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Year 12 *Trial Exam Paper*

2016

MATHEMATICAL METHODS

Written examination 1

Reading time: 15 minutes
Writing time: 1 hour

STUDENT NAME:

QUESTION AND ANSWER BOOK

Structure of book

Number of questions	*Number of questions to be answered*	*Number of marks*
10	10	40

- Students are permitted to bring the following items into the examination: pens, pencils, highlighters, erasers, sharpeners and rulers.
- Students are NOT permitted to bring blank sheets of paper, notes of any kind or correction liquid/tape into the examination.
- Calculators are NOT permitted in this examination.

Materials provided

- Question and answer book of 11 pages with a separate sheet of miscellaneous formulas.
- Working space is provided throughout this book.

Instructions

- Write your name in the box provided.
- Remove the formula sheet during reading time.
- Unless otherwise indicated, the diagrams in this book are not drawn to scale.
- You must answer the questions in English.

Students are NOT permitted to bring mobile phones or any other unauthorised electronic devices into the examination.

Instructions

Answer **all** questions in the spaces provided.

Provide exact answers to all questions where a numerical answer is required, unless otherwise stated.

In questions where more than one mark is available, appropriate working **must** be shown.

Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Question 1 (4 marks)

a. If $y = x^3 \cos(x)$, find $\frac{dy}{dx}$.

2 marks

b. If $f(x) = \sqrt{2 - x^2}$, find $f'(1)$.

2 marks

Question 2 (2 marks)

For $x > \frac{2}{3}$, let $\int_4^7 \frac{3}{3x-2}\,dx = \log_e(k)$.

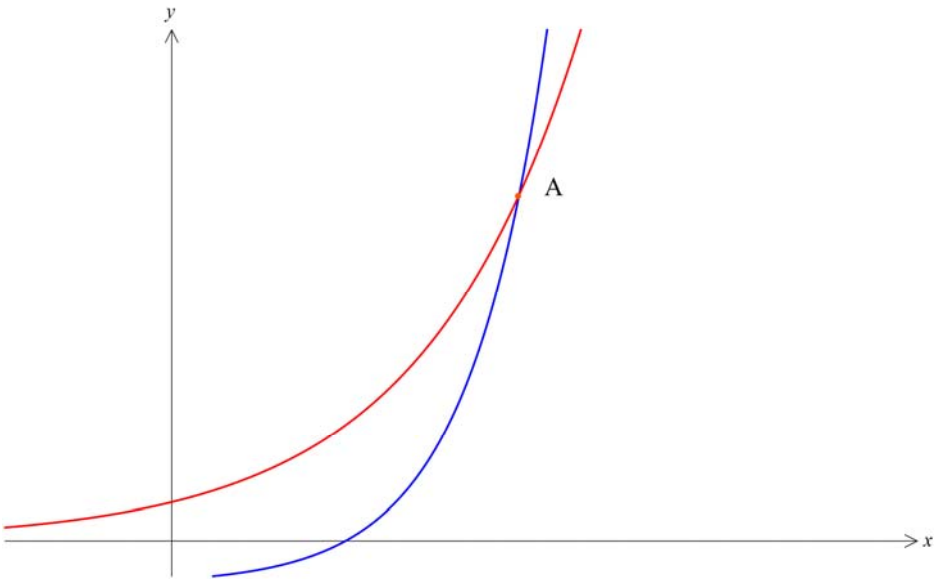
Find the value of *k*.

Question 3 (2 marks)

Solve $\sqrt{2}\sin\left(2\pi x + \frac{\pi}{4}\right) - 1 = 0$, where $0 \leq x \leq 1$.

Question 4 (3 marks)

Parts of the graphs of functions $f(x) = 9^x - 9$ and $g(x) = 8(3^x)$ are shown below.



The graphs intersect at the point labelled *A*. Find the coordinates of *A*.

Question 5 (2 marks)

Solve $\log_e(x) - 4 = \log_e(x+6)$ for *x*, where $x > 0$.

Question 6 (7 marks)

Consider the function $f:(-1,5] \to R,\ f(x) = \frac{1}{5}x^3(x-4)$.

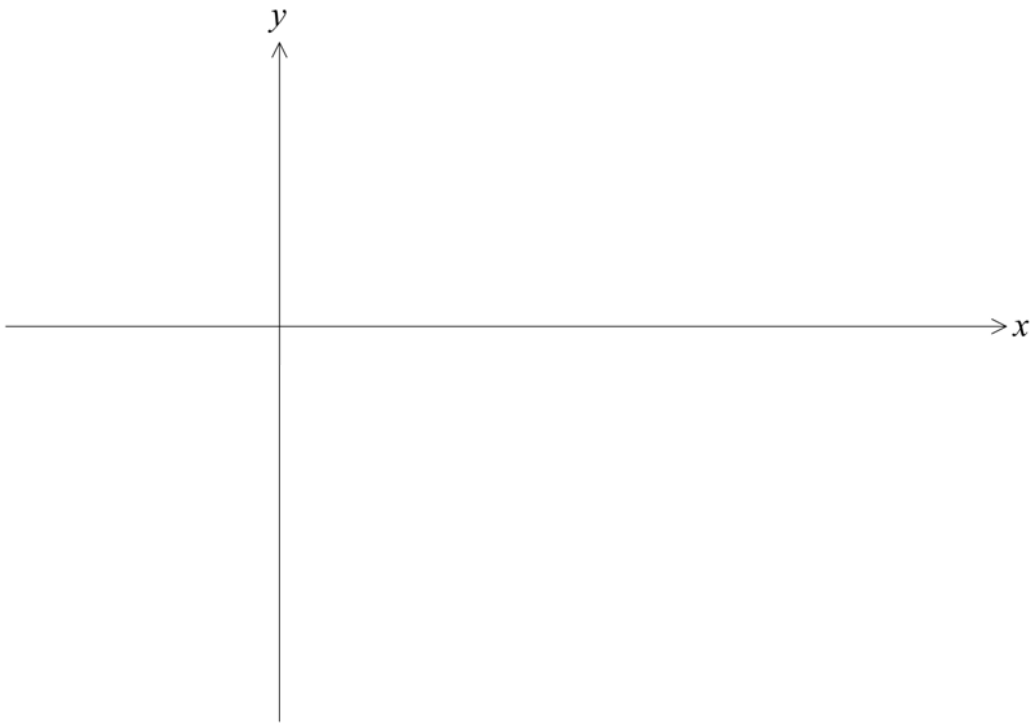
a. Find the coordinates of the stationary points of the function.

2 marks

b. On the axes below, sketch the graph of *f*.

Label end points, stationary points and axial intercepts with their coordinates.

3 marks



c. Find the equation of the tangent to the curve at $x = 1$.

2 marks

Question 7 (3 marks)

When $f'(x) = 3\sin(x) - \cos\left(\frac{x}{2}\right)$ and $f\left(\frac{\pi}{3}\right) = 1$, find $f(x)$.

Question 8 (6 marks)

A fish tank contains eight gold and eight red fish.

Let p be the proportion of red fish in the tank. A sample of three fish is taken from the tank without replacement.

a. What are the possible values of the sample proportion $\hat{p}$ of red fish in the sample?

1 mark

_____

_____

b. Determine the probability that the proportion of red fish in the sample is more than 0.25.

2 marks

_____

_____

_____

_____

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_____

c. Find $\Pr\left(\hat{p} = \frac{1}{3} \mid \hat{p} > \frac{1}{4}\right)$.

3 marks

Question 9 (5 marks)

A continuous random variable, X, has a probability density function

$$f(x) = \begin{cases} \frac{\pi}{2}\sin\left(\frac{\pi x}{2}\right) \text{ when } x \in [0, a] \\ 0, \text{ otherwise} \end{cases}.$$

a. Find the value of a.

2 marks

b. Given that $\frac{d}{dx}\left(x\cos\left(\frac{\pi x}{2}\right)\right) = \frac{-\pi x}{2}\sin\left(\frac{\pi x}{2}\right) + \cos\left(\frac{\pi x}{2}\right)$, find $E(X)$.

3 marks

Question 10 (6 marks)

a. The tangent to the graph $y = \sqrt{a + 2x}$ at $x = 4$ passes through the origin. Find the value of a.

3 marks

b. For this value of a, find the area bounded by the graph, the tangent and the x-axis.

3 marks

END OF QUESTION AND ANSWER BOOK