

## ***YEAR 12 Trial Exam Paper***

# **2016**

# **MATHEMATICAL METHODS**

## **Written examination 2**

### ***Worked solutions***

**This book presents:**

- worked solutions, giving you a series of points to show you how to work through the questions
- mark allocations
- tips on how to approach the exam

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## SECTION A – Multiple-choice questions

### Question 1

*Answer: C*

#### Explanatory notes

The point undergoes the transformations as such

$$(-3, 2) \rightarrow (-3, -2) \rightarrow (-3, 2)$$



#### Tip

- *Draw a diagram to help.*

### Question 2

*Answer: B*

#### Explanatory notes

Use the range to calculate endpoints

$$\begin{array}{ll} f(x) = 0 & f(x) = 6 \\ 2 - 2x = 0 & 2 - 2x = 6 \\ x = 1 & x = -2 \end{array}$$



#### Tip

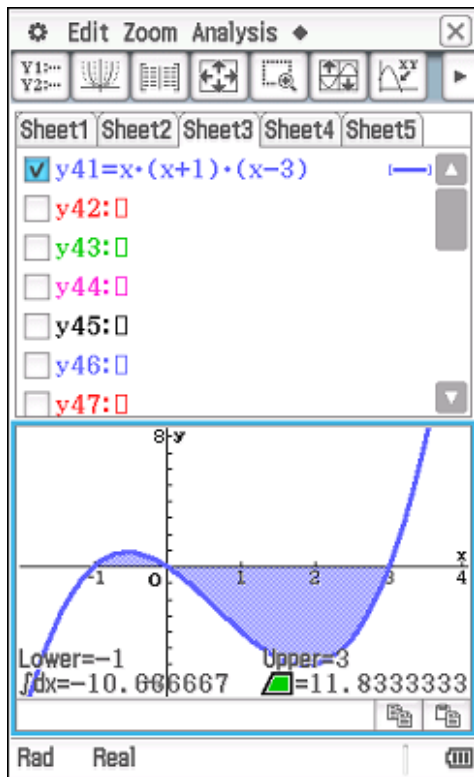
- *Use CAS to draw the graph if it helps.*

### Question 3

*Answer: D*

#### Explanatory notes

Use CAS to find the area.



### Question 4

*Answer: E*

#### Explanatory notes

The graph has a stationary point at  $x=4$  and the sign of the gradient doesn't change either side of this point so it is a stationary point of inflection.

**Question 5****Answer: C****Explanatory notes**

$$\begin{aligned}\Pr(X < 8) &= \Pr\left(Z < \frac{8-10}{2}\right) \\ &= \Pr(Z < -1) \\ &= \Pr(Z > 1) \text{ using symmetry}\end{aligned}$$

**Question 6****Answer: D****Explanatory notes**

To have an inverse function, the graph needs to be one-to-one.

**Tip**

- Draw a graph using CAS to decide where the graph is one-to-one.

**Question 7****Answer: D****Explanatory notes**

$$\begin{aligned}\int_1^5 (3 - 2f(x)) dx &= \int_1^5 3 dx - 2 \int_1^5 f(x) dx \\ &= [3x]_1^5 - 2 \times 4 \\ &= 15 - 3 - 8 = 4\end{aligned}$$

### Question 8

*Answer: A*

#### Explanatory notes

Swap  $x$  and  $y$ , then use CAS to rearrange to make  $y$  the subject. Check the range of the original because this becomes the domain of the inverse.

The screenshot shows a CAS calculator window titled "Edit Action Interactive". The input field contains the equation  $\text{solve}\left(x = \frac{1}{\sqrt{y}} + 2, y\right)$ . The output field displays the solution  $\left\{y = \frac{1}{x^2 - 4x + 4}\right\}$ . Below the input field is a small square icon. At the bottom of the window is a keypad with various mathematical functions and symbols, including Math1, Math2, Math3, Trig, Var, abc, and a numeric keypad. The mode is set to "Alg".



#### Tip

- Remember the range of the original becomes the domain of the inverse.

### Question 9

*Answer: C*

#### Explanatory notes

For  $f(f(x)) = x$ , the function must be the same as its inverse. In other words, when the graph of the function is reflected in the line  $y = x$ , the graph produced is a graph of itself. This is true of  $f(x) = 4 - x$ .

**Question 10****Answer: C****Explanatory notes**

$$\begin{aligned}\Pr(A) &= \Pr(A \cap B) + \Pr(A \cap B') \\ &= k + 3k - 1 = 4k - 1\end{aligned}$$

$$\begin{aligned}\Pr(B) &= \Pr(A' \cap B) + \Pr(A \cap B) \\ &= \frac{2k}{5} + k = \frac{7k}{5}\end{aligned}$$

since A and B are independent,  $\Pr(A) \times \Pr(B) = \Pr(A \cap B)$

$$\Rightarrow (4k - 1)\left(\frac{7k}{5}\right) = k$$

using CAS to solve gives  $k = \frac{3}{7}$

**Question 11****Answer: B****Explanatory notes**

Multiplying out the matrices gives

$$\left. \begin{array}{l} x' = -2x + 2 \\ y' = y - 1 \end{array} \right\} \Rightarrow \left. \begin{array}{l} x' - 2 = -2x \\ y' + 1 = y \end{array} \right\} \Rightarrow \begin{array}{l} x = \frac{x' - 2}{-2} \\ y = y' + 1 \end{array}$$

Substituting into the equation  $2x - y = 5$  gives

$$2\left(\frac{x' - 2}{-2}\right) - (y' + 1) = 5$$

$$2 - x - y - 1 = 5$$

$$-x - y = 4$$

$$x + y = -4$$

**Question 12****Answer: B****Explanatory notes**

The function  $y = \sin(x)$  is one-to-one for  $x \in \left[ \frac{-\pi}{2}, \frac{\pi}{2} \right]$ .

$$\text{and } \log_e(e^{\frac{-\pi}{2}}) = \frac{-\pi}{2} \quad \text{and} \quad \log_e(e^{\frac{\pi}{2}}) = \frac{\pi}{2}$$

**Question 13****Answer: A****Explanatory notes**

$$\begin{aligned} \Pr(X < 8 | X < 10) &= \frac{\Pr(X < 8 \cap X < 10)}{\Pr(X < 10)} \\ &= \frac{\Pr(X < 8)}{\Pr(X < 10)} = \frac{1-b}{1-a} = \frac{b-1}{a-1} \end{aligned}$$

**Question 14****Answer: C****Explanatory notes**

The volume function is  $V(x) = x(10 - 2x)(8 - 2x)$ .

Use CAS to find maximum value gives  $x = 1.47$ .

**Question 15****Answer: C****Explanatory notes**

The period of a tangent function is  $\frac{\pi}{n}$ . In this case  $n = 2\pi$  so the period is  $\frac{\pi}{2\pi} = \frac{1}{2}$ .

**Question 16****Answer: D****Explanatory notes**

$$\text{Var}(X) = E(X^2) - (E(X))^2$$

$$3 = E(X^2) - 25$$

$$E(X^2) = 28$$

$$\Rightarrow 2 E(X^2) = 56$$

**Question 17****Answer: B****Explanatory notes**

$$\sigma = \sqrt{\frac{p(1-p)}{n}}$$

$$\frac{1}{216} = \sqrt{\frac{\frac{1}{6} \times \frac{5}{6}}{n}}$$

Then use CAS to solve for  $n$ .

The screenshot shows a CAS interface with the following elements:

- Top bar: Edit Action Interactive
- Toolbar: 0.5, 1/2, solve, fdx, fdx/dx, Simp, fdx, and other mathematical symbols.
- Main display: solve  $\left( \frac{1}{216} = \sqrt{\frac{\frac{1}{6} \cdot \frac{5}{6}}{x}}, x \right)$  and the result  $\{x=6480\}$ .
- Bottom panel: Math1 (Line, fraction, sqrt, pi, arrow), Math2 (square, e, ln, log base m, root), Math3 (abs, x^2, x^-1, log base m, solve), Trig (trig functions, toDMS, braces, parentheses), Var (sin, cos, tan, degrees, r), abc (text input), and navigation buttons (left arrow, copy, paste, ans, EXE).
- Bottom status bar: Alg, Standard, Real, Rad, and a calculator icon.

**Question 18****Answer: B****Explanatory notes**The 90% confidence interval for  $p$  is

$$\left( \hat{p} - 1.65 \times \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}, \hat{p} + 1.65 \times \sqrt{\frac{\hat{p}(1-\hat{p})}{n}} \right)$$

**Tip**

- The 90% and 95% confidence intervals are worth remembering.

$$90\% \quad \left( \hat{p} - 1.65 \times \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}, \hat{p} + 1.65 \times \sqrt{\frac{\hat{p}(1-\hat{p})}{n}} \right)$$

$$95\% \quad \left( \hat{p} - 1.96 \times \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}, \hat{p} + 1.96 \times \sqrt{\frac{\hat{p}(1-\hat{p})}{n}} \right)$$



**Question 19****Answer: E****Explanatory notes**

$$\text{If } f(x) = g(x) - 10$$

$$\text{then } f'(x) = g'(x)$$

$$f(1) = -5 \text{ and } g(x) = -x f(x) \text{ gives } g(1) = -1 \times -5 = 5$$

$$f(x) = g(x) - 10$$

$$\text{then } f(1) = g(1) - 10 = 5 - 10 = -5 \text{ is true}$$

Or

$$\text{Since } f'(x) = g'(x)$$

$$\mathbf{A} \quad f'(x) = g'(x) - 10$$

$$\mathbf{B} \quad f'(x) = g''(x)$$

$$\mathbf{C} \quad f'(x) = g'(x) - 10$$

$$\mathbf{D} \quad f'(x) = g'(x) - 1$$

$$\mathbf{E} \quad f'(x) = g'(x)$$

 $\therefore$  Answer: E**Question 20****Answer: E****Explanatory notes**

$$\begin{aligned} \text{Average value} &= \frac{1}{14} \int_1^{15} h(x) \, dx \\ &= \frac{1}{14} [\text{area of trapezium} + \text{area of rectangle}] \\ &= \frac{1}{14} \left[ \frac{1}{2} (14 + 4) \times 6 + 14 \times 4 \right] = 7.86 \end{aligned}$$

**SECTION B****Question 1a.****Worked solution**

$$\text{period} = \frac{2\pi}{n} = \frac{2\pi}{\frac{\pi}{6}} = 12 \text{ hours}$$

amplitude = 11 cm.

**Mark allocation: 2 marks**

- 1 mark for period
- 1 mark for amplitude

**Question 1b.****Worked solution**

maximum = 98 cm

minimum = 76 cm

**Mark allocation: 2 marks**

- 1 mark for maximum
- 1 mark for minimum

**Question 1c.****Worked solution**

Use CAS to get  $d(10) = 87 - \frac{11\sqrt{3}}{2}$ .

The screenshot shows a CAS window titled "Edit Action Interactive". The main display area contains the expression  $87 + 11\sin\left(\frac{\pi \cdot 10}{6}\right)$  on the left and its simplified form  $\frac{-11 \cdot \sqrt{3}}{2} + 87$  on the right. Below the display is a toolbar with various mathematical functions and symbols, including Math1, Math2, Math3, Trig, Var, abc, and a numeric keypad. At the bottom, there are mode selection buttons for Alg, Standard, Real, and Rad.

**Mark allocation: 1 mark**

- 1 mark for correct answer (must be in exact form)

**Question 1d.****Worked solution**

Use CAS to solve

$$87 + 11 \sin\left(\frac{\pi t}{6}\right) = d(10) + 5$$

$$87 + 11 \sin\left(\frac{\pi t}{6}\right) = 82.47372$$

The screenshot shows a CAS interface with the following content:

- Input:  $87 + 11 \sin\left(\frac{\pi t}{6}\right)$
- Result: 77.47372056
- Input:  $\text{ans} + 5$
- Result: 82.47372056
- Input:  $\text{solve}\left(87 + 11 \cdot \sin\left(\frac{\pi \cdot x}{6}\right) = 82.47372\right)$
- Result:  $\{x = 6.809927895, x = 11.1900\}$

The screenshot shows a CAS interface with the following content:

- Input:  $\text{solve}\left(87 + 11 \cdot \sin\left(\frac{\pi \cdot x}{6}\right) = 82.47372\right)$
- Result:  $\{x = 6.809927895, x = 11.1900\}$
- Input:  $6.8099 + 12 - 11.1900$
- Result: 7.6199
- Input:  $\text{ans} / 12 * 100$
- Result: 63.49916667

So the depth of snow is above  $d(10) + 5$  for  $6.8099 + 12 - 11.1900 = 7.6199$  hours.

As a percentage  $\frac{7.6199}{12} \times 100 = 63.50\%$

**Mark allocation: 3 marks**

- 1 method mark for setting  $87 + 11 \sin\left(\frac{\pi t}{6}\right) = 82.47372$
- 1 answer mark for finding  $t = 6.8099$  and  $t = 11.1900$
- 1 answer for mark 63.50%

**Tip**

- You need to give your answer as a decimal correct to two decimal places. An exact value would not be accepted. Always work to a higher number of decimal places before rounding off.

**Question 2a.i.****Worked solution**

Use CAS to solve  $f(x) = 1$  gives  $x = -0.172$

**Mark allocation: 1 mark**

- 1 answer mark for  $x = -0.172$

**Question 2a.ii.****Worked solution**

Use CAS to find  $x$  such that  $f(x) = 1$  gives  $x = -0.172, 0.417, 1$

So the interval such that  $f(x) > 1$  is  $(-\infty, -0.172) \cup (0.417, 1.000)$

**Mark allocation: 2 marks**

- 1 method mark for finding  $x = -0.172, 0.417, 1$
- 1 answer mark for correct interval  $(-\infty, -0.172) \cup (0.417, 1.000)$

**Tip**

- *The question requires answers given to three decimal places. This means that 1 is written as 1.000.*

**Question 2a.iii.****Worked solution**

Use CAS to find the local maximum value at  $A$ .

The screenshot shows a CAS calculator interface with the following content:

- Window title: Edit Action Interactive
- Top toolbar:  $\frac{0.5}{1}$ ,  $\frac{1}{2}$ ,  $\frac{1}{3}$ ,  $\frac{1}{4}$ ,  $\frac{1}{5}$ ,  $\frac{1}{6}$ ,  $\frac{1}{7}$ ,  $\frac{1}{8}$ ,  $\frac{1}{9}$ ,  $\frac{1}{10}$ ,  $\frac{1}{11}$ ,  $\frac{1}{12}$ ,  $\frac{1}{13}$ ,  $\frac{1}{14}$ ,  $\frac{1}{15}$ ,  $\frac{1}{16}$ ,  $\frac{1}{17}$ ,  $\frac{1}{18}$ ,  $\frac{1}{19}$ ,  $\frac{1}{20}$ ,  $\frac{1}{21}$ ,  $\frac{1}{22}$ ,  $\frac{1}{23}$ ,  $\frac{1}{24}$ ,  $\frac{1}{25}$ ,  $\frac{1}{26}$ ,  $\frac{1}{27}$ ,  $\frac{1}{28}$ ,  $\frac{1}{29}$ ,  $\frac{1}{30}$ ,  $\frac{1}{31}$ ,  $\frac{1}{32}$ ,  $\frac{1}{33}$ ,  $\frac{1}{34}$ ,  $\frac{1}{35}$ ,  $\frac{1}{36}$ ,  $\frac{1}{37}$ ,  $\frac{1}{38}$ ,  $\frac{1}{39}$ ,  $\frac{1}{40}$ ,  $\frac{1}{41}$ ,  $\frac{1}{42}$ ,  $\frac{1}{43}$ ,  $\frac{1}{44}$ ,  $\frac{1}{45}$ ,  $\frac{1}{46}$ ,  $\frac{1}{47}$ ,  $\frac{1}{48}$ ,  $\frac{1}{49}$ ,  $\frac{1}{50}$ ,  $\frac{1}{51}$ ,  $\frac{1}{52}$ ,  $\frac{1}{53}$ ,  $\frac{1}{54}$ ,  $\frac{1}{55}$ ,  $\frac{1}{56}$ ,  $\frac{1}{57}$ ,  $\frac{1}{58}$ ,  $\frac{1}{59}$ ,  $\frac{1}{60}$ ,  $\frac{1}{61}$ ,  $\frac{1}{62}$ ,  $\frac{1}{63}$ ,  $\frac{1}{64}$ ,  $\frac{1}{65}$ ,  $\frac{1}{66}$ ,  $\frac{1}{67}$ ,  $\frac{1}{68}$ ,  $\frac{1}{69}$ ,  $\frac{1}{70}$ ,  $\frac{1}{71}$ ,  $\frac{1}{72}$ ,  $\frac{1}{73}$ ,  $\frac{1}{74}$ ,  $\frac{1}{75}$ ,  $\frac{1}{76}$ ,  $\frac{1}{77}$ ,  $\frac{1}{78}$ ,  $\frac{1}{79}$ ,  $\frac{1}{80}$ ,  $\frac{1}{81}$ ,  $\frac{1}{82}$ ,  $\frac{1}{83}$ ,  $\frac{1}{84}$ ,  $\frac{1}{85}$ ,  $\frac{1}{86}$ ,  $\frac{1}{87}$ ,  $\frac{1}{88}$ ,  $\frac{1}{89}$ ,  $\frac{1}{90}$ ,  $\frac{1}{91}$ ,  $\frac{1}{92}$ ,  $\frac{1}{93}$ ,  $\frac{1}{94}$ ,  $\frac{1}{95}$ ,  $\frac{1}{96}$ ,  $\frac{1}{97}$ ,  $\frac{1}{98}$ ,  $\frac{1}{99}$ ,  $\frac{1}{100}$
- Main display:
  - fMax( $x^2 \cdot e^{-3 \cdot x + 3}$ , x, 0, 0.5)
  - {MaxValue=1.120422268, x=0}
  - fMax( $x^2 \cdot e^{-3 \cdot x + 3}$ , x, 0, 50)
  - {MaxValue= $\frac{4 \cdot e}{9}$ , x= $\frac{2}{3}$ }
- Bottom toolbar: Math1, Math2, Math3, Trig, Var, abc,  $\frac{1}{2}$ ,  $\frac{1}{3}$ ,  $\frac{1}{4}$ ,  $\frac{1}{5}$ ,  $\frac{1}{6}$ ,  $\frac{1}{7}$ ,  $\frac{1}{8}$ ,  $\frac{1}{9}$ ,  $\frac{1}{10}$ ,  $\frac{1}{11}$ ,  $\frac{1}{12}$ ,  $\frac{1}{13}$ ,  $\frac{1}{14}$ ,  $\frac{1}{15}$ ,  $\frac{1}{16}$ ,  $\frac{1}{17}$ ,  $\frac{1}{18}$ ,  $\frac{1}{19}$ ,  $\frac{1}{20}$ ,  $\frac{1}{21}$ ,  $\frac{1}{22}$ ,  $\frac{1}{23}$ ,  $\frac{1}{24}$ ,  $\frac{1}{25}$ ,  $\frac{1}{26}$ ,  $\frac{1}{27}$ ,  $\frac{1}{28}$ ,  $\frac{1}{29}$ ,  $\frac{1}{30}$ ,  $\frac{1}{31}$ ,  $\frac{1}{32}$ ,  $\frac{1}{33}$ ,  $\frac{1}{34}$ ,  $\frac{1}{35}$ ,  $\frac{1}{36}$ ,  $\frac{1}{37}$ ,  $\frac{1}{38}$ ,  $\frac{1}{39}$ ,  $\frac{1}{40}$ ,  $\frac{1}{41}$ ,  $\frac{1}{42}$ ,  $\frac{1}{43}$ ,  $\frac{1}{44}$ ,  $\frac{1}{45}$ ,  $\frac{1}{46}$ ,  $\frac{1}{47}$ ,  $\frac{1}{48}$ ,  $\frac{1}{49}$ ,  $\frac{1}{50}$ ,  $\frac{1}{51}$ ,  $\frac{1}{52}$ ,  $\frac{1}{53}$ ,  $\frac{1}{54}$ ,  $\frac{1}{55}$ ,  $\frac{1}{56}$ ,  $\frac{1}{57}$ ,  $\frac{1}{58}$ ,  $\frac{1}{59}$ ,  $\frac{1}{60}$ ,  $\frac{1}{61}$ ,  $\frac{1}{62}$ ,  $\frac{1}{63}$ ,  $\frac{1}{64}$ ,  $\frac{1}{65}$ ,  $\frac{1}{66}$ ,  $\frac{1}{67}$ ,  $\frac{1}{68}$ ,  $\frac{1}{69}$ ,  $\frac{1}{70}$ ,  $\frac{1}{71}$ ,  $\frac{1}{72}$ ,  $\frac{1}{73}$ ,  $\frac{1}{74}$ ,  $\frac{1}{75}$ ,  $\frac{1}{76}$ ,  $\frac{1}{77}$ ,  $\frac{1}{78}$ ,  $\frac{1}{79}$ ,  $\frac{1}{80}$ ,  $\frac{1}{81}$ ,  $\frac{1}{82}$ ,  $\frac{1}{83}$ ,  $\frac{1}{84}$ ,  $\frac{1}{85}$ ,  $\frac{1}{86}$ ,  $\frac{1}{87}$ ,  $\frac{1}{88}$ ,  $\frac{1}{89}$ ,  $\frac{1}{90}$ ,  $\frac{1}{91}$ ,  $\frac{1}{92}$ ,  $\frac{1}{93}$ ,  $\frac{1}{94}$ ,  $\frac{1}{95}$ ,  $\frac{1}{96}$ ,  $\frac{1}{97}$ ,  $\frac{1}{98}$ ,  $\frac{1}{99}$ ,  $\frac{1}{100}$
- Bottom status bar: Alg, Standard, Real, Rad,  $\frac{1}{2}$

**Mark allocation: 1 mark**

- 1 mark for  $\left(\frac{2}{3}, \frac{4e}{9}\right)$

**Tip**

- *An exact answer is required; so remember to have your CAS set to give an exact value.*

**Question 2b.****Worked solution**

Let  $a$  be the  $x$  value of the point on the graph. The coordinates of this point are  $(a, a^2e^{-3a+3})$ .

If the tangent line passes through the origin then the gradient of this line is  $m = \frac{a^2e^{-3a+3}}{a} = ae^{-3a+3}$

$$\begin{aligned} \text{Also } f(x) &= x^2e^{-3x+3}, \quad f'(x) = 2xe^{-3x+3} - 3x^2e^{-3x+3} \\ &= x(2-3x)(e^{-3x+3}) \end{aligned}$$

$$\text{and } f'(a) = a(2-3a)(e^{-3a+3})$$

so if  $f'(a) = m$

$$(2-3a) = 1$$

$$a = \frac{1}{3}$$

$$f\left(\frac{1}{3}\right) = \frac{1}{9}e^2$$

Point is  $\left(\frac{1}{3}, \frac{1}{9}e^2\right)$

**Mark allocation: 3 marks**

- 1 method mark for finding  $m = \frac{a^2e^{-3a+3}}{a} = ae^{-3a+3}$
- 1 answer mark for finding  $f'(a) = a(2-3a)(e^{-3a+3})$
- 1 answer mark for  $\left(\frac{1}{3}, \frac{1}{9}e^2\right)$

**Tip**

- You can check your answer by using CAS to find the equation of the tangent at the point you have found and see if it goes through the origin.

**Question 2c.i.****Worked solution**

$$f(x) = x^m e^{-nx+n}$$

$$\begin{aligned} f'(x) &= mx^{m-1} e^{-nx+n} + -nx^m e^{-nx+n} \\ &= x^{m-1} e^{-nx+n} (m - nx) \end{aligned}$$

$$\text{let } f'(x) = 0 \Rightarrow x^{m-1} = 0 \text{ or } m - nx = 0$$

$$x = 0 \text{ (not possible) or } x = \frac{m}{n}$$

**Mark allocation: 3 marks**

- 1 answer mark for finding  $f'(x)$
- 1 method mark for setting  $f'(x) = 0$
- 1 answer mark for getting  $x = \frac{m}{n}$



**Question 2c.ii.****Worked solution**

Let  $x = a$  lie on the curve. The coordinates of the point are  $(a, a^m e^{-an+n})$ .

If the tangent drawn at  $x = a$  passes through the origin then

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{a^m e^{-an+n} - 0}{a - 0} = \frac{a^m e^{-an+n}}{a} = a^{m-1} e^{-an+n}$$

and

$$f'(x) = x^{m-1} e^{-nx+n} (m - nx)$$

$$\text{at } x = a, f'(a) = a^{m-1} e^{-na+n} (m - na)$$

Let  $f'(a) = m$

$$a^{m-1} e^{-na+n} (m - na) = a^{m-1} e^{-na+n}$$

$$a^{m-1} e^{-na+n} (m - na) - a^{m-1} e^{-na+n} = 0$$

$$a^{m-1} e^{-na+n} (m - na - 1) = 0$$

$$\text{so } a^{m-1} = 0 \text{ or } m - na - 1 = 0$$

$$a = 0 \text{ or } a = \frac{m-1}{n}$$

**Mark allocation: 3 marks**

- 1 method mark for finding  $f'(a)$
- 1 method mark for setting  $f'(a) = m$
- 1 answer mark for solving equation to obtain correct answer

**Question 3a.****Worked solution**

$$X_c \sim N(\mu = 15, \sigma = 4)$$

$$\Pr(X_c > c) = 0.15$$

Use CAS to find  $c$  gives  $c = 19.145$

So  $c = 191$  mm

**Mark allocation: 1 mark**

- 1 answer mark for 191 mm

**Tip**

- *Take note of the units required. Here the question is given in cm and the answer is required in mm.*

**Question 3b.****Worked solution**

$$\Pr(X_c < 9) = 0.0668$$

$$0.0668 \times 4000 = 267$$

**Mark allocation: 2 marks**

- 1 answer mark for 0.0668
- 1 answer mark for 267 plants.

**Tip**

- *Remember to give your answer as the number of plants, not just the probability.*

**Question 3c.****Worked solution**

$$k \int_6^{16} \left( \sin \left( \frac{\pi(x-6)}{10} \right) \right) dx = 1$$

Using CAS to evaluate the definite integral gives

$$k \times \frac{20}{\pi} = 1$$

$$k = \frac{\pi}{20}$$

**Mark allocation: 2 marks**

- 1 answer mark for setting  $k \int_6^{16} \left( \sin \left( \frac{\pi(x-6)}{10} \right) \right) dx = 1$
- 1 answer mark for  $\int_6^{16} \left( \sin \left( \frac{\pi(x-6)}{10} \right) \right) dx = \frac{20}{\pi}$  leading to correct value for  $k$ .

**Tip**

- *The area under a probability density function is always 1.*

**Question 3d.****Worked solution**

Using the symmetry of the sine curve gives mean as half way between 6 and 16 as 11.

**Mark allocation: 1 mark**

- 1 answer mark for the answer of 11.

**Tip**

- *When dealing with a symmetrical distribution such as sine, make good use of the symmetry for determining the mean.*

**Question 3e.****Worked solution**

$$\Pr(X_r < c) = 0.1$$

$$0.1 = \int_6^c \frac{\pi}{20} \sin\left(\frac{\pi(x-6)}{10}\right) dx$$

Use CAS to solve gives

$$c = 8.04$$

$$c = 80 \text{ mm}$$

**Mark allocation: 2 marks**

- 1 method mark for writing the integral from 6 to  $c$  equal to 0.1
- 1 answer mark for 80 mm

**Question 3f.****Worked solution**

There are six ways that she could choose exactly two tall plants from the next four.

STTS, SSTT, STST, TSST, TTSS, TSTS

$$\begin{aligned} \Pr(STTS) &= \frac{3}{4} \times \frac{1}{3} \times \frac{1}{4} \times \frac{3}{4} & \Pr(TSST) &= \frac{1}{4} \times \frac{3}{4} \times \frac{2}{3} \times \frac{1}{3} \\ \Pr(SSTT) &= \frac{3}{4} \times \frac{2}{3} \times \frac{1}{3} \times \frac{1}{4} & \Pr(TTSS) &= \frac{1}{4} \times \frac{1}{4} \times \frac{3}{4} \times \frac{2}{3} \\ \Pr(STST) &= \frac{3}{4} \times \frac{1}{3} \times \frac{3}{4} \times \frac{1}{3} & \Pr(TSTS) &= \frac{1}{4} \times \frac{3}{4} \times \frac{1}{3} \times \frac{3}{4} \end{aligned}$$

The sum of these probabilities is  $\frac{13}{48}$

**Mark allocation: 2 marks**

- 1 method mark for determining the six ways: STTS, SSTT, STST, TSST, TTSS and TSTS
- 1 answer mark for  $\frac{13}{48}$  must be an exact value

**Tip**

- There are six ways of choosing two tall plants from four because  ${}^4C_2 = 6$ .

**Question 3g.i.****Worked solution**

If the sample proportion is  $\hat{p} = 0.3$  and the sample size is 20, then the number of diseased plants in the sample is  $0.3 \times 20 = 6$ .

Thus

$$\begin{aligned}\Pr(\hat{P} = 0.3) &= \Pr(X = 6) \\ &= \binom{20}{6} (0.3)^6 (0.7)^{14} \\ &= 0.1916\end{aligned}$$

**Mark allocation: 2 marks**

- 1 method mark for finding  $X = 6$
- 1 answer mark for 0.1916

**Question 3g.ii.****Worked solution**

$$\begin{aligned}sd(\hat{P}) &= \sqrt{\frac{p(1-p)}{n}} \\ &= \sqrt{\frac{0.3 \times 0.7}{20}} = 0.1025\end{aligned}$$

Since

$$0.3 - 2 \times 0.1025 = 0.095$$

$$0.3 + 2 \times 0.1025 = 0.505$$

we find

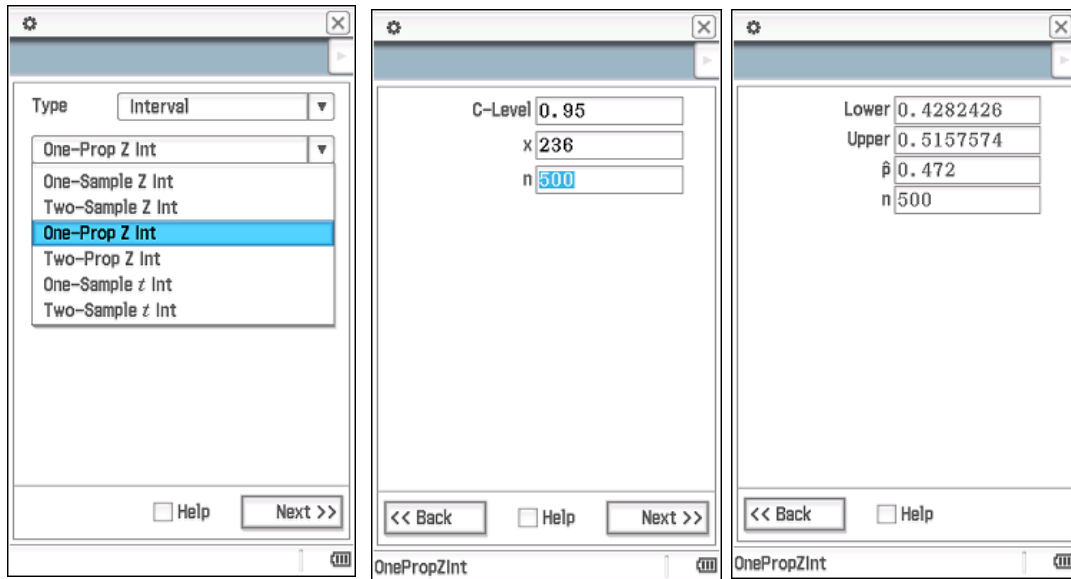
$$\begin{aligned}\Pr(0.095 \leq \hat{P} \leq 0.505) \\ &= \Pr(1.9 \leq X \leq 10.1) \quad \text{and} \quad X \sim Bi(n = 20, p = 0.3) \\ &= \Pr(2 \leq X \leq 10) \\ &= 0.9752 \text{ using CAS}\end{aligned}$$

**Mark allocation: 3 marks**

- 1 method mark for finding standard deviation of 0.1025
- 1 method mark for finding the endpoints of 0.095 and 0.505
- 1 answer mark for 0.9752

**Question 3h.****Worked solution**

This can be done using CAS.



The 95% confidence interval is (0.428, 0.516).

**Mark allocation: 1 mark**

- 1 answer mark for (0.428, 0.516)

**Question 3i.****Worked solution**

This experiment is a binomial distribution with  $Y \sim Bi(n = 5, p = \frac{26}{27})$ .

Using CAS  $\Pr(X = 3) = 0.0122$ .

**Mark allocation: 2 marks**

- 1 method mark for recognising the binomial distribution
- 1 answer mark for 0.0122

**Question 4a.****Worked solution**

$$\begin{aligned}
 &64x - x^4 \\
 &= x(64 - x^3) \\
 &= x(4 - x)(x^2 + 4x + 16) \\
 &= x(4 - x)((x + 2)^2 + 12)
 \end{aligned}$$

**Mark allocation: 2 marks**

- 1 method mark for recognising the difference of 2 cubes
- 1 answer mark for getting to the correct form

**Tip**

- *Always look to use common factor factorisation.*

**Question 4b.****Worked solution**

It is observed that  $g(-(x + 2)) = f(x)$ , so the graph of  $g(x)$  is reflected in the  $y$ -axis and translated 2 units to the left.

**Mark allocation: 1 mark**

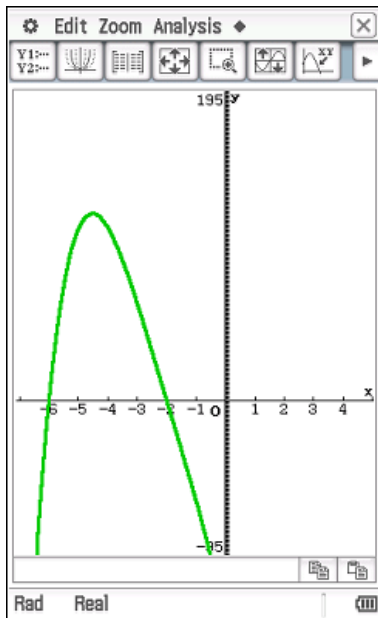
- 1 mark for correct transformation in correct order

**Tip**

- *Use CAS to sketch the graphs of  $f(x)$  and  $g(x)$  to help to see the transformations.*

**Question 4c.i.****Worked solution**

Looking at the graph, it can be seen that the graph can be shifted at most two units to the right. So  $d < 2$ .

**Mark allocation: 1 mark**

- 1 answer mark for  $d < 2$

**Question 4c.ii.****Worked solution**

If the graph is shifted more than two units to the right then the graph will have at least one positive  $x$ - intercept. So  $d > 2$ .

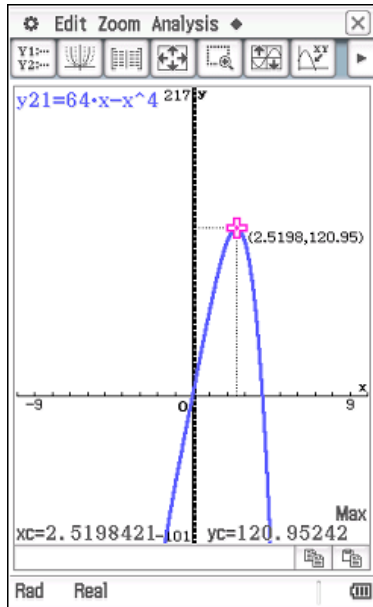
**Mark allocation: 1 mark**

- 1 mark for correct answer  $d > 2$



**Question 4d.****Worked solution**

Using CAS the maximum turning point of the graph occurs at  $(2.52, 120.95)$ . So the equation  $g(x) = n$  would have one solution if  $n = 120.95$ .

**Mark allocation: 1 marks**

- 1 answer mark for  $n = 120.95$

**Tip**

- Questions like part c. and d. are best undertaken using a graphical approach.

**Question 4e.****Worked solution**

$$h'(x) = k - 4x^3$$

$$h'(x) = 0$$

$$\text{Let } \Rightarrow 4x^3 = k$$

$$\Rightarrow x = \sqrt[3]{\frac{k}{4}} = \left(\frac{k}{4}\right)^{\frac{1}{3}}$$

$$\begin{aligned} h\left(\left(\frac{k}{4}\right)^{\frac{1}{3}}\right) &= k\left(\left(\frac{k}{4}\right)^{\frac{1}{3}}\right) - \left(\frac{k}{4}\right)^{\frac{4}{3}} \\ &= \frac{4k}{4}\left(\left(\frac{k}{4}\right)^{\frac{1}{3}}\right) - \left(\frac{k}{4}\right)^{\frac{4}{3}} \\ &= 4\left(\left(\frac{k}{4}\right)^{\frac{4}{3}}\right) - \left(\frac{k}{4}\right)^{\frac{4}{3}} = 3\left(\frac{k}{4}\right)^{\frac{4}{3}} \end{aligned}$$

**Mark allocation: 3 marks**

- 1 answer mark for finding  $x = \sqrt[3]{\frac{k}{4}} = \left(\frac{k}{4}\right)^{\frac{1}{3}}$
- 1 method mark for substituting into  $h(x)$
- 1 answer mark for obtaining  $3\left(\frac{k}{4}\right)^{\frac{4}{3}}$

**Tip**

- *With 'show that' questions, make sure each step in the solution process is clear.*

**Question 4f.i.****Worked solution**

Use CAS. The area is 307.2 square units.

**Mark allocation: 1 mark**

- 1 mark for correct answer of 307.2

**Question 4f.ii.****Worked solution**

The graph of  $y = f\left(\frac{-x}{2}\right)$  is the graph of  $y = f(x)$  reflected in the  $y$ -axis and then dilated by a factor of 2 in the  $x$ -direction.

The area bounded by the graph of  $y = f\left(\frac{-x}{2}\right)$  will be equal to  $2 \times$  area bounded by  $g(x)$  .

Area is  $2 \times 307.2 = 614.4$  square units.

**Mark allocation: 1 mark**

- 1 answer mark for 614.4, but it must be obtained as 2 times the previous answer

**Tip**

- Remember for 'hence' questions you must use the previous answer and it must be clear how you have used it.

**Question 5a.****Worked solution**

$$V = \pi r^2 h$$

$$h = \frac{V}{\pi r^2}$$

**Mark allocation: 1 mark**

- 1 answer mark for  $h = \frac{V}{\pi r^2}$

**Question 5b.****Worked solution**

$$\begin{aligned} A &= 2\pi r^2 + 2\pi r h \\ &= 2\pi r^2 + 2\pi r \left( \frac{V}{\pi r^2} \right) \\ &= 2\pi r^2 + \left( \frac{2V}{r} \right) \end{aligned}$$

**Mark allocation: 2 marks**

- 1 method mark for substituting into  $A = 2\pi r^2 + 2\pi r h$
- 1 answer mark for obtaining  $A = \frac{2V}{r} + 2\pi r^2$

**Tip**

- Remember when differentiating this,  $V$  is a constant and is treated as a number not as a variable.

**Question 5c.****Worked solution**

Minimum surface area occurs when  $\frac{dA}{dr} = 0$

$$\begin{aligned}\frac{dA}{dr} &= 4\pi r - 2Vr^{-2} \\ &= 4\pi r - \frac{2V}{r^2}\end{aligned}$$

$$\text{let } \frac{dA}{dr} = 0$$

$$\Rightarrow 4\pi r = \frac{2V}{r^2}$$

$$r^3 = \frac{2V}{4\pi} = \frac{V}{2\pi}$$

$$r = \sqrt[3]{\frac{V}{2\pi}}$$

**Mark allocation: 2 marks**

- 1 method mark for finding  $\frac{dA}{dr}$  and setting it equal to zero
- 1 answer mark for  $r = \sqrt[3]{\frac{V}{2\pi}}$

**Question 5d.****Worked solution**

Using CAS evaluate  $A$  when  $r = \sqrt[3]{\frac{V}{2\pi}}$

The screenshot shows a CAS calculator interface. The main display area contains the following text and mathematical expressions:

$$\frac{5 \cdot 2^3}{\pi^3}$$

simplify  $\left( \frac{2v}{x} + 2 \cdot \pi \cdot x^2 \mid x = \sqrt[3]{\frac{v}{2\pi}} \right)$

$$3 \cdot v^{\frac{2}{3}} \cdot (2 \cdot \pi)^{\frac{1}{3}}$$

The interface includes a toolbar with buttons for 'Simp', 'f(x)', and 'f(x)'. Below the display is a keypad with rows of variables (a-f, g-l, m-r, s-x, y-z), operators, and a mode selector at the bottom (Alg, Standard, Real, Rad).

So  $a = 3$  and  $b = 2$

**Mark allocation: 3 marks**

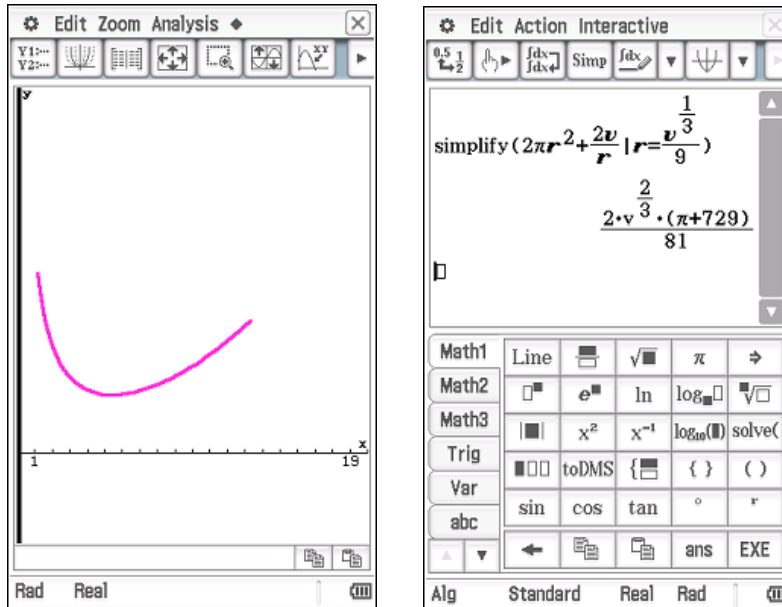
- 1 method mark for substituting  $r = \sqrt[3]{\frac{V}{2\pi}}$  into  $A$
- 1 mark for  $a = 3$
- 1 mark for  $b = 2$

**Question 5e.****Worked solution**

The maximum surface area will occur at the endpoints of the function.

Use CAS and let  $V = 1000$  to assist with the sketching.

The maximum surface occurs at the left most endpoint; that is,  $r = \frac{1}{9}V^{\frac{1}{3}}$



So the maximum surface area is  $\frac{2V^{\frac{2}{3}}(\pi + 729)}{81}$

**Mark allocation: 2 marks**

- 1 answer mark for determining that maximum surface area occurs at  $r = \frac{1}{9}V^{\frac{1}{3}}$
- 1 answer mark for  $\frac{2V^{\frac{2}{3}}(\pi + 729)}{81}$

**END OF WORKED SOLUTIONS**