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YEAR 12 *Trial Exam Paper*

2016
MATHEMATICAL METHODS

Written examination 2

Worked solutions

This book presents:

- worked solutions, giving you a series of points to show you how to work through the questions
- mark allocations
- tips on how to approach the exam

SECTION A – Multiple-choice questions

Question 1

Answer: C

Explanatory notes

The point undergoes the transformations as such

$(-3,\ 2) \rightarrow (-3,\ -2) \rightarrow (-3,\ 2)$

Tip

- *Draw a diagram to help.*

Question 2

Answer: B

Explanatory notes

Use the range to calculate endpoints

$$\begin{aligned} f(x) &= 0 \qquad & f(x) &= 6 \\ 2 - 2x &= 0 \qquad & 2 - 2x &= 6 \\ x &= 1 \qquad & x &= -2 \end{aligned}$$

Tip

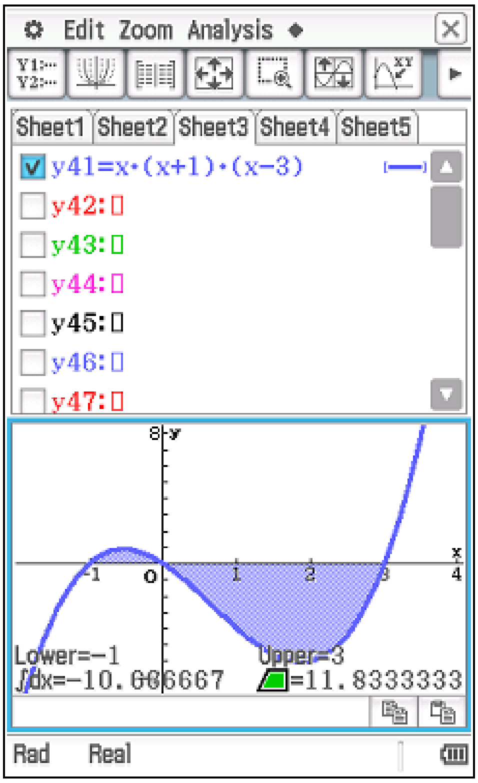
- *Use CAS to draw the graph if it helps.*

Question 3

Answer: D

Explanatory notes

Use CAS to find the area.



Question 4

Answer: E

Explanatory notes

The graph has a stationary point at $x=4$ and the sign of the gradient doesn't change either side of this point so it is a stationary point of inflection.

Question 5

Answer: C

Explanatory notes

$$\begin{aligned}\Pr(X<8) &= \Pr(Z<\frac{8-10}{2})\\ &= \Pr(Z<-1)\\ &= \Pr(Z>1) \ \text{ using symmetry}\end{aligned}$$

Question 6

Answer: D

Explanatory notes

To have an inverse function, the graph needs to be one-to-one.

Tip

- *Draw a graph using CAS to decide where the graph is one-to-one.*

Question 7

Answer: D

Explanatory notes

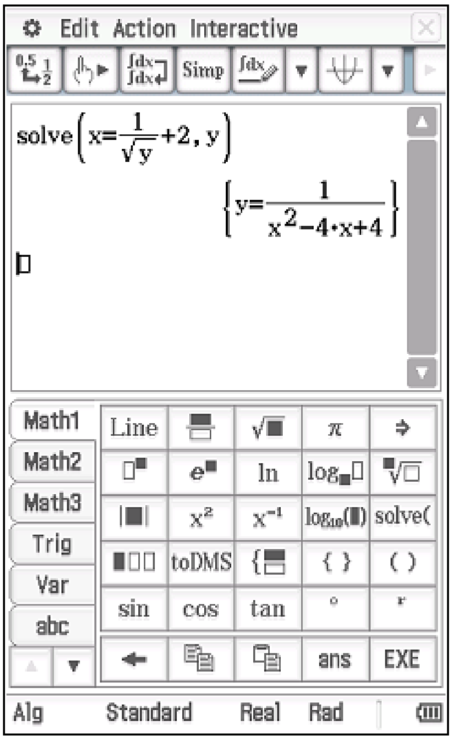
$$\begin{aligned}\int_1^5 (3-2f(x))dx &= \int_1^5 3\ dx - 2\int_1^5 f(x)\ dx\\ &= [3x]_1^5 - 2\times 4\\ &= 15-3-8=4\end{aligned}$$

Question 8

Answer: A

Explanatory notes

Swap x and y, then use CAS to rearrange to make y the subject. Check the range of the original because this becomes the domain of the inverse.



Tip

- *Remember the range of the original becomes the domain of the inverse.*

Question 9

Answer: C

Explanatory notes

For $f(f(x)) = x$, the function must be the same as its inverse. In other words, when the graph of the function is reflected in the line $y = x$, the graph produced is a graph of itself. This is true of $f(x) = 4 - x$.

Question 10

Answer: C

Explanatory notes

$$\begin{aligned}\Pr(A) &= \Pr(A \cap B) + \Pr(A \cap B') \\ &= k + 3k - 1 = 4k - 1\end{aligned}$$

$$\begin{aligned}\Pr(B) &= \Pr(A' \cap B) + \Pr(A \cap B) \\ &= \frac{2k}{5} + k = \frac{7k}{5}\end{aligned}$$

since A and B are independent, $\Pr(A) \times \Pr(B) = \Pr(A \cap B)$

$$\Rightarrow \ (4k-1)\left(\frac{7k}{5}\right) = k$$

using CAS to solve gives $k = \frac{3}{7}$

Question 11

Answer: B

Explanatory notes

Multiplying out the matrices gives

$$\left.\begin{aligned} x' &= -2x + 2 \\ y' &= y - 1 \end{aligned}\right\} \Rightarrow \quad \left.\begin{aligned} x' - 2 &= -2x \\ y' + 1 &= y \end{aligned}\right\} \Rightarrow \quad \begin{aligned} x &= \frac{x' - 2}{-2} \\ y &= y' + 1 \end{aligned}$$

Substituting into the equation $2x - y = 5$ gives

$$\begin{aligned} &2\left(\frac{x'-2}{-2}\right) - (y'+1) = 5 \\ &2 - x - y - 1 = 5 \\ &-x - y = 4 \\ &x + y = -4 \end{aligned}$$

Question 12

Answer: B

Explanatory notes

The function $y = \sin(x)$ is one-to-one for $x \in \left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$.

and $\log_e(e^{\frac{-\pi}{2}}) = \frac{-\pi}{2}$ and $\log_e(e^{\frac{\pi}{2}}) = \frac{\pi}{2}$

Question 13

Answer: A

Explanatory notes

$$\Pr(X < 8 \mid X < 10) = \frac{\Pr(X < 8 \cap X < 10)}{\Pr(X < 10)}$$

$$= \frac{\Pr(X < 8)}{\Pr(X < 10)} = \frac{1-b}{1-a} = \frac{b-1}{a-1}$$

Question 14

Answer: C

Explanatory notes

The volume function is $V(x) = x(10-2x)(8-2x)$.

Use CAS to find maximum value gives $x = 1.47$.

Question 15

Answer: C

Explanatory notes

The period of a tangent function is $\frac{\pi}{n}$. In this case $n = 2\pi$ so the period is $\frac{\pi}{2\pi} = \frac{1}{2}$.

Question 16

Answer: D

Explanatory notes

$$Var(X) = E(X^2) - (E(X))^2$$

$$3 = E(X^2) - 25$$

$$E(X^2) = 28$$

$$\Rightarrow 2\ E(X^2) = 56$$

Question 17

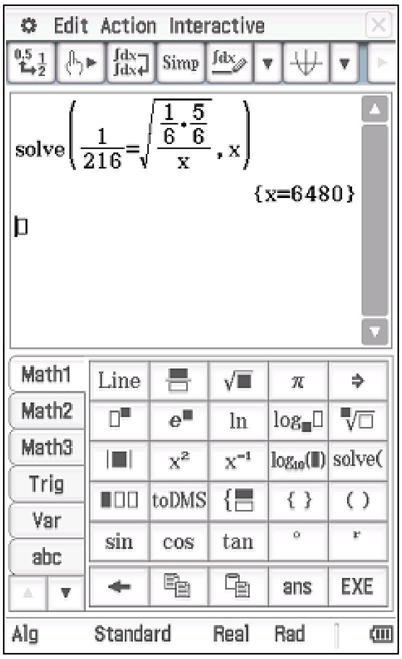
Answer: B

Explanatory notes

$$\sigma = \sqrt{\frac{p(1-p)}{n}}$$

$$\frac{1}{216} = \sqrt{\frac{\frac{1}{6}\times\frac{5}{6}}{n}}$$

Then use CAS to solve for n.



Question 18

Answer: B

Explanatory notes

The 90% confidence interval for p is

$$\left(\hat{p}-1.65\times\sqrt{\frac{\hat{p}(1-\hat{p})}{n}},\ \hat{p}+1.65\times\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}\right)$$

Tip

- *The 90% and 95% confidence intervals are worth remembering.*

90% $\left(\hat{p}-1.65\times\sqrt{\frac{\hat{p}(1-\hat{p})}{n}},\ \hat{p}+1.65\times\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}\right)$

95% $\left(\hat{p}-1.96\times\sqrt{\frac{\hat{p}(1-\hat{p})}{n}},\ \hat{p}+1.96\times\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}\right)$

Question 19

Answer: E

Explanatory notes

If $f(x) = g(x) - 10$
then $f'(x) = g'(x)$

$f(1) = -5$ and $g(x) = -x\,f(x)$ gives $g(1) = -1 \times -5 = 5$

$f(x) = g(x) - 10$
then $f(1) = g(1) - 10 = 5 - 10 = -5$ is true

Or

Since $f'(x) = g'(x)$

A $f'(x) = g'(x) - 10$

B $f'(x) = g''(x)$

C $f'(x) = g'(x) - 10$

D $f'(x) = g'(x) - 1$

E $f'(x) = g'(x)$

$\therefore$ Answer: E

Question 20

Answer: E

Explanatory notes

$$\text{Average value} = \frac{1}{14}\int_{1}^{15} h(x)\,dx$$
$$= \frac{1}{14}\left[\text{area of trapezium + area of rectangle}\right]$$
$$= \frac{1}{14}\left[\frac{1}{2}(14+4)\times 6 + 14\times 4\right] = 7.86$$

SECTION B

Question 1a.

Worked solution

$$\text{period} = \frac{2\pi}{n} = \frac{2\pi}{\frac{\pi}{6}} = 12 \text{ hours}$$

amplitude = 11 cm.

Mark allocation: 2 marks

- 1 mark for period
- 1 mark for amplitude

Question 1b.

Worked solution

maximum = 98 cm

minimum = 76 cm

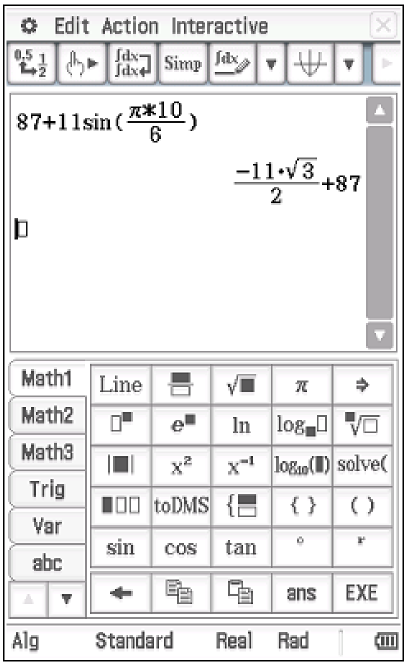
Mark allocation: 2 marks

- 1 mark for maximum
- 1 mark for minimum

Question 1c.

Worked solution

Use CAS to get $d(10) = 87 - \dfrac{11\sqrt{3}}{2}$.



Mark allocation: 1 mark

- 1 mark for correct answer (must be in exact form)

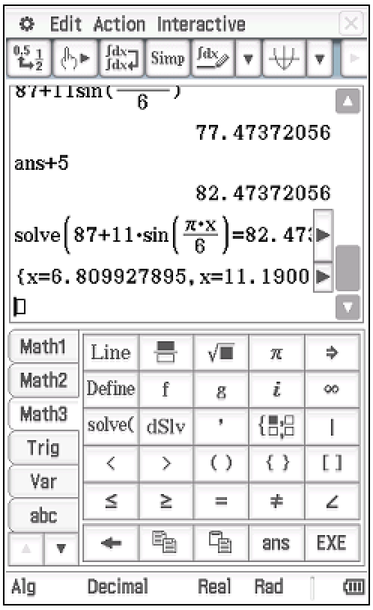
Question 1d.

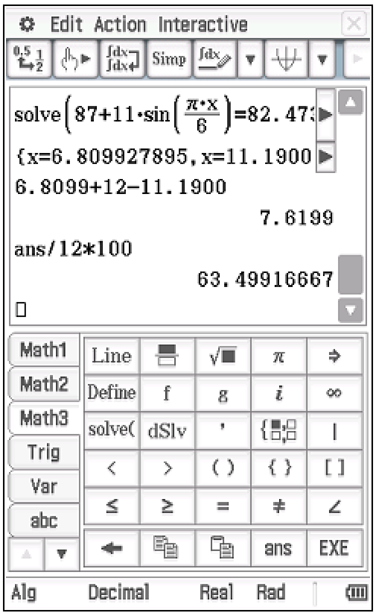
Worked solution

Use CAS to solve

$$87+11\sin\left(\frac{\pi t}{6}\right)=d(10)+5$$

$$87+11\sin\left(\frac{\pi t}{6}\right)=82.47372$$





So the depth of snow is above $d(10)+5$ cm for $6.8099+12-11.1900=7.6199$ hours.

As a percentage $\frac{7.6199}{12}\times100=63.50\%$

Mark allocation: 3 marks

- 1 method mark for setting $87+11\sin\left(\frac{\pi t}{6}\right)=82.47372$
- 1 answer mark for finding $t=6.8099$ *and* $t=11.1900$
- 1 answer for mark 63.50%

Tip

- *You need to give your answer as a decimal correct to two decimal places. An exact value would not be accepted. Always work to a higher number of decimal places before rounding off.*

Question 2a.i.

Worked solution

Use CAS to solve $f(x)=1$ gives $x=-0.172$

Mark allocation: 1 mark

- 1 answer mark for $x=-0.172$

Question 2a.ii.

Worked solution

Use CAS to find x such that $f(x)=1$ gives $x=-0.172,\ 0.417,\ 1$

So the interval such that $f(x)>1$ is $(-\infty,-0.172)\cup(0.417,\ 1.000)$

Mark allocation: 2 marks

- 1 method mark for finding $x=-0.172,\ 0.417,\ 1$
- 1 answer mark for correct interval $(-\infty,\ -0.172)\cup(0.417,\ 1.000)$

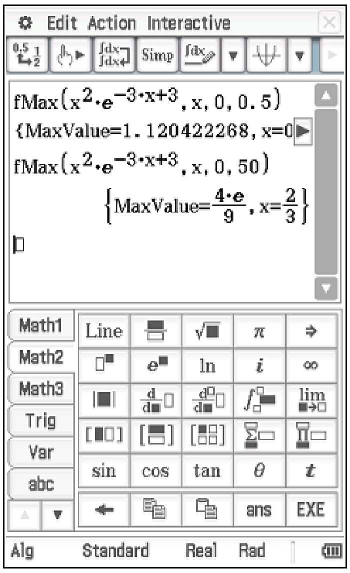
Tip

- *The question requires answers given to three decimal places. This means that 1 is written as 1.000.*

Question 2a.iii.

Worked solution

Use CAS to find the local maximum value at *A*.



Mark allocation: 1 mark

- 1 mark for $\left(\frac{2}{3}, \frac{4e}{9}\right)$

Tip

- *An exact answer is required; so remember to have your CAS set to give an exact value.*

Question 2b.

Worked solution

Let a be the x value of the point on the graph. The coordinates of this point are $(a,\, a^2e^{-3a+3})$.

If the tangent line passes through the origin then the gradient of this line is $m = \dfrac{a^2e^{-3a+3}}{a} = ae^{-3a+3}$

Also $f(x) = x^2e^{-3x+3}$, $f'(x) = 2xe^{-3x+3} - 3x^2e^{-3x+3}$

$$= x(2-3x)(e^{-3x+3})$$

and $f'(a) = a(2-3a)(e^{-3a+3})$

so if $f'(a) = m$

$(2-3a) = 1$

$a = \dfrac{1}{3}$

$f\left(\dfrac{1}{3}\right) = \dfrac{1}{9}e^2$

Point is $\left(\dfrac{1}{3}, \dfrac{1}{9}e^2\right)$

Mark allocation: 3 marks

- 1 method mark for finding $m = \dfrac{a^2e^{-3a+3}}{a} = ae^{-3a+3}$
- 1 answer mark for finding $f'(a) = a(2-3a)(e^{-3a+3})$
- 1 answer mark for $\left(\dfrac{1}{3}, \dfrac{1}{9}e^2\right)$

Tip

- *You can check your answer by using CAS to find the equation of the tangent at the point you have found and see if it goes through the origin.*

Question 2c.i.

Worked solution

$f(x) = x^m e^{-nx+n}$

$f'(x) = mx^{m-1}e^{-nx+n} + -nx^m e^{-nx+n}$

$\quad = x^{m-1}e^{-nx+n}\left(m - nx\right)$

let $f'(x) = 0 \quad \Rightarrow x^{m-1} = 0$ or $m - nx = 0$

$x = 0$ (not possible) or $x = \dfrac{m}{n}$

Mark allocation: 3 marks

- 1 answer mark for finding $f'(x)$
- 1 method mark for setting $f'(x) = 0$
- 1 answer mark for getting $x = \dfrac{m}{n}$

Question 2c.ii.

Worked solution

Let $x = a$ lie on the curve. The coordinates of the point are $\left(a,\ a^m e^{-an+n}\right)$.

If the tangent drawn at $x = a$ passes through the origin then

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{a^m e^{-an+n} - 0}{a - 0} = \frac{a^m e^{-an+n}}{a} = a^{m-1}e^{-an+n}$$

and

$$f'(x) = x^{m-1}e^{-nx+n}(m - nx)$$
$$at\ x = a,\ \ f'(a) = a^{m-1}e^{-na+n}(m - na)$$

Let $f'(a) = m$

$$a^{m-1}e^{-na+n}(m - na) = a^{m-1}e^{-na+n}$$
$$a^{m-1}e^{-na+n}(m - na) - a^{m-1}e^{-na+n} = 0$$
$$a^{m-1}e^{-na+n}(m - na - 1) = 0$$
$$\text{so } a^{m-1} = 0\ \ or\ \ m - na - 1 = 0$$
$$a = 0 \quad or \quad a = \frac{m-1}{n}$$

Mark allocation: 3 marks

- 1 method mark for finding $f'(a)$
- 1 method mark for setting $f'(a) = m$
- 1 answer mark for solving equation to obtain correct answer

Question 3a.

Worked solution

$X_c \sim N(\mu = 15, \sigma = 4)$

$\Pr(X_c > c) = 0.15$

Use CAS to find c gives c = 19.145

So c = 191 mm

Mark allocation: 1 mark

- 1 answer mark for 191 mm

Tip

- *Take note of the units required. Here the question is given in cm and the answer is required in mm.*

Question 3b.

Worked solution

$\Pr(X_c < 9) = 0.0668$

$0.0668 \times 4000 = 267$

Mark allocation: 2 marks

- 1 answer mark for 0.0668
- 1 answer mark for 267 plants.

Tip

- *Remember to give your answer as the number of plants, not just the probability.*

Question 3c.

Worked solution

$$k\int_{6}^{16}\left(\sin\left(\frac{\pi(x-6)}{10}\right)\right)dx=1$$

Using CAS to evaluate the definite integral gives

$$k\times\frac{20}{\pi}=1$$

$$k=\frac{\pi}{20}$$

Mark allocation: 2 marks

- 1 answer mark for setting $k\int_{6}^{16}\left(\sin\left(\frac{\pi(x-6)}{10}\right)\right)dx=1$
- 1 answer mark for $\int_{6}^{16}\left(\sin\left(\frac{\pi(x-6)}{10}\right)\right)dx=\frac{20}{\pi}$ leading to correct value for k.

Tip

- *The area under a probability density function is always 1.*

Question 3d.

Worked solution

Using the symmetry of the sine curve gives mean as half way between 6 and 16 as 11.

Mark allocation: 1 mark

- 1 answer mark for the answer of 11.

Tip

- *When dealing with a symmetrical distribution such as sine, make good use of the symmetry for determining the mean.*

Question 3e.

Worked solution

$\Pr(X_r < c) = 0.1$

$$0.1 = \int_6^c \frac{\pi}{20}\sin\left(\frac{(\pi(x-6))}{10}\right)dx$$

Use CAS to solve gives

$$c = 8.04$$

$$c = 80 \text{ mm}$$

Mark allocation: 2 marks

- 1 method mark for writing the integral from 6 to *c* equal to 0.1
- 1 answer mark for 80 mm

Question 3f.

Worked solution

There are six ways that she could choose exactly two tall plants from the next four.

STTS, SSTT, STST, TSST, TTSS, TSTS

$$\Pr(STTS) = \frac{3}{4}\times\frac{1}{3}\times\frac{1}{4}\times\frac{3}{4} \qquad \Pr(\text{TSST}) = \frac{1}{4}\times\frac{3}{4}\times\frac{2}{3}\times\frac{1}{3}$$

$$\Pr(SSTT) = \frac{3}{4}\times\frac{2}{3}\times\frac{1}{3}\times\frac{1}{4} \qquad \Pr(\text{TTSS}) = \frac{1}{4}\times\frac{1}{4}\times\frac{3}{4}\times\frac{2}{3}$$

$$\Pr(STST) = \frac{3}{4}\times\frac{1}{3}\times\frac{3}{4}\times\frac{1}{3} \qquad \Pr(\text{TSTS}) = \frac{1}{4}\times\frac{3}{4}\times\frac{1}{3}\times\frac{3}{4}$$

The sum of these probabilities is $\frac{13}{48}$

Mark allocation: 2 marks

- 1 method mark for determining the six ways: STTS, SSTT, STST, TSST, TTSS and TSTS
- 1 answer mark for $\frac{13}{48}$ must be an exact value

Tip

- *There are six ways of choosing two tall plants from four because* ${}^4C_2 = 6$.

Question 3g.i.

Worked solution

If the sample proportion is $\hat{p} = 0.3$ and the sample size is 20, then the number of diseased plants in the sample is $0.3\times$ 20 =6.

Thus

$$\begin{aligned}\Pr(\hat{P} = 0.3) &= \Pr(X = 6)\\ &= \binom{20}{6}(0.3)^6(0.7)^{14}\\ &= 0.1916\end{aligned}$$

Mark allocation: 2 marks

- 1 method mark for finding $X = 6$
- 1 answer mark for 0.1916

Question 3g.ii.

Worked solution

$$\begin{aligned}sd(\hat{P}) &= \sqrt{\frac{p(1-p)}{n}}\\ &= \sqrt{\frac{0.3\times 0.7}{20}} = 0.1025\end{aligned}$$

Since

$$0.3 - 2\times 0.1025 = 0.095$$
$$0.3 + 2\times 0.1025 = 0.505$$

we find

$$\begin{aligned}&\Pr(0.095 \le \hat{P} \le 0.505)\\ &= \Pr(1.9 \le X \le 10.1)\\ &= \Pr(2 \le X \le 10)\\ &= 0.9752 \text{ using CAS}\end{aligned} \qquad \text{and} \qquad X \sim Bi(n = 20,\ p = 0.3)$$

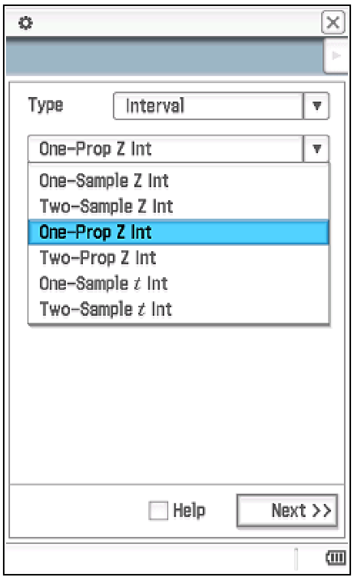
Mark allocation: 3 marks

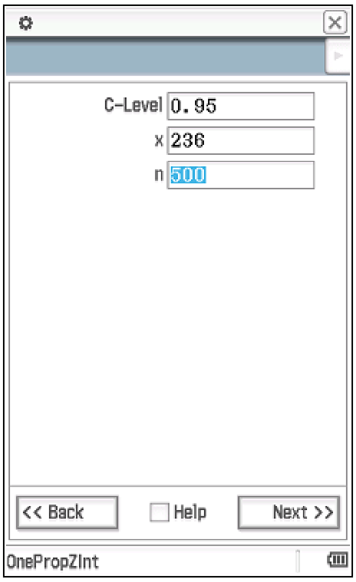
- 1 method mark for finding standard deviation of 0.1025
- 1 method mark for finding the endpoints of 0.095 and 0.505
- 1 answer mark for 0.9752

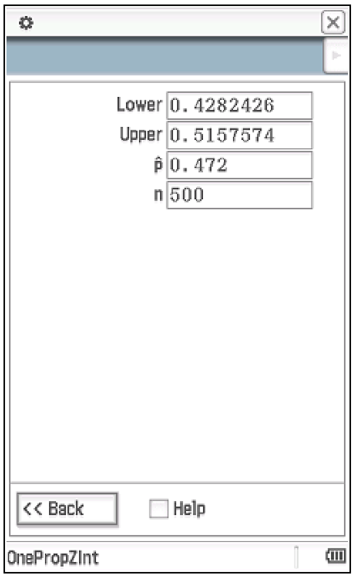
Question 3h.

Worked solution

This can be done using CAS.







The 95% confidence interval is $(0.428,\ 0.516)$.

Mark allocation: 1 mark

- 1 answer mark for $(0.428,\ 0.516)$

Question 3i.

Worked solution

This experiment is a binomial distribution with $Y \sim Bi(n=5,\ p=\frac{26}{27})$.

Using CAS $\Pr(X=3)=0.0122$.

Mark allocation: 2 marks

- 1 method mark for recognising the binomial distribution
- 1 answer mark for 0.0122

Question 4a.

Worked solution

$64x - x^4$

$= x(64 - x^3)$

$= x(4 - x)(x^2 + 4x + 16)$

$= x(4 - x)((x + 2)^2 + 12)$

Mark allocation: 2 marks

- 1 method mark for recognising the difference of 2 cubes
- 1 answer mark for getting to the correct form

Tip

- *Always look to use common factor factorisation.*

Question 4b.

Worked solution

It is observed that $g(-(x+2)) = f(x)$, so the graph of $g(x)$ is reflected in the *y*-axis and translated 2 units to the left.

Mark allocation: 1 mark

- 1 mark for correct transformation in correct order

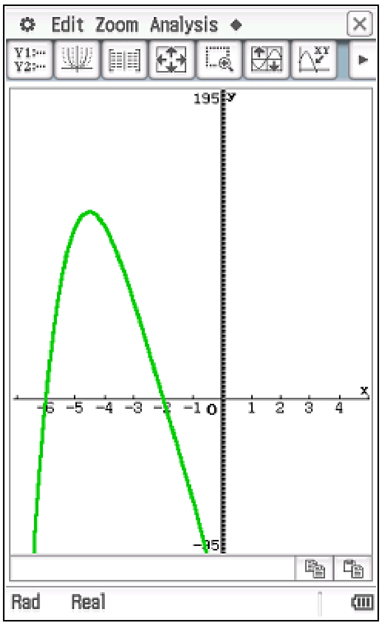
Tip

- *Use CAS to sketch the graphs of* $f(x)$ *and* $g(x)$ *to help to see the transformations.*

Question 4c.i.

Worked solution

Looking at the graph, it can be seen that the graph can be shifted at most two units to the right. So $d < 2$.



Mark allocation: 1 mark

- 1 answer mark for $d < 2$

Question 4c.ii.

Worked solution

If the graph is shifted more than two units to the right then the graph will have at least one positive x- intercept. So $d > 2$.

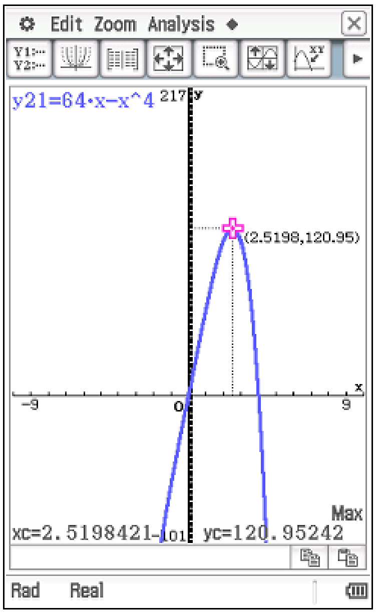
Mark allocation: 1 mark

- 1 mark for correct answer $d > 2$

Question 4d.

Worked solution

Using CAS the maximum turning point of the graph occurs at $(2.52, 120.95)$. So the equation $g(x) = n$ would have one solution if $n = 120.95$.



Mark allocation: 1 marks

- 1 answer mark for $n = 120.95$

Tip

- *Questions like part c. and d. are best undertaken using a graphical approach.*

Question 4e.

Worked solution

$h'(x) = k - 4x^3$

$h'(x) = 0$

Let $\Rightarrow 4x^3 = k$

$\Rightarrow x = \sqrt[3]{\frac{k}{4}} = \left(\frac{k}{4}\right)^{\frac{1}{3}}$

$$h\left(\left(\frac{k}{4}\right)^{\frac{1}{3}}\right) = k\left(\left(\frac{k}{4}\right)^{\frac{1}{3}}\right) - \left(\frac{k}{4}\right)^{\frac{4}{3}}$$

$$= \frac{4k}{4}\left(\left(\frac{k}{4}\right)^{\frac{1}{3}}\right) - \left(\frac{k}{4}\right)^{\frac{4}{3}}$$

$$= 4\left(\left(\frac{k}{4}\right)^{\frac{4}{3}}\right) - \left(\frac{k}{4}\right)^{\frac{4}{3}} = 3\left(\frac{k}{4}\right)^{\frac{4}{3}}$$

Mark allocation: 3 marks

- 1 answer mark for finding $x = \sqrt[3]{\frac{k}{4}} = \left(\frac{k}{4}\right)^{\frac{1}{3}}$
- 1 method mark for substituting into $h(x)$
- 1 answer mark for obtaining $3\left(\frac{k}{4}\right)^{\frac{4}{3}}$

Tip

- *With 'show that' questions, make sure each step in the solution process is clear.*

Question 4f.i.

Worked solution

Use CAS. The area is 307.2 square units.

Mark allocation: 1 mark

- 1 mark for correct answer of 307.2

Question 4f.ii.

Worked solution

The graph of $y = f\left(\frac{-x}{2}\right)$ is the graph of $y = f(x)$ reflected in the *y*-axis and then dilated by a factor of 2 in the *x*-direction.

The area bounded by the graph of $y = f\left(\frac{-x}{2}\right)$ will be equal to $2 \times \text{area bounded by } g(x)$.

Area is $2 \times 307.2 = 614.4$ square units.

Mark allocation: 1 mark

- 1 answer mark for 614.4, but it must be obtained as 2 times the previous answer

Tip

- *Remember for 'hence' questions you must use the previous answer and it must be clear how you have used it.*

Question 5a.

Worked solution

$V = \pi r^2 h$

$h = \dfrac{V}{\pi r^2}$

Mark allocation: 1 mark

- 1 answer mark for $h = \dfrac{V}{\pi r^2}$

Question 5b.

Worked solution

$$\begin{aligned} A &= 2\pi r^2 + 2\pi rh \\ &= 2\pi r^2 + 2\pi r\left(\frac{V}{\pi r^2}\right) \\ &= 2\pi r^2 + \left(\frac{2V}{r}\right) \end{aligned}$$

Mark allocation: 2 marks

- 1 method mark for substituting into $A = 2\pi r^2 + 2\pi rh$
- 1 answer mark for obtaining $A = \dfrac{2V}{r} + 2\pi r^2$

Tip

- *Remember when differentiating this, V is a constant and is treated as a number not as a variable.*

Question 5c.

Worked solution

Minimum surface area occurs when $\dfrac{dA}{dr} = 0$

$$\frac{dA}{dr} = 4\pi r - 2Vr^{-2}$$

$$= 4\pi r - \frac{2V}{r^2}$$

$$let\ \frac{dA}{dr} = 0$$

$$\Rightarrow 4\pi r = \frac{2V}{r^2}$$

$$r^3 = \frac{2V}{4\pi} = \frac{V}{2\pi}$$

$$r = \sqrt[3]{\frac{V}{2\pi}}$$

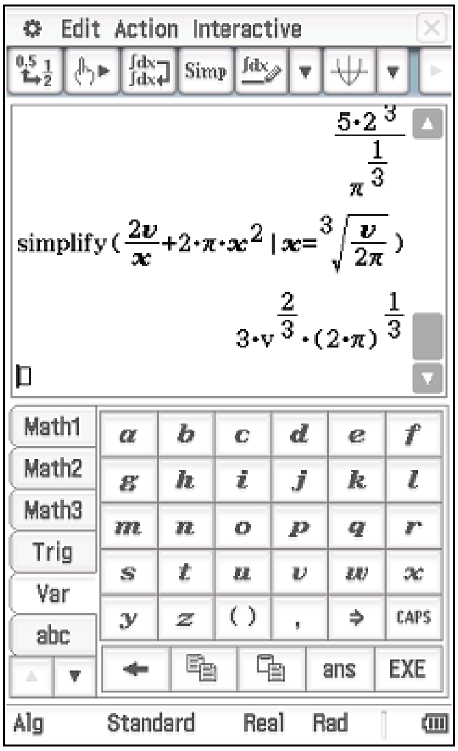
Mark allocation: 2 marks

- 1 method mark for finding $\dfrac{dA}{dr}$ and setting it equal to zero
- 1 answer mark for $r = \sqrt[3]{\dfrac{V}{2\pi}}$

Question 5d.

Worked solution

Using CAS evaluate A *when* $r = \sqrt[3]{\dfrac{V}{2\pi}}$



So $a = 3$ *and* $b = 2$

Mark allocation: 3 marks

- 1 method mark for substituting $r = \sqrt[3]{\dfrac{V}{2\pi}}$ into A
- 1 mark for $a = 3$
- 1 mark for $b = 2$

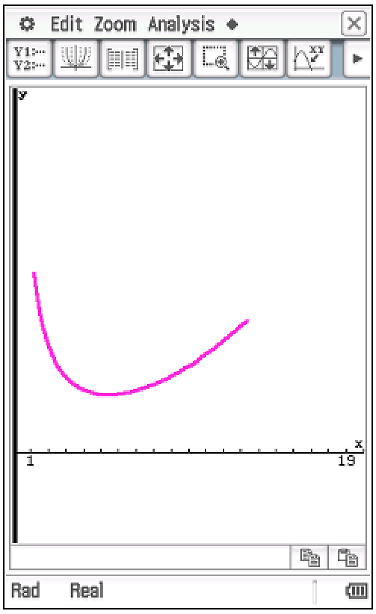
Question 5e.

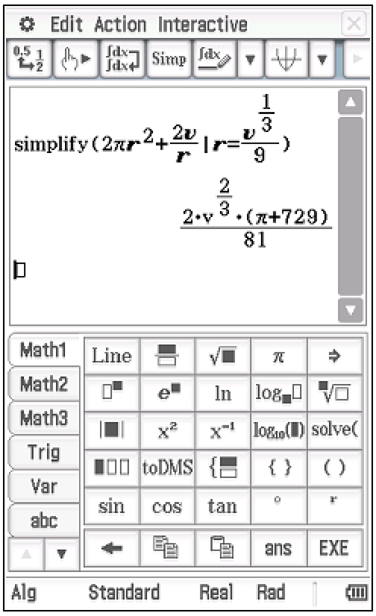
Worked solution

The maximum surface area will occur at the endpoints of the function.

Use CAS and let $V = 1000$ to assist with the sketching.

The maximum surface occurs at the left most endpoint; that is, $r = \dfrac{V^{\frac{1}{3}}}{9}$





So the maximum surface area is $\dfrac{2V^{\frac{2}{3}}(\pi + 729)}{81}$

Mark allocation: 2 marks

- 1 answer mark for determining that maximum surface area occurs at $r = \dfrac{V^{\frac{1}{3}}}{9}$
- 1 answer mark for $\dfrac{2V^{\frac{2}{3}}(\pi + 729)}{81}$

END OF WORKED SOLUTIONS