



Trial Examination 2015

# VCE Mathematical Methods (CAS) Units 3&4

Written Examination 1

## Question and Answer Booklet

Reading time: 15 minutes

Writing time: 1 hour

Student's Name: \_\_\_\_\_

Teacher's Name: \_\_\_\_\_

### Structure of Booklet

<i>Number of questions</i>	<i>Number of questions to be answered</i>	<i>Number of marks</i>
10	10	40

Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers.

Students are NOT permitted to bring into the examination room: notes of any kind, blank sheets of paper, white out liquid/tape or a calculator of any type.

### Materials supplied

Question and answer booklet of 13 pages and a sheet of miscellaneous formulas.

Working space is provided throughout the booklet.

### Instructions

Write your **name** and **teacher's name** in the space provided above on this page.

All written responses must be in English.

**Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.**

Students are advised that this is a trial examination only and cannot in any way guarantee the content or the format of the 2015 VCE Mathematical Methods (CAS) Units 3&4 Written Examination 1.

Neap Trial Exams are licensed to be photocopied or placed on the school intranet and used only within the confines of the school purchasing them, for the purpose of examining that school's students only. They may not be otherwise reproduced or distributed. The copyright of Neap Trial Exams remains with Neap. No Neap Trial Exam or any part thereof is to be issued or passed on by any person to any party inclusive of other schools, non-practising teachers, coaching colleges, tutors, parents, students, publishing agencies or websites without the express written consent of Neap.

**Instructions**

Answer **all** questions in the spaces provided.

In all questions where a numerical answer is required, an exact value must be given unless otherwise specified.

In questions where more than one mark is available, appropriate working **must** be shown.

Unless otherwise indicated, the diagrams in this booklet are **not** drawn to scale.

**Question 1 (3 marks)**

If  $h(x) = x \sin(x^2)$ , find

a.  $h'(x)$ .

2 marks

---



---



---



---



---



---

b.  $h'\left(\sqrt{\frac{\pi}{2}}\right)$ .

1 mark

---



---



---



---



---



---



**Question 3 (4 marks)**

a. Solve  $2 \sin\left(2x - \frac{\pi}{6}\right) = \sqrt{3}$  for  $-\pi < x < \pi$ .

2 marks

---



---



---



---



---



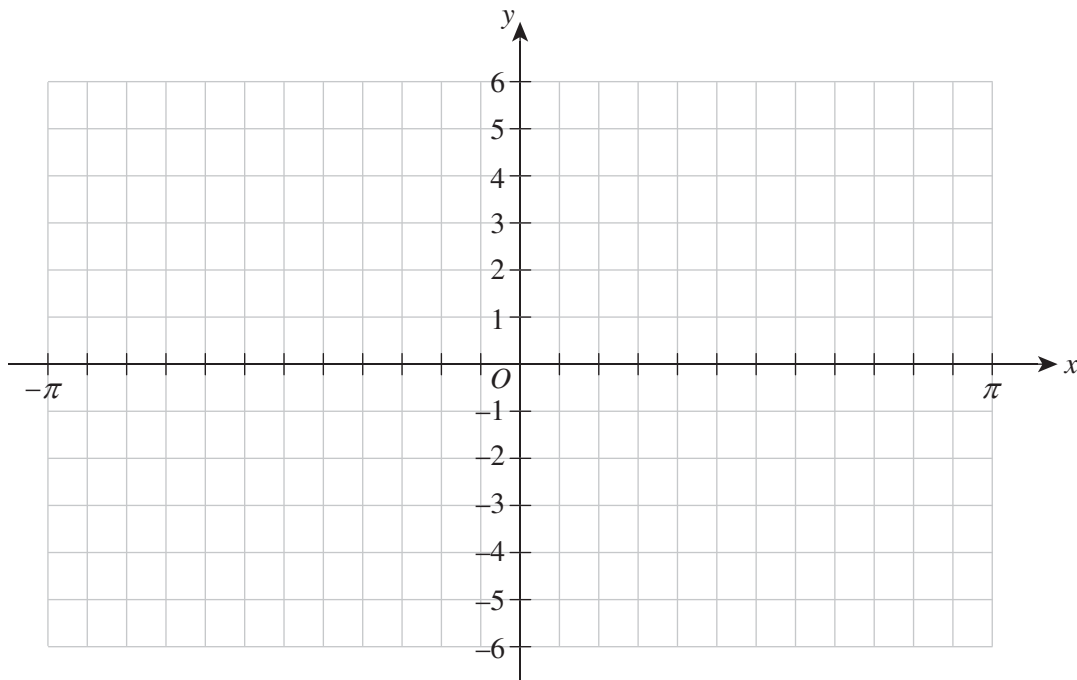
---



---

b. Hence sketch the graph of  $y = 2 \sin\left(2x - \frac{\pi}{6}\right) - \sqrt{3}$  for  $-\pi < x < \pi$ , showing all axial intercepts and endpoints.

2 marks





**Question 5 (5 marks)**

Consider  $f: [-2, 3] \rightarrow \mathbb{R}$ ,  $f(x) = 6 + x - x^2$  and  $g: (-3, 3) \rightarrow \mathbb{R}$ ,  $g(x) = |x|$ .

- a.** Find the coordinates of the maximum point of  $f$ . 2 marks

---



---



---



---



---

- b. i.** Find the function  $f(g(x))$ . 1 mark

---



---

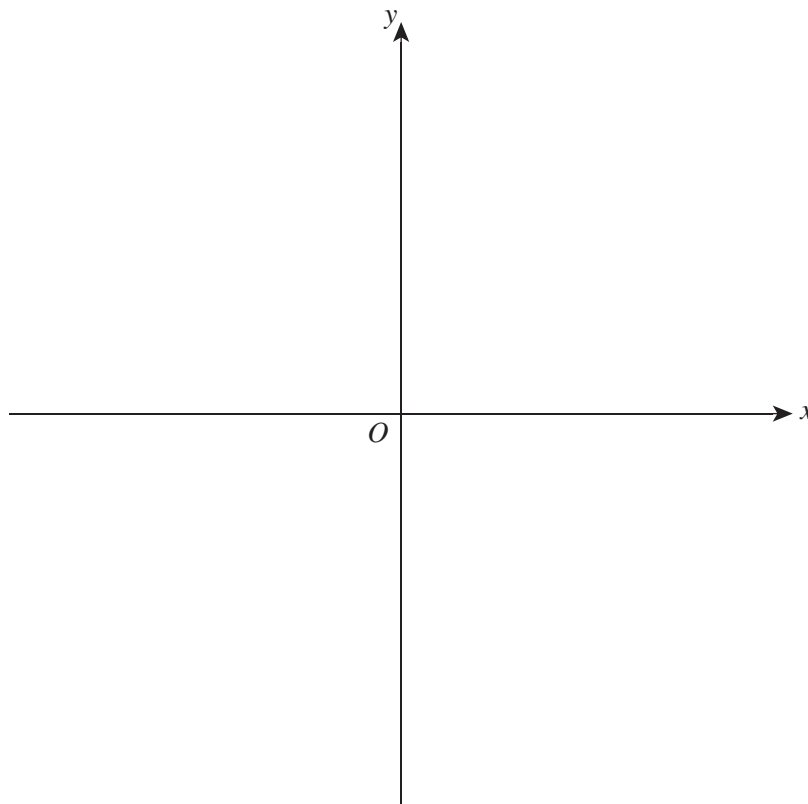


---



---

- ii.** Sketch the graph of  $f(g(x))$ , clearly labelling any axial intercepts and turning points with their coordinates. 2 marks





**Question 7 (4 marks)**

A farmer is building a storage shed for wheat. He decides on a square-shaped floor of side length  $s$  metres, vertical walls of height  $h$  metres and a flat roof. The floor is to be built from concrete at a cost of  $\frac{\$80}{9}$  per square metre. The roof is to be made of steel at a cost of  $\frac{\$40}{9}$  per square metre. The walls will be built of reinforced concrete at a cost of \$10 per square metre and will require foundations around the perimeter of the square floor at a cost of \$10 per lineal metre.

- a. Find an expression for  $C$ , the total cost of the project. 1 mark

---

---

---

---

---

---

---

---

- b. Given that the total cost of the shed is to be \$4800, deduce an expression for  $h$  as a function of  $s$ . 1 mark

---

---

---

---

---

---

---

---

- c. Hence, find an expression for the volume,  $V$ , of the shed that depends only on  $s$ . 1 mark

---

---

---

---

---

---

---

---



- d.** Find the height  $h$  and the side length  $s$  that will result in the maximum volume. 1 mark

---

---

---

---

---

---

---

---

---

---

**Question 8 (3 marks)**

A transformation  $T: \mathbb{R}^2 \rightarrow \mathbb{R}^2$  that maps the graph of  $g: \mathbb{R} \setminus \left\{-\frac{1}{3}\right\} \rightarrow \mathbb{R}$ ,  $g(x) = \frac{1}{(3x+1)^2} + 2$  to the

graph  $h: \mathbb{R} \setminus \{0\} \rightarrow \mathbb{R}$ ,  $h(x) = \frac{1}{x^2}$  has the rule  $T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} a & 0 \\ 0 & 1 \end{bmatrix} \left(\begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} b \\ c \end{bmatrix}\right)$ , where  $a$ ,  $b$  and  $c$  are non-zero

real numbers.

Find the values of  $a$ ,  $b$  and  $c$ .

---

---

---

---

---

---

---

---

**Question 9 (5 marks)**

The number of customers,  $X$ , waiting to be served in a bakery at 9:00 am has the probability distribution given in the table below.

$X$	0	1	2	3	4
$p(x)$	$\frac{3k^2 - 1}{7}$	$\frac{3k}{7}$	$\frac{4k}{7}$	$\frac{2k}{7}$	$\frac{k}{7}$

- a. Find the value of  $k$ . 3 marks

---

---

---

---

---

---

---

---

- b. Calculate the probability that there is at least one customer in the shop at 9:00 am. 2 marks

---

---

---

---

---

---

---

---

**Question 10 (6 marks)**

A continuous random variable has the following probability density function:

$$f(x) = \begin{cases} kx^2 & 0 \leq x \leq 1, \text{ where } k \text{ is a constant} \\ \frac{7k-3x}{4} & 1 < x \leq \frac{7}{3} \\ 0 & \text{elsewhere} \end{cases}$$

- a. Find the value of  $k$ .

2 marks

---

---

---

---

---

---

---

---

---

---

- b. Find the exact value of the mode.

2 marks

---

---

---

---

---

---

---

---

c. Find the exact value of the mean.

2 marks

---

---

---

---

---

---

---

---

---

**END OF QUESTION AND ANSWER BOOKLET**