TSSM™
Creating VCE Success

‘2016 Examination Package’ - Trial Examination 5 of 5

THIS BOX IS FOR ILLUSTRATIVE PURPOSES ONLY

STUDENT NUMBER

Figures										Letter
Words										

MATHEMATICAL METHODS

Units 3 & 4 – Written examination 2

(TSSM’s *2015 trial exam updated for the current study design)*

Reading time: 15 minutes
Writing time: 2 hours

QUESTION & ANSWER BOOK

Structure of book

Section	*Number of questions*	*Number of questions to be answered*	*Number of marks*
1	22	22	22
2	5	5	58
			Total 80

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers, a protractor, set-squares, aids for curve sketching, one bound reference, one approved CAS calculator (memory DOES NOT need to be cleared) and, if desired, one scientific calculator.
- Students are NOT permitted to bring into the examination room: blank sheets of paper and/or white out liquid/tape.

Materials supplied

- Question and answer book of 21 pages including answer sheet for multiple-choice questions.

Instructions

- Print your name in the space provided on the top of this page and the multiple-choice answer sheet.
- All written responses must be in English.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic communication devices into the examination room.

SECTION 1 – Multiple-choice questions

Instructions for Section 1

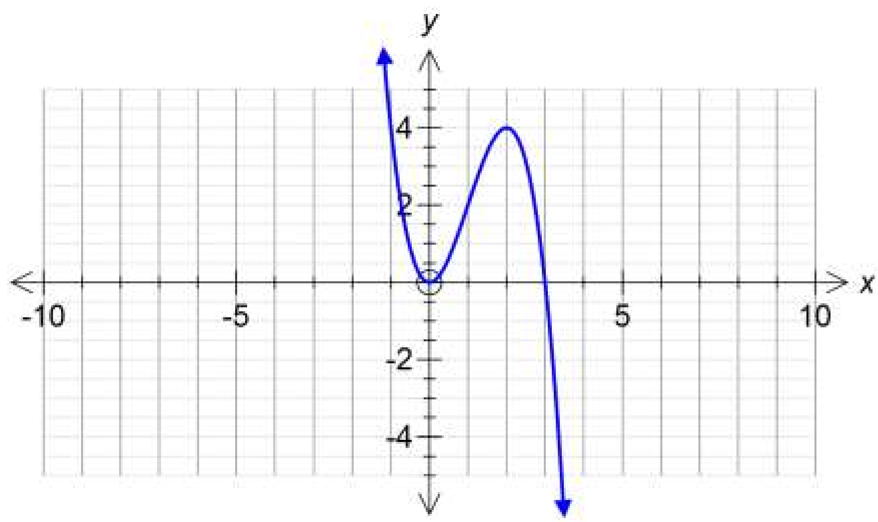
Answer all questions on the answer sheet provided for multiple choice questions.
Choose the response that is **correct** for the question.
A correct answer scores 1, an incorrect answer scores 0.
Marks will **not** be deducted for incorrect answers.
No marks will be given if more than one answer is completed for any question.

Question 1

The *x*-value of the point of intersection of the graphs of $y = 3$ and $y = \frac{5}{2}x + 1$ is:

A. 0.8
B. 1.6
C. 3
D. 5
E. 8

Question 2



The function that represents the graph above is:

A. $y = x^3 - x^2$
B. $y = 3x^3 - x^2$
C. $y = 3x^2 - x^3$
D. $y = x^2 - 3x^3$
E. $y = 3x^4 - x^2$

SECTION 1 – continued

Question 3

If $f(x) = x^2 + 2x + 5$ and $g(x) = x - 1$, then $f(g(x))$ is:

A. $x^2 + 3x + 4$

B. $x^2 - 4$

C. $x^2 + 4x - 1$

D. $x^2 - 1$

E. $x^2 + 4$

Question 4

The function $f(x) = 2\sqrt{x} + 1$ is dilated by a factor of ½ unit from the x-axis to transform to the function $g(x)$. The rule for $g(x)$ is:

A. $g(x) = \sqrt{x} + 1$

B. $g(x) = \sqrt{\frac{x}{2}} + 1$

C. $g(x) = \sqrt{2x} + 1$

D. $g(x) = \sqrt{x} + \frac{1}{2}$

E. $g(x) = \sqrt{\frac{x}{2}} + \frac{1}{2}$

Question 5

The domain and range of the function $f(x) = \sqrt{4 - x}$ respectively are:

A. $[4, \infty)$ *and* $[2, \infty)$

B. $(-\infty, 4]$ *and* R

C. $(-\infty, 4]$ *and* $[0, \infty)$

D. $[4, \infty)$ *and* $(-\infty, 0]$

E. R *and* $[0, \infty)$

SECTION 1 - continued

TURN OVER

Question 6

The average rate of change for the function $g(x) = \sqrt{2x}$ between $x = 2\ \ and\ x = 8$ is:

A. $\frac{1}{3}$

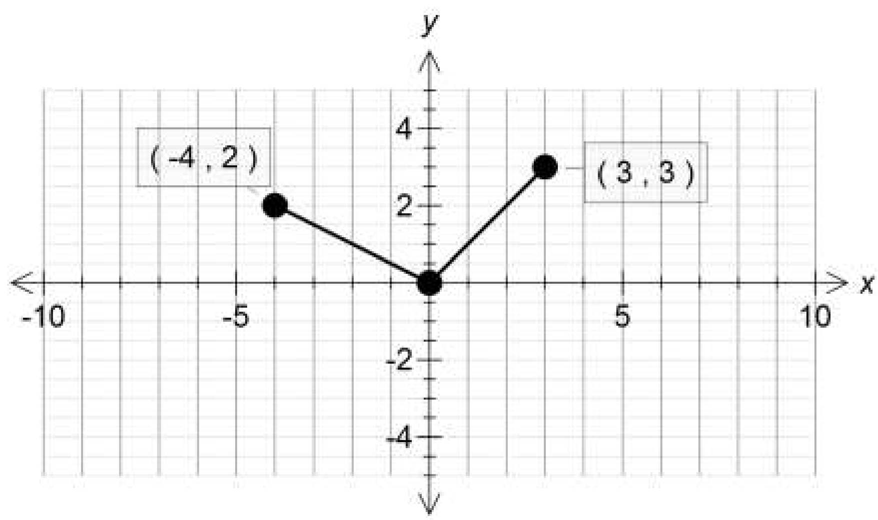
B. $-\frac{3}{10}$

C. 2

D. 3

E. 7

Question 7



The function that describes the graph of the hybrid function $y = f(x)$ shown above is:

A. $f(x) = \begin{cases} \frac{1}{2}x, & -4 < x < 0 \\ x, & 0 \leq x \leq 4 \end{cases}$

B. $f(x) = \begin{cases} \frac{1}{2}x, & -4 < x < 0 \\ \frac{1}{2}x, & 0 \leq x \leq 4 \end{cases}$

C. $f(x) = \begin{cases} -\frac{1}{2}x, & -4 < x < 0 \\ x, & 0 \leq x \leq 3 \end{cases}$

D. $f(x) = \begin{cases} \frac{-1}{2}x, & -4 < x < 0 \\ x, & 0 < x < 4 \end{cases}$

E. $f(x) = \begin{cases} \frac{-1}{2}x, & -4 < x < 0 \\ 2x, & 0 \leq x \leq 3 \end{cases}$

SECTION 1 - continued

Question 8

The domain of the composite function fog where $f(x) = \frac{x}{x+6}$ and $g(x) = \frac{18}{x+2}$ is:

A. R

B. $R\backslash\{-2\}$

C. $R\backslash\{-2, -5\}$

D. $R\backslash\{-2, -6\}$

E. $R\backslash\{0, -2, -5\}$

Question 9

Which of the following functions is one-to-one?

A. $f: R \to R,\ f(x) = x^2$

B. $f: [-1, \infty) \to R,\ f(x) = x^2$

C. $f: [-1, \infty) \to R,\ f(x) = \frac{1}{x^2}$

D. $f: (-\infty, 3] \to R,\ f(x) = 1 - x^2$

E. $f: [1, \infty) \to R,\ f(x) = \frac{1}{x^2}$

Question 10

The amplitude and period of $y = 2\cos\left(\frac{x}{5}\right)$ respectively are:

A. $5\ and\ 4\pi$

B. $2\ and\ \frac{2\pi}{5}$

C. $\frac{1}{2}\ and\ \frac{2\pi}{5}$

D. $2\ and\ 10\pi$

E. $\frac{1}{2}\ and\ 10\pi$

SECTION 1 - continued

TURN OVER

Question 11

The gradient of the tangent line to the graph of $y = -x^2 + 4\sqrt{x}$ at $x = 4$ is:

A. -8

B. -9

C. -10

D. -5

E. -7

Question 12

Let $f(x) = x^3$

Let A_1 be the area bounded by the graphs of $y = -1$ *and* $y = f(x)$ between $x = -1$ *and* $x = 0$ and let A_2 be the area bounded by the graphs of $y = 1$ *and* $y = f(x)$ between $x = 0$ *and* $x = 1$. The total area of these two regions can be represented in the integral form as:

A. $\int_{-1}^{1}(1 - x^3)\,dx$

B. $2\int_{0}^{1}(1 - x^3)\,dx$

C. $2\int_{0}^{1}(1 + x^3)\,dx$

D. $\int_{-1}^{1}(1 + x^3)\,dx$

E. $\int_{0}^{1}(-1 - x^3)\,dx$

Question 13

The equation of the normal line to the graph of $y = 2x\sqrt{x^2 + 8} + 2$ at the point (0, 2) is:

A. $x - 4\sqrt{2}y + 8\sqrt{2} = 0$

B. $x + 4\sqrt{2}y = 8\sqrt{2}$

C. $4\sqrt{2}x + y = 2$

D. $-4\sqrt{2}x + y = 2$

E. $x + 4\sqrt{2}y = 2$

Question 14

Which of the following functions satisfy the condition $\left(f(x)\right)^2 \times \left(f(y)\right)^2 = f(2x + 2y)$?

A. $f(x) = x + y$

B. $f(x) = \sqrt{x}$

C. $f(x) = e^x$

D. $f(x) = log_e x$

E. $f(x) = \frac{1}{x}$

Question 15

The function $f(x) = x^3$ has an average value of 9 on the closed interval [0, *k*].
The value of *k* is:

A. $\sqrt[3]{36}$

B. $\sqrt{3}$

C. $\sqrt[3]{18}$

D. 6

E. 3

Question 16

A test consists of 10 multiple choice questions with five choices for each question. If you guess on each and every answer without even reading the questions, the probability of getting exactly six questions correct is:

A. 0.999

B. 0.006

C. 0.088

D. 0.12

E. 0.046

SECTION 1 - continued

TURN OVER

Question 17

Heights of college women have a distribution that can be approximated by a normal curve with a mean of 165cm and a standard deviation equal to 7.62cm. The proportion of college women between 165 and 170 tall is closest to:

A. 0.50

B. 0.75

C. 0.24

D. 0.15

E. 0.17

Question 18

The probability that a certain machine will produce a defective item is 0.20. If a random sample of 6 items is taken from the output of this machine, the probability that there will be 5 or more defectives in the sample is:

A. 0.0016

B. 0.0154

C. 0.0015

D. 0.2458

E. 0.0001

Question 19

Tina's score on her midterm exam was at the 50th percentile. The grades were normally distributed and the exam average was 78 and the standard deviation was 6. Tina's score on the exam was:

A. 90

B. 50

C. 84

D. 78

E. 72

Question 20

The maximum value of $f(x) = \sin(x) + \cos(2x)$ over $0 \leq x \leq 2\pi$ is:

A. 0

B. 1

C. 1.125

D. 2.5

E. 2.7

SECTION 1 - continued

Question 21

For the following discrete probability distribution, the value of E(X) is closest to:

x	0	2	4	6
Pr(x)	0.15	0.20	k	0.45

A. 0.2

B. 1

C. 3.9

D. 7.8

E. $4k + 0.8$

Question 22

Two successive asymptotes of the function $f(x) = \tan(nx)$ have equations $x = 3$ *and* $x = 6$. The value of n is:

A. π

B. $\frac{\pi}{3}$

C. $\frac{\pi}{2}$

D. $\frac{3\pi}{2}$

E. $\frac{\pi}{6}$

END OF SECTION 1

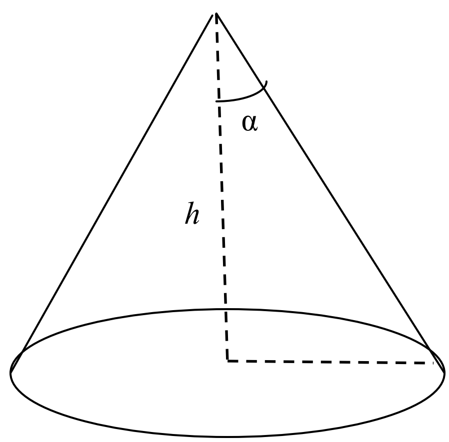
TURN OVER

SECTION 2 – Analysis questions

Instructions for Section 2
Answer **all** questions in the spaces provided.
A decimal approximation will not be accepted if an **exact** answer is required to a question.
In questions where more than one mark is available, appropriate working **must** be shown.
Where an instruction to **use calculus** is stated for a question, you must show an appropriate derivative or anti-derivative.
Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Question 1 (9 marks)

An inverted cone of radius r cm and height h cm is shown below.



Let the slant height of the cone be l cm and the semi-vertical angle be α.

a. Find the radius and the height of the cone in terms of l and α.

___

___

___

2 marks

SECTION 2 - continued

b. Show that the volume of the cone can be expressed in terms of l and α as

$$V = \frac{\pi}{3} l^3 \sin^2(\alpha)\cos(\alpha)$$

1 mark

c. Find the stationary point(s) of $V(\alpha)$.

4 marks

d. If the slant height of the cone is $6cm$ find the maximum volume of the cone.

2 marks

SECTION 2 - continued

TURN OVER

Question 2 (9 marks)

The population of foxes in a certain forest varies according to the rule

$$P(t) = 500 - 300cos\left(\frac{\pi}{2.2}(t - 2.9)\right)$$

where $P(t)$ is the number of foxes t years after 2011.

a. Find the period and amplitude of this function.

2 marks

b. Find the maximum and minimum number of foxes in this forest.

2 marks

c. How many **months** does the population of foxes take to reach its first maximum?

2 marks

SECTION 2 – continued

Foxes are declared a vulnerable species if their population drops below 300.

d. Over which two time intervals will the population of foxes become vulnerable in the first 10 years? Give your answer correct to one decimal place.

3 marks

e. Give the interval(s) for which the population of foxes is strictly increasing, during the first 5 years.

3 marks

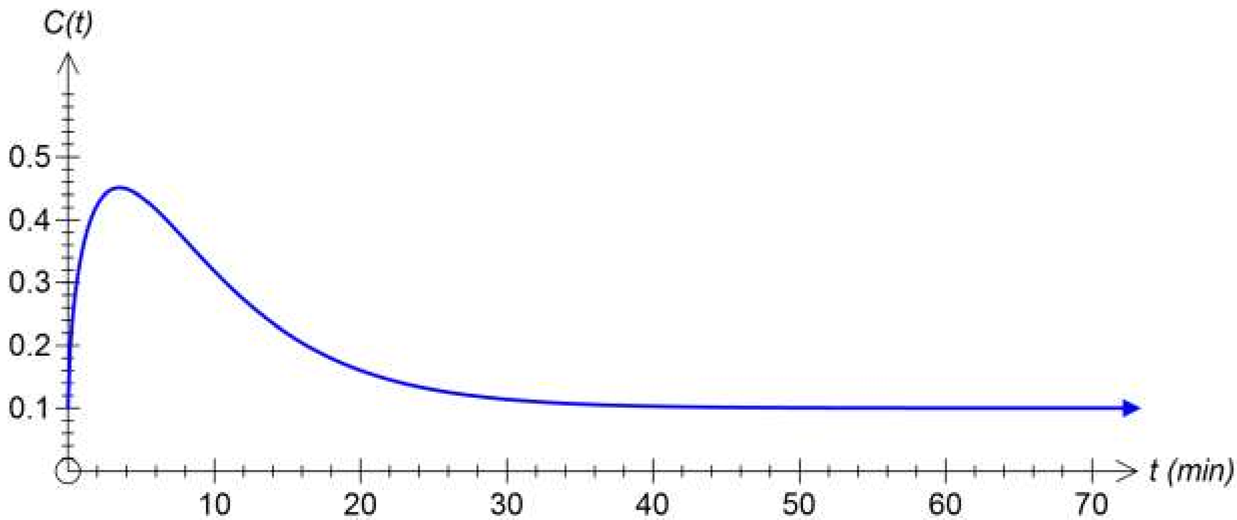
SECTION 2 - continued

TURN OVER

Question 3 (11 marks)

While smoking tobacco, the body absorbs many chemical compounds in addition to nicotine, including cyanide (which is highly toxic to humans).

The function $C(t) = 0.1 + 0.3t^{0.6}e^{-0.17t}$ is a reasonable model of the measured blood cyanide concentrations in µg/mL after *t* minutes, which is shown in the figure below.



a. Find the maximum concentration of blood cyanide, correct to 2 decimal places.

1 mark

b. When, to the nearest tenth of a minute, is the concentration a maximum?

1 mark

c. Find the concentration, correct to two decimal places, after 10 minutes.

2 marks

SECTION 2 - continued

d. Find the value of the average rate of change of concentration over the time interval $\left[\frac{3}{2}, 5\right]$ correct to four decimal places.

2 marks

e. Find the time interval, correct to 2 decimal places, when the rate of change of concentration is negative?

2 marks

Another function $C_1(t) = ate^{-bt}$ is used to measure the concentration of another substance in the blood in μg/mL after *t* minutes.

f. Find the values of a and b be if the maximum amount of this substance in the blood was 120 μg/mL after 2 hours?

3 marks

Question 4 (11 marks)

Consider the function $f(x) = x^2 + bx + 3, where\ b > 0\ \ and\ \ g(x) = x^2$.

a. If the function $f(x)$ can be written as $f(x) = (x + 5)^2 - 22$, find the value of *b*.

2 marks

SECTION 2 – continued

b. State the transformations that transformed the graph of $y = f(x)\ to\ y = g(x)$.

2 marks

c. Explain why $f\big(g(x)\big)$ exists and find the rule for $f\big(g(x)\big)$.

3 marks

Let $h(x) = f\big(g(x)\big)$.

d. Find the equation of the tangent to the graph of $y = h(x)$ at the point $x = k$.

2 marks

SECTION 2 - continued

TURN OVER

e. Write down the integral that will find the area bounded by the graph of $y = h(x)$ and the x-axis between $x = 0$ and $x = 3$.

2 marks

Question 5 (15 marks)

The distance, in kilometres, that Jack walks each week follows a normal distribution with a mean of 7.5 km and standard deviation of 2.5km.

a. Find the probability, correct to four decimal places, that in a particular week Jack walks

i. less than 11km.

ii. between 5.5km and 10.5km

2 marks

SECTION 2 – continued

b. 10% of the time Jack walks less than d km.
Find the value of *d*, correct to one decimal place.

2 marks

Jack observes his pattern of walking over a 6 week time period.

c. Find, to 4 decimal places, the probability that in 4 out of 6 weeks, he walks for at least 6.8km.

3 marks

Julie also walks each week following the normal pattern with mean distance of 6.4km. She has 65% chance of walking at least 5km each week.

d. Find, to 2 decimal places, the standard deviation of the distance she covers each week.

3 marks

SECTION 2 - continued

TURN OVER

Jack and Julie live in the community of Walkalot. The town council conducted a survey of 200 people and found that 80% walk over 5kms each week.

e. Find an approximate 95% confidence interval for the proportion p of residents who walk over 5 kms per week. Answer correct to 3 decimal places.

3 marks

f. Determine the sample size required in order to achieve a margin of error of 2% in an approximate 95% confidence interval for the proportion of residents of Walkalot who walk over 5kms per week, if the sample proportion remains at 0.8.

2 marks

END OF QUESTION AND ANSWER BOOK

MULTIPLE CHOICE ANSWER SHEET

Student Name:__________________________________

Circle the letter that corresponds to each correct answer.

Question					
1	A	B	C	D	E
2	A	B	C	D	E
3	A	B	C	D	E
4	A	B	C	D	E
5	A	B	C	D	E
6	A	B	C	D	E
7	A	B	C	D	E
8	A	B	C	D	E
9	A	B	C	D	E
10	A	B	C	D	E
11	A	B	C	D	E
12	A	B	C	D	E
13	A	B	C	D	E
14	A	B	C	D	E
15	A	B	C	D	E
16	A	B	C	D	E
17	A	B	C	D	E
18	A	B	C	D	E
19	A	B	C	D	E
20	A	B	C	D	E
21	A	B	C	D	E
22	A	B	C	D	E