions in the spaces provided.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

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Section B: Multiple Choice **Calculators and notes allowed**

THESE QUESTIONS MUST BE ANSWERED ON THE ANSWER SHEET PROVIDED

1. The maximal domain of the function f with rule $f(x) = \sqrt{5x - 7}$ is:

A $(0, \infty)$

B $(-\frac{7}{5}, \infty)$

C $(-7, \infty)$

D $[-3, \infty)$

E $[\frac{7}{5}, \infty)$

2. The discriminant of $x^2 + x - 10$ is:

A 42

B –39

C 41

D 65

E 60

3. When $2x^3 - x^2$ is divided by $2x - 1$ the remainder is:

A -2

B 2

C $\frac{1}{2}$

D $-\frac{1}{2}$

E 0

4. If P has coordinates $\left(a, \frac{a}{2}\right)$ and Q has coordinates $\left(-a, \frac{a}{3}\right)$, the coordinates of the midpoint of PQ are:

A $\left(a, \frac{a}{5}\right)$

B $\left(0, \frac{5a}{12}\right)$

C $\left(0, \frac{a}{12}\right)$

D $\left(0, \frac{a}{2}\right)$

E $\left(a, \frac{5a}{12}\right)$

5. For f: $(a, b] \rightarrow R$, $f(x) = 5 - x$, where $a < b$, the range is:

A $(5 - a, 5 - b)$

B $(5 - a, 5 - b]$

C $(5 - b, 5 - a)$

D $(5 - b, 5 - a]$

E $[5 - b, 5 - a)$

6. If events X and Y are independent then:

A $\Pr(X \mid Y) = \Pr(X \cup Y) - \Pr(X) - \Pr(Y)$

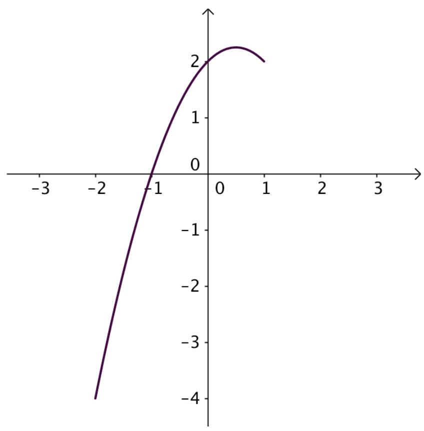
B $\Pr(X \mid Y) = \dfrac{\Pr(X)}{\Pr(Y)}$

C $\Pr(X \mid Y) = \Pr(X) + \Pr(Y) - \Pr(X \cap Y)$

D $\Pr(X \mid Y) = \Pr(Y)$

E $\Pr(X \mid Y) = \Pr(X)$

7. Is the graph to the right representing:



A one to one function

B many to one function

C relation not a function

D function not relation

E none of the above

8. For two events A and B, $\Pr(B') = 0.4$, $\Pr(A) = 0.3$ and $\Pr(A \cup B) = 0.1$, then $\Pr(A \cap B)$ is equal to:

A 0.12

B 0.18

C 0.9

D 0.8

E 0

9. If $5x^2 - t = 4x$, and x and t are both positive real numbers, then x is equal to:

A t

B $\frac{2}{5} + \frac{\sqrt{4+5t}}{5}$

C $-\frac{1}{5}$ or 1

D 1

E $\frac{2}{5} \pm \frac{\sqrt{4+5t}}{5}$

10. The function $f(x) = x^2$ is dilated by a factor $\frac{1}{2}$ from the y-axis, translated 2 units in the positive direction of the x-axis and translated one unit in the negative direction of the y-axis. The equation of the transformed function is:

A $y = \frac{1}{2}(x-2)^2 - 1$

B $y = \frac{1}{2}(x+2)^2 - 1$

C $y = 4(x+2)^2 - 1$

D $y = 4(x-2)^2 - 1$

E $y = 2(x-2)^2 - 1$

11. The line with equation $y = ax + b$ is perpendicular to the line with equation $y = 1 - \frac{x}{3}$.
The value of a is:

A -3

B $\frac{1}{3}$

C 3

D $-\frac{1}{3}$

E $-\frac{1}{4}$

12. The equation of the parabola that passes through the point (0, 13) and has its vertex at (3, −5) is:

A $y = -2(x-3)^2 - 5$

B $y = (x+3)^2 - 5$

C $y = (x-3)^2 - 5$

D $y = -(x-3)^2 - 5$

E $y = 2x^2 - 12x + 13$

13. If for two events A and B, $\Pr(A) = \frac{1}{6}$ and $\Pr(B) = \frac{1}{3}$, and A and B are mutually exclusive, then $\Pr(A \cup B)$ is equal to:

A 0

B $\frac{1}{3}$

C $\frac{1}{2}$

D $\frac{1}{18}$

E $\frac{2}{9}$

14. A possible rule of the graph shown is:

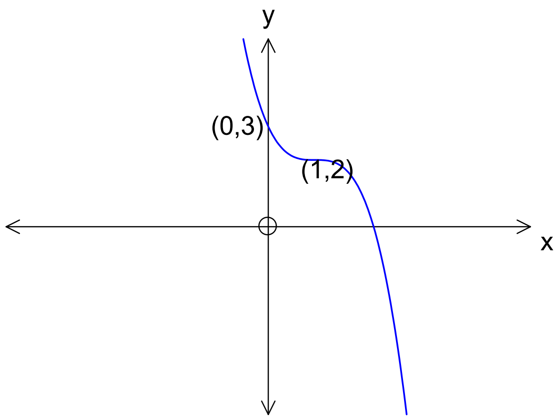
A $y = 2(1-x)^3 + 2$

B $y = (1-x)^3 + 2$

C $y = (x-1)^3 + 2$

D $y = (1-x)^3 - 1$

E $y = 2(x-1)^3 + 2$



15. A **biased** die when tossed has six outcomes where
$\Pr(1) = \Pr(2) = \Pr(3) = 0.1$, $\Pr(4) = \Pr(5) = 0.2$ and $\Pr(6) = 0.3$.
If the die is tossed twice, find the probability that both numbers are the same.

A 0.02

B 0.2

C 0.002

D $\frac{1}{6}$

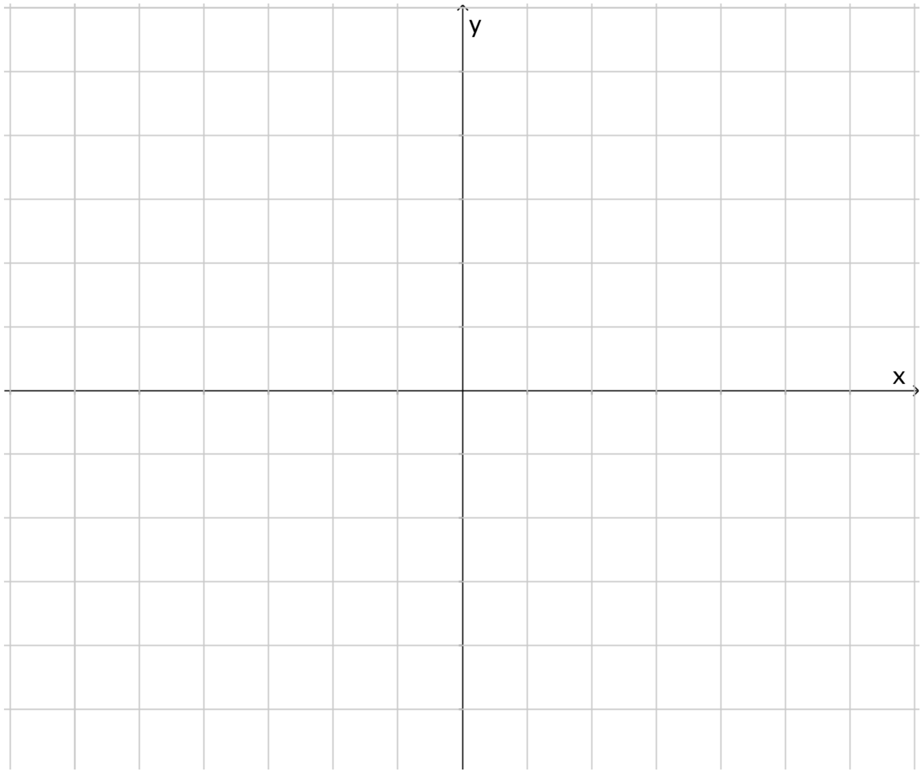
E $\frac{3}{5}$

End of Section B

SECTION C: Analysis
Calculators and notes allowed. Exact answers should be given unless instructed otherwise.

Question 1 (17 marks)
A triangle *ABC* has vertices with coordinates $A(1, 3)$, $B(-3, 1)$ and $C(5, -2)$.

a Construct the triangle *ABC* on the axes below clearly labelling coordinates of the vertices.



(2 marks)

b Find the equation of the line connecting the vertices *A* and *C*.

(3 marks)

c Using the midpoint formula, find the midpoint *M* of the line *AC*.

(2 marks)

d Find the equation of the line perpendicular to AC which goes through the midpoint M.

(3 marks)

e Find the coordinates of the y-intercept for the line found in **d** and sketch this line on the axes in part **a**.

(2 marks)

f Find the exact coordinates of the point of intersection of the line sketched in part **d** and the line BC.

(3 marks)

g Using the distance formula to find the length of the side AB (answer to two decimal places).

(2 marks)

Work Space Page

Examination Continues Next Page
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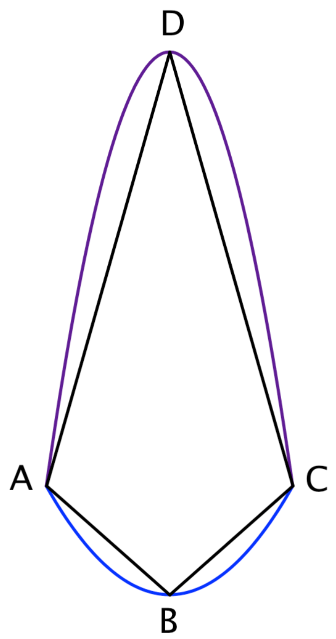
Question 2 (15 marks)

4. An organisation ran a competition for a new logo. Lara entered the logo shown in this competition. She used a set of coordinate axes to plan her design.

The curve ABC is a parabola with equation $y = \frac{3}{4}x^2 - 6x + 12$.

B is the **turning point** of the curve.



a Show, by completing a square or otherwise, that the coordinates of B are (4, 0)

(2 marks)

The curve ADC is a parabola with equation $y = -3(x-b)^2 + c$.

b The turning point is when $x = 4$. State the value of b.

(1 mark)

c Points A and C are (2, 3) and (6, 3) respectively. Find the value of c for the parabola.

(2 marks)

d Hence state the coordinates of D.

(1 mark)

e Write the appropriate domain Lara needed for each of the curves in order to draw her logo design.

(2 marks)

The points A, B, C and D are joined with straight lines as shown in the diagram.

f Find the angle between AB and BC. Give the answer correct to the nearest degree.

(4 marks)

g The quadrilateral ABCD is to be made of yellow plastic. What area of plastic is needed?

(2 marks)

h Give the most precise mathematical name for the quadrilateral ABCD.

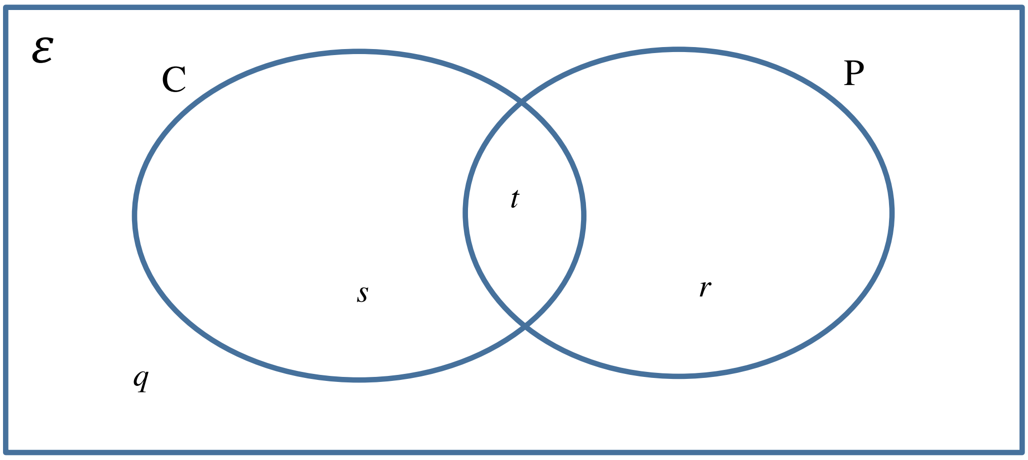
(1 mark)

Question 3 (14 marks)

a A drawer contains 8 red socks and 4 blue socks. A sock is drawn without looking and then another one is drawn without looking. Find the probability that a matching pair is drawn.

(3 marks)

b In a group of 115 VCE students, 51 study Chemistry, 73 study Physics and 19 study neither. Use this information to find *s, t, r* and *q* in the following Venn diagram.



(4 marks)

c If a fair die is rolled twice

i find the probability that the first roll is an even number, and the second roll is odd.

(1 mark)

ii find the probability that one roll is a two, and the other roll is not a two

(2 marks)

iii find the probability that the sum of the rolls is nine.

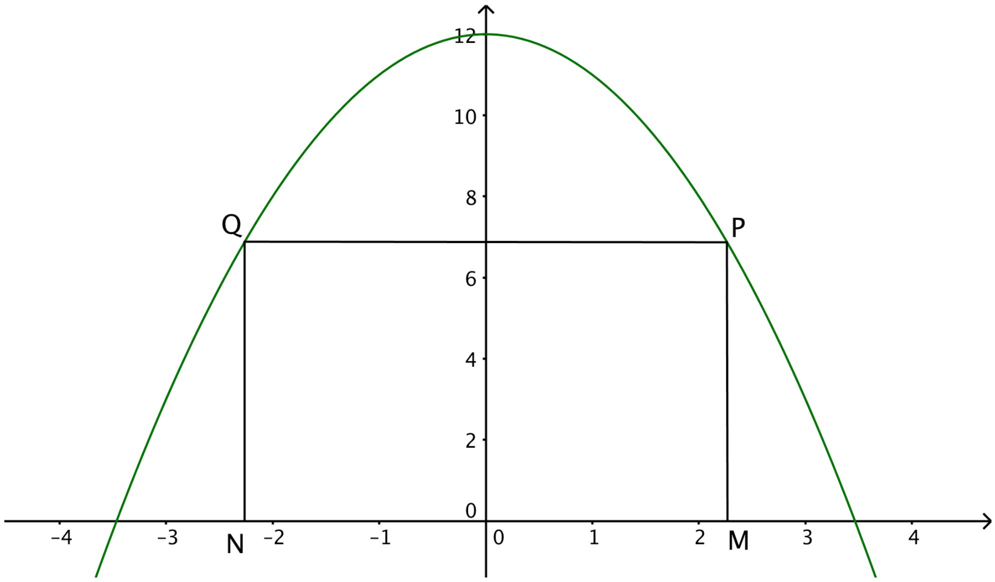
(2 marks)

iv find the probability that, given that the sum of the rolls is nine, one of the rolls is a three.

(2 marks)

Question 4 (11 marks)

A rectangle NMPQ drawn such that NM lies on the x-axis and vertices P and Q are points on the curve $y = 12 - x^2$. This is shown in the diagram below. All units are in centimetres.



The length of NM is 2x centimetres

a Given M = (x, 0) find the length of MP in terms of x.

(1 mark)

b If the area of the rectangle is A cm^2 show that $A = 24x - 2x^3$

(1 mark)

c **i** Find algebraically the exact coordinates, in the simplest form, of the x-intercepts for $y = 12 - x^2$.

(2 marks)

ii Hence state a reasonable domain for the area function found in part **b**.

(1 mark)

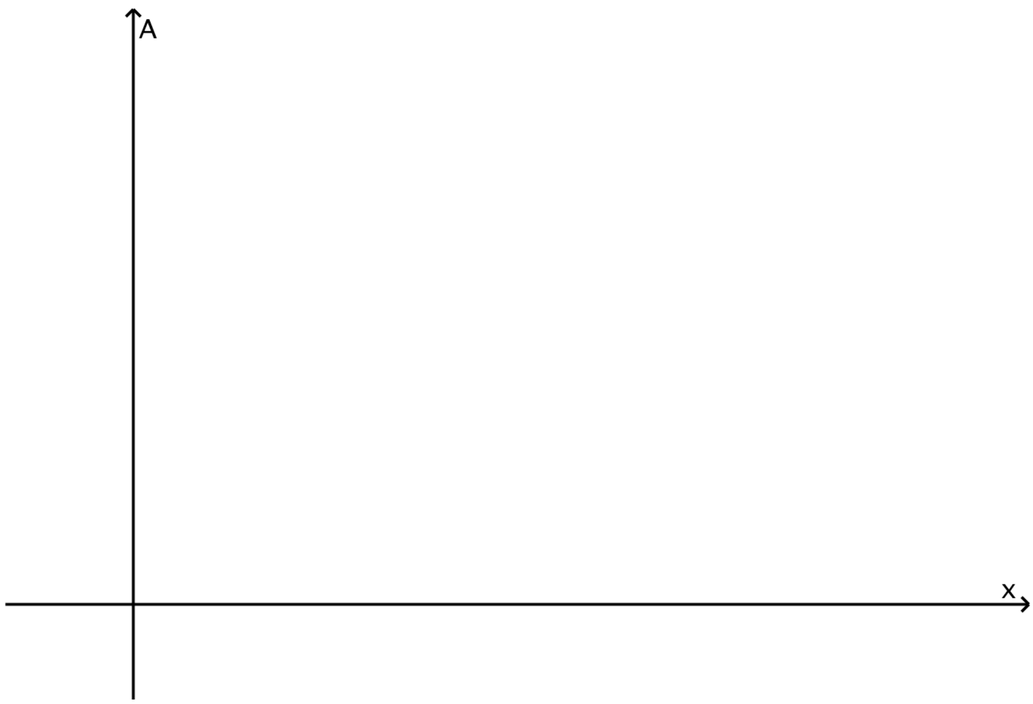
d **i** Using your calculator, find the maximum possible area of the rectangle.

(1 mark)

ii Find the length and width of the rectangle when it has the maximum area.

(2 marks)

e Sketch a graph of A against x over a suitable domain. Label the coordinates of the axes intercepts and turning point. Any approximate values should be given to two decimal places.



(3 marks)

End of Section C

End of the Examination