VCE Mathematical Methods – Unit 4 – 2016

Probability SAC : Part B
Technology Active

Student Name

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Writing Time: 75 minutes writing

QUESTION AND ANSWER BOOKLET

Structure of book

Number of Tasks	Total Marks
Short Answer: 2	**38**

Time Allocated: 75 minutes writing

Materials

Question and answer booklet. Pens, pencils, erasers.
Students are permitted to bring one bound reference and calculators.

All written responses should be in English.
All answers to be exact values, unless otherwise stated.
Appropriate working must be shown to achieve full marks on questions worth more than one mark.

Question 1

On October 12th 2017, news broke that Lebron James had quit professional basketball! He knew he was the best player in the world and had nothing left to prove. So he decided to follow his life-long dream of becoming an anaesthetist. During his medical training he found that the amount of the anaesthetic *Sleepwell (SW*) required to achieve surgical anaesthesia in an 85 kg adult male is a normally distributed random variable with a mean of 60 mg and a standard deviation of 11 mg.

a) Find, correct to four decimal places, the probability that an 85 kg adult male will require an amount of *SW*:

i. between 43 mg and 64 mg for surgical anaesthesia.

(2 marks)

ii. less than 55 mg for surgical anaesthesia.

(2 marks)

An 85 kg adult male is considered *SW-sensitive* if the amount of anaesthetic required to achieve surgical anaesthesia is less than 35 mg, *SW-resistant* if the amount is greater than *a* mg, and *SW-typical* otherwise. The probability of a patient being *SW-resistant* is equal to 0.18 .

b) **i.** Find, correct to the nearest mg, the value of *a*.

(2 marks)

ii. Find, correct to four decimal places, the probability that an 85 kg adult male patient who is *SW-sensitive* requires more than 30 mg to achieve surgical anaesthesia.

(3 marks)

c) Find, correct to four decimal places, the probability that in a random group of sixteen 85 kg adult males:

i. exactly six of the adults will require between 46 mg and 58 mg of *SW* to achieve surgical anaesthesia.

(3 marks)

ii. more than ten of the adults are *SW-resistant*.

(2 marks)

iii. at most 1 adult is *SW-sensitive.*

(2 marks)

The amount of *SW* required to achieve surgical anaesthesia in a 25 kg child is a normally distributed random variable with a standard deviation of 6 mg. The probability of requiring less than 24 mg is 0.28.

e) Find, correct to two decimal places, the mean amount of *SW* required to achieve surgical anaesthesia in a 25 kg child.

(3 marks)

Question 2

Lebron James other dream is to continue to make his line of basketball shoes for his fans. Lebron buys the shoe boxes he uses for his shoes from a local manufacturer.

It is known that, in the process of manufacturing, 4% of the shoe boxes will be rejected because of a faulty lid, independent of each other. After manufacturing, the shoe boxes are packed in containers of 200.

a) If a container of shoe boxes is randomly selected, find the probability (to 4 decimal places) that no more than 6 shoe boxes are faulty in the container.

(2 marks)

Kobe Bryant was beginning to become bored in his retirement, so he decided he wanted to work with his long-time friend, Lebron James. Kobe became a quality control officer for the local shoe box manufacturer. If he selects 15 containers at random to inspect.

b) Find the probability (to 4 decimal places) that at least 10 out of fifteen containers have no more than 6 faulty shoe boxes per container.

(2 marks)

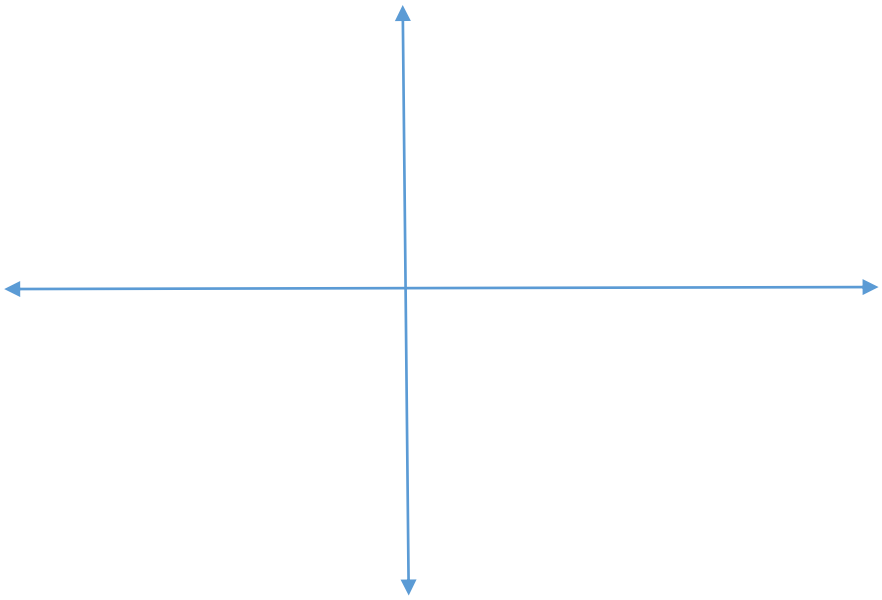
The cost of producing the shoe boxes ($*x*) is a continuous random variable X with a probability density function (pdf) given by

$$C(x) = \begin{cases} \frac{6}{5}\left(x^2 - x\right) & 1 \le x \le 2 \\ 0 & \textit{elsewhere} \end{cases}$$

c) Show that $C(x)$ is a valid probability density function.

(1 mark)

d) Sketch the graph of $y = C(x)$.



(2 marks)

e) Find the probability (correct to 4 decimal places) that the cost of production of the shoe boxes will be less than $1.40.

(1 mark)

f) Find the median cost. Give your answer correct to 3 decimal places.

(2 marks)

g) Use **calculus** to show that the mean cost is 1.7

(2 marks)

The weights *W* (grams) of shoe boxes are normally distributed with a mean of 48 g. It is known that 3.6% of the shoe boxes are over 57 g.

h) Find the standard deviation, correct to 2 decimal places.

(2 marks)

Question 3

Jason, a friend of Michael and a leading NBL player, runs a Basketball Clinics for 6-12 years old boys. The trainings often go over time and parents of the boys need to wait in their cars.
The waiting time is a continuous random variable Y, with a probability density function $y(t)$ defined below:

$$y(t) = \begin{cases} k\,t\,e^{-\frac{t}{4}}, & t > 0 \\ 0, & elsewhere \end{cases}$$

where t is the time in minutes.

(a) Find the value of k.

(1 mark)

(b) Find the probability, to 3 decimal places, that a parent will wait longer than 10 minutes.

(1 mark)

(c) Find the mean waiting time, correct to the nearest minute.

(2 marks)

(d) Find the median waiting time, correct to the nearest second.

(2 marks)