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NAME: Solutions. YEAR/HOUSE: ________________

TEACHER'S NAME: (Students please circle) Mr Jones Mr James Mrs Itter

SEMESTER 2 EXAMINATIONS NOVEMBER 2016

Year Eleven Mathematical Methods – Exam 1

Reading time: 15 Minutes
Writing time: 60 Minutes

Marks Allocated:

Section	Number of Questions	Number of Marks
Short Answer	10 Questions	40 Marks

Specific Instructions

- Calculators, summary books or aids of any kind are NOT permitted in this exam.
- Answer all questions in the spaces provided.
- In all questions where a numerical answer is required, an exact value must be given unless otherwise specified.
- In questions where more than one mark is available, appropriate working must be shown.
- Unless otherwise indicated, the diagrams in this book are not drawn to scale.

Supplies and Equipment

Please use only permitted supplies and equipment, as described above, in taking your examination. Any other items not listed on the examination instructions are prohibited from being used unless specific permission is given by the examination supervisor.

At the end of the examination

Wait quietly for specific instruction as to how you will be dismissed. Leave your examination paper on your table. Pick up unwanted papers around you, push your chair under the table, and put your rubbish in the bin on your way out of the examination room.

Question 1 (3 marks)

Solve the following for x.

a. $5^{2x+3} = \frac{1}{25}$ 1 mark

$5^{2x+3} = \frac{1}{5^2}$ $\rightarrow$ $2x + 3 = -2$

$5^{2x+3} = 5^{-2}$ $\quad 2x = -5$

$x = -5/2$

b. $\log_2(3x) + 2\log_2(x) - \log_2(6) = -1$ 2 marks

$\log_2 3x + \log_2 x^2 - \log_2 6 = -1$

$\log_2 \frac{3x \times x^2}{6} = -1$ $\rightarrow$ $1/2 = 3x^3/6$

$\log_2 3x^3/6 = -1$ $\quad 1/2 = x^3/2$

$2^{-1} = 3x^3/6$ $\quad x^3 = 1$

$x = 1$

Question 2 (6 marks)

Consider the polynomial $P(x) = x^3 - x^2 - 2x + 2$. The equation $P(x) = 0$ has three real solutions.

a. Show that $x - 1$ is a linear factor. 1 mark

if $(x-1)$ is a linear factor, then

$P(1) = 0$.

$P(1) = 1^3 - 1^2 - 2(1) + 2$

$= 1 - 1 - 2 + 2$

$= 0$

$\therefore$ $x - 1$ is a linear factor of $P(x) = x^3 - x^2 - 2x + 2$

b. Hence or otherwise find all the linear factors of $P(x)$. 2 marks

$$
\begin{array}{r}
x^2 + 0x^2 - 2 \\
x-1 \;\big)\; x^3 - x^2 - 2x + 2 \\
-(x^3 - x^2) \\
\hline
0 - 2x + 2 \\
-(-2x + 2) \\
\hline
0
\end{array}
$$

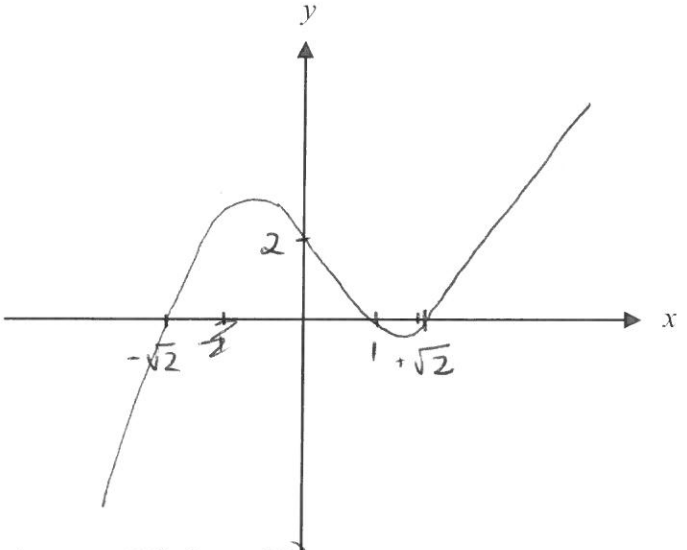
$(x-1)(x^2-2)$

$(x-1)(x-\sqrt{2})(x+\sqrt{2})$

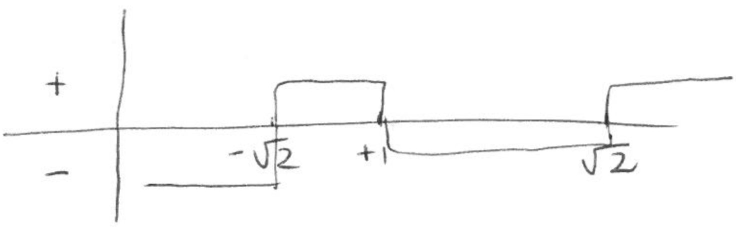
c. Sketch the graph of $P(x) = x^3 - x^2 - 2x + 2$ on the set of axes below.
Indicate clearly all axes intercepts. It is not necessary to find the turning points. 3 marks

y-int, $x=0$

$P(0) = 2$



$P(x) = (x-1)(x-\sqrt{2})(x+\sqrt{2})$



Question 3 (3 marks)

a. If $y = 3x^2 - 5x - 2$, find $\dfrac{dy}{dx}$. 1 mark

$dy/dx = 6x - 5$

b. Given that $g(x) = 4\sqrt{x} - \dfrac{3}{x^2}$, find $g'(x)$. 2 marks

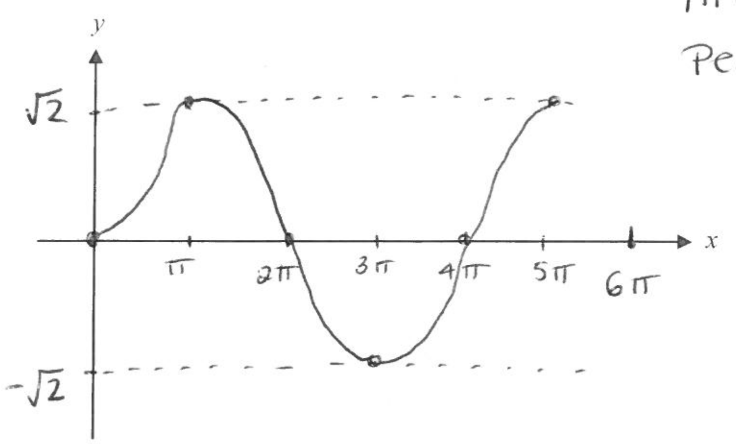
$g(x) = 4x^{1/2} - 3x^{-2}$

$g'(x) = 2x^{-1/2} + 6x^{-3}$

$g'(x) = \dfrac{2}{\sqrt{x}} + \dfrac{6}{x^3}$

Question 4 (6 marks)

a. On the set of axes below, sketch the graph of $y = \sqrt{2}\sin\left(\dfrac{x}{2}\right)$ for $x \in [0, 6\pi]$. Indicate clearly any axes intercepts and endpoints as well as the amplitude of the graph. 3 marks



Amp $= \sqrt{2}$

Period $= \dfrac{2\pi}{1/2}$

$= 2\pi \times \dfrac{2}{1}$

$= 4\pi$

b. Solve the equation $\sqrt{2}\sin\left(\frac{x}{2}\right)=1$ for $x \in \underline{[0,6\pi]}$. 3 marks

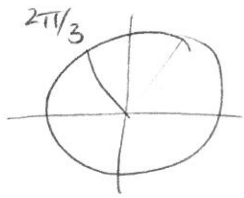
$\sqrt{2}\sin\left(\frac{x}{2}\right) = 1$ $\qquad 0 \leq x \leq 6\pi$

$\sin\left(\frac{x}{2}\right) = \frac{1}{\sqrt{2}}$ $\qquad \therefore 0 \leq \frac{x}{2} \leq 3\pi$

$\frac{x}{2} = \pi/4,\ 3\pi/4,\ 9\pi/4,\ 11\pi/4$

$x = \pi/2,\ 3\pi/2,\ 9\pi/2,\ 11\pi/2$

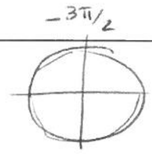
Question 5 (4 marks)



a. Evaluate

i. $\cos\left(\frac{2\pi}{3}\right)$ 1 mark

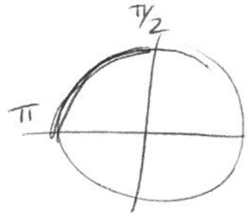
$= -1/2$



ii. $\sin\left(-\frac{3\pi}{2}\right)$ 1 mark

$= 1$

b. Given $\sin(\theta) = 0.6$ and $\frac{\pi}{2} < \theta < \pi$, evaluate



i. $\cos(\theta)$ 1 mark

$\sin^2\theta + \cos^2\theta = 1$

$0.6^2 + \cos^2\theta = 1$ $\qquad \cos^2\theta = 0.64$

$0.36 = \cos^2\theta = 1$ $\qquad \cos\theta = 0.8$

2nd Quadrat, $\cos\theta = \underline{-0.8}$.

ii. $\tan(\theta)$ 1 mark

$\tan\theta = \frac{\sin\theta}{\cos\theta}$

$= \frac{0.6}{-0.8}$

$= -6/8$

$= -3/4$

Question 6 (3 Marks)

A particular weekly flight is late to leave 30% of the time. Find the probability that in the coming four weeks this particular flight is late

a. on all four occasions.

$(0\cdot 3)^4 = \left(\frac{3}{10}\right)^4$

$= 3^4/10^4$ or $0\cdot 0081$

$= 81/10000$

1 mark

b. on at least one occasion.

$1 - 0\cdot 7^4 = 1 - \left(\frac{7}{10}\right)^4$

$= 1 - \frac{2401}{10000}$

$= \frac{7599}{10000}$

$= 0\cdot 7599.$

2 marks

Question 7 (4 marks)

A group of two boys and three girls line up in a straight line.

c. In how many different ways can the children be arranged in this line? 1 mark

$5! = 120$

Two of the children are randomly selected from the group.

d. How many different selections can be made? 1 mark

${}^5C_2 = \frac{5!}{2!\,3!} = \frac{20}{2}$

$= \frac{5\times 4\times \cancel{3}\times \cancel{2}\times \cancel{1}}{2\times 1\times \cancel{3}\times \cancel{2}\times \cancel{1}} = 10$

i. What is the probability that one boy and one girl are selected? 2 marks

$\frac{{}^2C_1 \times {}^3C_1}{{}^5C_1} = \frac{2\times 3}{10}$

$= \frac{6}{10}$ $3/5$

$= 0\cdot 6$ or

Question 8 (4 marks)

The gradient of a straight line is 2. The straight line is a tangent to the curve with equation $y = x^2 - 4x + 1$. Find the equation of the straight line.

$dy/dx = 2x - 4$

$2 = 2x - 4$

$6 = 2x$

$x = 3.$

when $x = 3$, $y = 3^2 - 4 \times 3 + 1$

$y = -2$

$m = 2, \quad (3, -2)$

$y - y_1 = m(x - x_1)$

$y - -2 = 2(x - 3)$

$y + 2 = 2x - 6$

$y = 2x - 8$

Question 9 (3 marks)

a. Show that the stationary points of the graph of $y = x^4 - 5x^3 - 2x^2$ occur at the points where $x = -\frac{1}{4}$, $x = 0$ and $x = 4$. 2 marks

$dy/dx = 4x^3 - 15x^2 - 4x$

$0 = 4x^3 - 15x^2 - 4x$

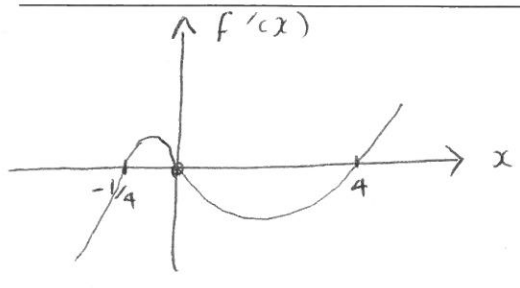
$0 = x(4x^2 - 15x - 4)$

$0 = x(4x + 1)(x - 4)$

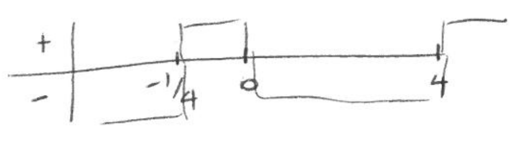
S.P. occur when $x = 0, -1/4$ or 4

b. Find the values of x for which the function $f: [-1, 1] \to R$, $f(x) = x^4 - 5x^3 - 2x^2$ has a positive gradient. 1 mark

$(-1/4, 0)$

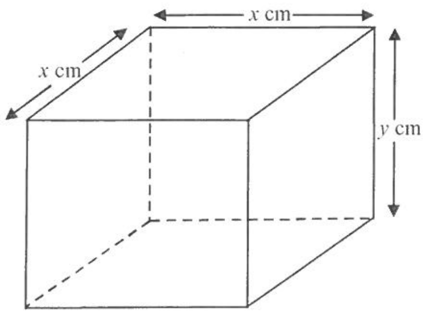


$f'(x) = x(4x+1)(x-4)$



Question 10 (4 marks)

A cardboard box is in the shape of a square prism with side lengths of x cm and y cm as shown in the diagram below.



The sum of all the side lengths of the box is 120 cm.

a. Show that the total surface area A, in cm^2, of the box is given by $A = -6x^2 + 120x$. 2 marks

$120 = 8x + 4y$

$4y = 120 - 8x$

$y = 30 - 2x$

$A = 2x^2 + 4xy$

$A = 2x^2 + 4x(30 - 2x)$

$A = 2x^2 + 120x - 8x^2$

$A = -6x^2 + 120x$

b. Find the maximum surface area of the box and the value of x when this maximum occurs. 2 marks

$dA/dx = -12x + 120$

$0 = -12x + 120$

$12x = 120$

$x = 10$

$x = 10\text{cm}$

Max Surface Area:

when $x = 10$

$A = -6 \times 10^2 + 120 \times 10$

$= -6 \times 100 + 1200$

$= 1200 - 600$

$= 600\text{ cm}^2$

END