Year 12 *Trial Exam Paper*

2016

MATHEMATICAL METHODS

Written examination 2

Reading time: 15 minutes
Writing time: 2 hours

STUDENT NAME:

QUESTION AND ANSWER BOOK

Structure of book

Number of questions	*Number of questions to be answered*	*Number of marks*
20	20	20
5	5	60
		Total 80

- Students are permitted to bring the following items into the examination: pens, pencils, highlighters, erasers, sharpeners, rulers, a protractor, set squares, aids for curve sketching, one bound reference, one approved technology (calculator or software) and, if desired, one scientific calculator. Calculator memory DOES NOT need to be cleared. For approved computer-based CAS, full functionality may be used.
- Students are NOT permitted to bring blank sheets of paper and/or correction liquid/tape into the examination.

Materials provided

- Question and answer book of 23 pages.
- Formula sheet.
- Answer sheet for multiple-choice questions.

Instructions

- Write your **name** in the box provided above on this page, and on your answer sheet for multiple-choice questions.
- Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.
- You must answer the questions in English.

Students are NOT permitted to bring mobile phones or any other unauthorised electronic devices into the examination.

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SECTION A – Multiple-choice questions

Instructions for Section A

Answer **all** questions in pencil on the multiple-choice answer sheet.

Select the response that is **correct** for the question.

A correct answer scores 1 mark, whereas an incorrect answer scores 0.

Marks will **not** be deducted for incorrect answers.

If more than one answer is selected, no marks will be awarded.

Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Question 1

The point P (–3, 2) lies on the graph of the function f. The graph of the function f is reflected in the x-axis and then translated 4 units vertically up.

The coordinates of the final image of P are

A. $(3, 6)$

B. $(-1, 2)$

C. $(-3, 2)$

D. $(-3, 6)$

E. $(-3, -6)$

Question 2

The linear function $f : D \to R,\ f(x) = 2 - 2x$ has range $[0, 6)$.

The domain D of the function is

A. $(-10, 2]$

B. $(-2, 1]$

C. $[-2, 1]$

D. $[-2, 1)$

E. $(-1, 2)$

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Question 3

The area of the region enclosed by the graph of $y = x(x+1)(x-3)$ and the x-axis is

A. $\frac{-32}{3}$ square units

B. $\frac{32}{3}$ square units

C. $\frac{7}{12}$ square units

D. $\frac{71}{6}$ square units

E. $\frac{45}{4}$ square units

Question 4

Let f be a function with domain R such that $f'(4) = 0$ and $f'(x) > 0$ when $x \neq 4$.

At $x = 4$, the graph of f has a

A. local maximum.

B. local minimum.

C. gradient of 4.

D. gradient of $\frac{1}{4}$.

E. stationary point of inflection.

Question 5

The random variable X has a normal distribution with mean 10 and variance of 4.

If Z has a standard normal distribution, then the probability that X is less than 8 is equal to

A. $\Pr(Z > -1)$

B. $\Pr(Z < -0.5)$

C. $\Pr(Z > 1)$

D. $\Pr(Z > 0.5)$

E. $\Pr(Z < 1)$

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Question 6

The function $f: D \to R$ with rule $f(x) = x^2 e^x$ will have an inverse function for

A. $D = R$

B. $D = R^-$

C. $D = [-2, \infty)$

D. $D = (-\infty, -2)$

E. $D = (-2, \infty)$

Question 7

If $\int_1^5 f(x)dx = 4$, then $\int_1^5 (3 - 2f(x))dx$ is equal to

A. 7

B. –6

C. –20

D. 4

E. –5

Question 8

The inverse of $f: R^+ \to R,\ f(x) = \frac{1}{\sqrt{x}} + 2$ is

A. $f^{-1}: (2, \infty) \to R, \quad f^{-1}(x) = \frac{1}{(x-2)^2}$

B. $f^{-1}: R^+ \to R, \quad f^{-1}(x) = \frac{1}{x^2} + 2$

C. $f^{-1}: R^+ \to R, \quad f^{-1}(x) = (x+2)^2$

D. $f^{-1}: (-2, \infty) \to R, \quad f^{-1}(x) = \frac{1}{(x+2)^2}$

E. $f^{-1}: (-\infty, 2) \to R, \quad f^{-1}(x) = \frac{1}{(x-2)^2}$

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Question 9

Which one of the following satisfies the functional equation $f(f(x)) = x$ for every value of x?

A. $f(x) = 4\sqrt{x}$

B. $f(x) = 4x^2$

C. $f(x) = 4 - x$

D. $f(x) = 4 + x$

E. $f(x) = 4x$

Question 10

For events A and B, $\Pr(A \cap B) = k$, $\Pr(A' \cap B) = \dfrac{2k}{5}$ and $\Pr(A \cap B') = 3k - 1$.

If A and B are independent, then the value of k is

A. 0

B. $\dfrac{1}{2}$

C. $\dfrac{3}{7}$

D. $\dfrac{5}{13}$

E. $\dfrac{3}{8}$

Question 11

The transformation $T: R^2 \to R^2$ with the rule

$$T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} -2 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} 2 \\ -1 \end{bmatrix}$$

maps the line with equation $2x - y = 5$ onto the line with equation

A. $x - y = 4$

B. $x + y = -4$

C. $4x + y = 0$

D. $-4x - y = 5$

E. $-x + y = -4$

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Question 12

The domain of the function *h*, where $h(x) = \sin(\log_e(x))$ is chosen such that *h* is a one-to-one function.

Which one of the following could be the domain?

A. $\left[0, e^{\frac{\pi}{2}}\right]$

B. $\left[e^{-\frac{\pi}{2}}, e^{\frac{\pi}{2}}\right]$

C. $\left[e^{-\pi}, e^{\pi}\right]$

D. $\left[-e^{\pi}, e^{\pi}\right]$

E. $\left[-e^{\frac{\pi}{2}}, e^{\frac{\pi}{2}}\right]$

Question 13

If X is a continuous random variable such that $\Pr(X > 8) = b$ and $\Pr(X > 10) = a$, then $\Pr(X < 8 \mid X < 10)$ is

A. $\dfrac{b-1}{a-1}$

B. $\dfrac{b}{a}$

C. $\dfrac{b-1}{1-a}$

D. $\dfrac{b-1}{a}$

E. $\dfrac{b}{a-1}$

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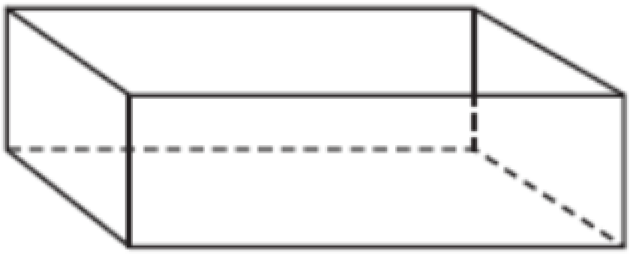
Question 14

Ben has a rectangular piece of cardboard that is 10 cm long and 8 cm wide. Ben cuts 4 identical squares, one from each corner, of side length x centimetres as shown in the diagram below.

Ben turns up the sides to form an open box.



The value of x for which the volume of the box is a maximum is closest to

A. 52.51

B. 52.52

C. 1.47

D. 1.48

E. 1.46

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Question 15

The function with rule $f(x) = -4\tan\left(2\pi x - \frac{\pi}{2}\right)$ has period

A. $\frac{2}{\pi}$

B. 2

C. $\frac{1}{2}$

D. $\frac{1}{4}$

E. 2π

Question 16

The continuous random variable X with probability density function $p(x)$ has mean 5 and variance 3.

The value of $\int_{-\infty}^{\infty} 2x^2\, p(x)\, dx$ is

A. 16

B. 28

C. 68

D. 56

E. 8

Question 17

Let the random variable $\hat{P}$ represent a sample proportion observed in an experiment.

If $p = \frac{1}{6}$, what is the smallest integer value of the sample size such that the standard deviation of $\hat{P}$ is less than or equal to $\frac{1}{216}$?

A. 30

B. 6480

C. 180

D. 1600

E. 16

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Question 18

An exit poll of 1000 voters found that 620 favoured candidate A.

An approximate 90% confidence interval for the proportion of voters from the total population in favour of candidate A is

A. $\left(0.62-\sqrt{\dfrac{0.62\times0.38}{1000}},\ 0.62+\sqrt{\dfrac{0.62\times0.38}{1000}}\right)$

B. $\left(0.62-1.65\sqrt{\dfrac{0.62\times0.38}{1000}},\ 0.62+1.65\sqrt{\dfrac{0.62\times0.38}{1000}}\right)$

C. $\left(0.62-2.58\sqrt{\dfrac{0.62\times0.38}{1000}},\ 0.62+2.58\sqrt{\dfrac{0.62\times0.38}{1000}}\right)$

D. $\left(620-1.96\sqrt{\dfrac{0.62\times0.38}{1000}},\ 620+1.96\sqrt{\dfrac{0.62\times0.38}{1000}}\right)$

E. $\left(0.62-\sqrt{\dfrac{0.62\times0.38}{620}},\ 0.62+\sqrt{\dfrac{0.62\times0.38}{620}}\right)$

Question 19

Assume that $f'(x)=g'(x)$ with $f(1)=-5$ and $g(x)=-x\,f(x)$. Then $f(x)=$

A. $g(x)-10x-10$

B. $g'(x)-10$

C. $g(x)-10x$

D. $g(x)-x$

E. $g(x)-10$

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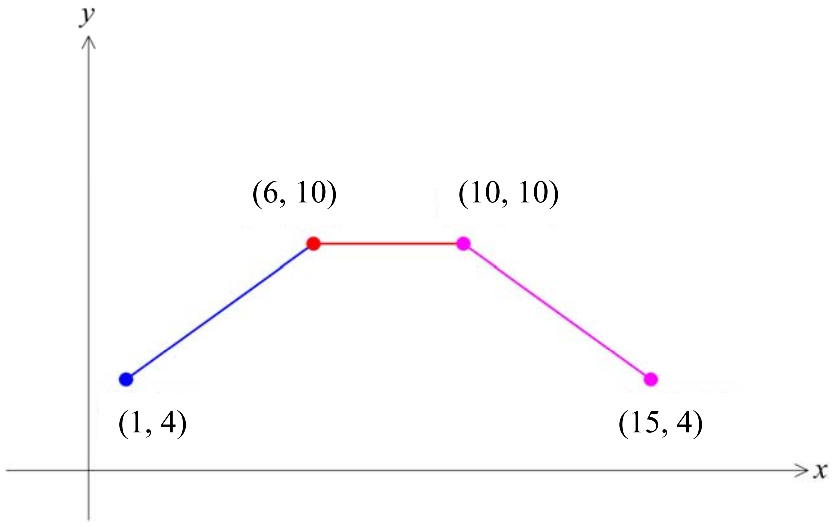
Question 20

The graph of h is shown below.



The average value of h is closest to

A. 9.86

B. 8.86

C. 5.86

D. 6.86

E. 7.86

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END OF SECTION A

SECTION B

Instructions for Section B

Answer **all** questions in the spaces provided.

In all questions where a numerical answer is required, an exact value must be given unless otherwise specified.

For questions where more than one mark is available, appropriate working must be shown.

Unless otherwise stated, diagrams are not drawn to scale.

Question 1 (8 marks)

The depth of snow covering a particular section of the Victorian Alps during a short time in June can be modelled by the equation $d(t) = 87 + 11\sin\left(\frac{\pi t}{6}\right)$, where d is the depth of the snow in centimetres and t is the number of hours from 6 am on 1 June 2015.

a. Find the period and the amplitude of the function *d*.

2 marks

b. Find the maximum and minimum snow depth in this location.

2 marks

c. Find $d(10)$.

1 mark

d. Over the 12 hours from 6 am on 1 June 2015, find the percentage of time when the depth of the snow is more than 5 cm greater than the value of $d(10)$. Give your answer correct to two decimal places.

3 marks

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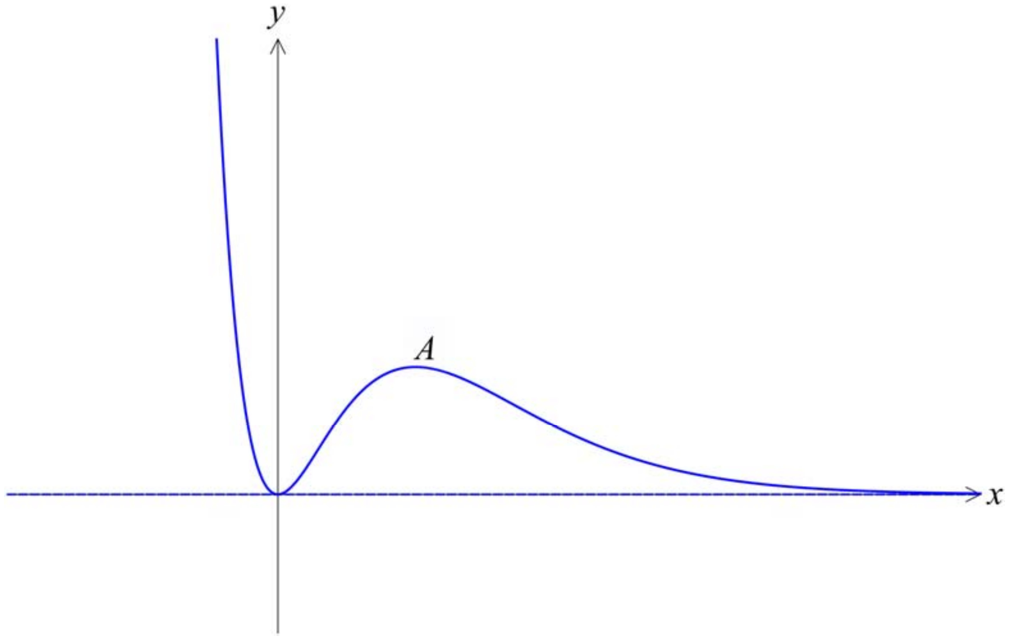
Question 2 (13 marks)

The function f is defined as $f : R \to R$, where $f(x) = x^m e^{(-nx+n)}$ and m and n are positive integers.

The graph of $y = f(x)$ is shown below.



a. Let $m = 2$ and $n = 3$.

i. Find the value of x, for $x < 0$, such that $f(x) = 1$. Give your answer correct to three decimal places.

1 mark

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ii. State the values of x, correct to 3 decimal places, such that $f(x) > 1$.

2 marks

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iii. Find the coordinates of the stationary point marked as A.

1 mark

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b. Find the coordinates of the point on the graph at which the tangent drawn to *f* passes through the origin.

3 marks

c. For $f : R \to R$, where $f(x) = x^m e^{(-nx+n)}$ and *m* and *n* are unknown,

i. Show that the stationary point at *A* occurs at $x = \frac{m}{n}$.

3 marks

ii. Show that the only tangents drawn to curve that pass through the origin are at $x = 0$ and $x = \frac{m-1}{n}$.

3 marks

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Question 3 (18 marks)

Tom is a keen gardener and owns a garden nursery. He specialises in tomato plants and grows two varieties—cherry tomatoes and roma tomatoes.

The height in centimetres of the cherry tomato plants that Tom is selling is normally distributed with a mean of 15 cm and a standard deviation of 4 cm. There are 4000 cherry tomato plants in his nursery.

a. Tom classifies the tallest 15% of his cherry tomato plants as advanced.

What is the minimum height of an advanced tomato plant, correct to the nearest millimetre?

1 mark

Tom thinks that some of his cherry tomato plants are not growing quickly enough and decides to move them to a special greenhouse. He will move the cherry tomato plants that are less than 9 cm in height.

b. How many cherry tomato plants will Tom move to the greenhouse? Give your answer correct to the nearest whole number.

2 marks

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The height, x centimetres, of the roma tomatoes in the nursery follows the probability distribution density function where

$$h(x)=\begin{cases} k\sin\left(\dfrac{\pi(x-6)}{10}\right), & 6\le x\le 16 \\ 0, & \text{otherwise}\end{cases}$$

c. Show that $k=\dfrac{\pi}{20}$.

2 marks

d. State the mean height of a roma tomato plant at the nursery.

1 mark

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Tom thinks that the smallest 10% of the roma plants should be given a fertilizer.

e. Find the maximum height correct to the nearest millimetre of a roma tomato plant that should be given a fertilizer.

2 marks

Tom classifies his roma tomato plants as either standard or tall. He knows that 20% of his roma tomato plants are tall.

A customer, Katie, selects five roma tomato plants. She chooses each plant individually and finds that her decision on which plant to choose depends on her previous choice. If she chooses a tall plant then she has a probability of $\frac{1}{4}$ of choosing a tall plant as her next plant. If she chooses a standard plant then she has a probability of $\frac{2}{3}$ of choosing a standard plant as her next plant. The first one she chooses is tall.

f. What is the probability that of the next four plants he chooses exactly two tall plants?

2 marks

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Tom also has parsley plants in his nursery. He obtains his parsley plants from growers all over Australia. It is known that 30% will be prone to a particular leaf disease.

Tom decides to test his plants for the leaf disease. He takes a random sample of 20 parsley plants.

g. **i.** What is the probability that the sample proportion is equal to the population proportion of 0.3? Give your answer correct to four decimal places. Do not use a normal approximation.

2 marks

ii. What is the probability that the sample proportion lies within two standard deviations of the population proportion? Give your answer correct to four decimal places. Do not use a normal approximation.

3 marks

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While testing for the leaf disease, Tom sees that some of his plants are infested with a leaf moth. He finds that 236 plants out of 500 are infested with the leaf moth.

h. Find an approximate 95% confidence interval for the proportion of plants infested with the leaf moth. Give your answer correct to three decimal places.

1 mark

Another 26 nursery owners from around Australia independently sample parsley plants from their stocks and test for leaf disease. Each calculates an approximate 95% confidence interval for p, the proportion of plants in the population infested with the leaf moth. It is subsequently found that of these 27 confidence intervals exactly one does not contain the value of p.

Researchers investigating the prevalence of leaf moth randomly select five of the confidence intervals calculated by the nursery owners.

i. What is the probability that exactly three of the selected confidence intervals contain the value of p? Give your answer correct to four decimal places.

2 marks

Question 4 (11 marks)

Let $f: R \to R,\ f(x) = -\left(x+2\right)\left(6+x\right)\left(x^2+12\right)$ and $g: R \to R,\ g(x) = 64x - x^4$.

a. Express $64x - x^4$ in the form $x(a-x)\left(\left(x+b\right)^2+c\right)$.

2 marks

b. Describe the transformations that map the graph of $y = g(x)$ onto the graph of $y = f(x)$.

1 mark

c. Find the values of d such that the graph of $y = f(x-d)$ has

i. only negative x-intercepts.

1 mark

ii. at least one positive x-intercept.

1 mark

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d. Find the value of n, correct to two decimal places, for which the equation $g(x) = n$ has one solution.

1 mark

Let $h(x) = kx - x^4$

e. Show that for $n = 3\left(\frac{k}{4}\right)^{\frac{4}{3}}$, the equation $h(x) = n$ has exactly one solution.

3 marks

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f. **i.** Find the area bounded by the graph of $y = g(x)$ and the x-axis.

1 mark

ii. Hence find the area bounded by the x-axis and the graph of $y = f\left(\frac{-x}{2}\right)$.

1 mark

SECTION B – continued
TURN OVER

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Question 5 (10 marks)

A soft drink company manufactures soft drink in cylindrical cans of volume V cm^3.

The height of the can is h cm and the radius of the can is r cm where $\frac{V^{\frac{1}{3}}}{9} \le r \le \frac{4V^{\frac{1}{3}}}{3}$.

a. Express h in terms of r and V.

1 mark

b. Show that the total surface area A cm^2 of the can is given by $A = \frac{2V}{r} + 2\pi r^2$.

2 marks

c. Find, in terms of V, the value of r so that the surface area of the can is a minimum. You do not need to justify the nature of the stationary point.

2 marks

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d. The minimum surface area can be written in the form $aV^{\frac{2}{3}}(b\pi)^{\frac{1}{3}}$.

State the values of *a* and *b*.

3 marks

e. Find, in terms of *V*, the maximum possible surface area of the can.

2 marks

END OF QUESTION AND ANSWER BOOK

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