Year 12 *Trial Exam Paper*

2015

MATHEMATICAL METHODS (CAS)

Written examination 1

Reading time: 15 minutes
Writing time: 1 hour

STUDENT NAME:

QUESTION AND ANSWER BOOK

Structure of book

Number of questions	*Number of questions to be answered*	*Number of marks*
10	10	40

- Students are permitted to bring the following items into the examination: pens, pencils, highlighters, erasers, sharpeners and rulers.
- Students are NOT permitted to bring blank sheets of paper, notes of any kind or white out liquid/tape into the examination.
- Calculators are NOT permitted in this examination.

Materials provided

- The question and answer book of 15 pages with a separate sheet of miscellaneous formulas.
- Working space is provided throughout this book.

Instructions

- Write your **name** in the box provided.
- Remove the formula sheet during reading time.
- You must answer the questions in English.

Students are NOT permitted to bring mobile phones or any other unauthorised electronic devices into the examination.

Instructions

Answer **all** questions in the spaces provided.

Please provide **exact** answers to all questions where a numerical answer is required, unless otherwise stated.

In questions where more than one mark is available, show appropriate working.

Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Question 1 (7 marks)

a. Find $\frac{d}{dx}\left(e^{x}\sin(2x)\right)$.

2 marks

b. For $f(x)=e^{\cos(x)}$, find $f'\left(\frac{\pi}{2}\right)$.

2 marks

QUESTION 1 – continued

c. The average value of the function $f:\left(-\frac{3}{2},\infty\right)\to R,\ f(x)=\frac{1}{3+2x}$

over the interval $[1,\ k]$ is $\frac{1}{10}\log_e(3)$. Find the value of k.

3 marks

TURN OVER

Question 2 (2 marks)

Consider these simultaneous linear equations.

$$ax - 2y = a$$
$$5x + y = 7$$

Find the value(s) of a for which the equations have a unique solution.

CONTINUES OVER PAGE

TURN OVER

Question 3 (4 marks)

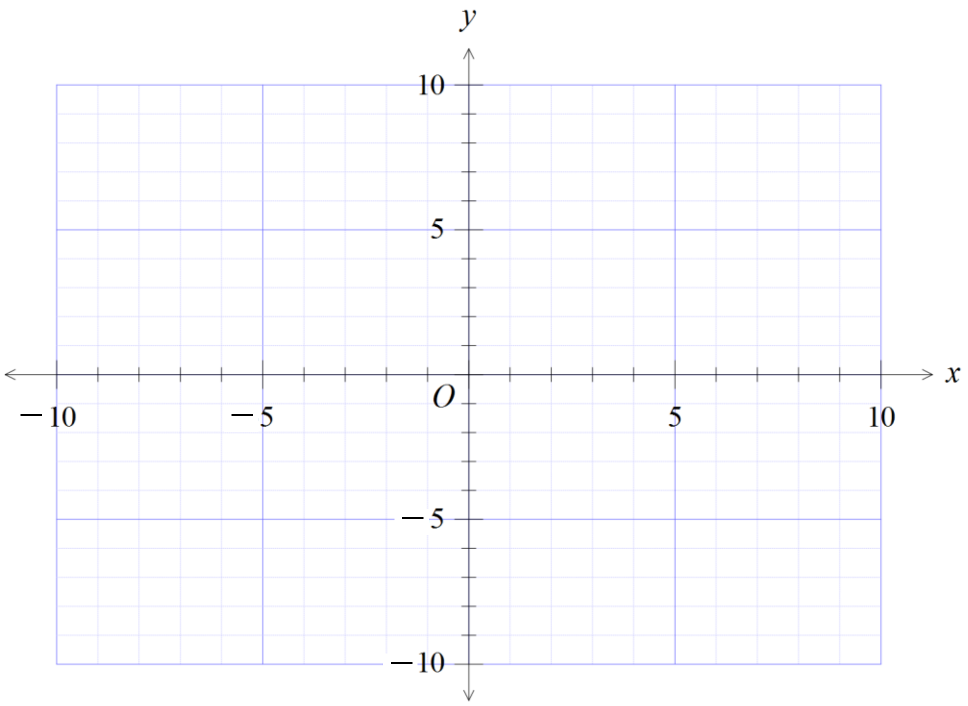
For the function $f:(2,\infty)\to R,\ f(x)=\frac{1}{3}\log_e\left(\frac{x-2}{3}\right)$

a. Find the rule for the inverse function, f^{-1}.

2 marks

b. Sketch the graph of $y=f^{-1}\left(f(x)\right)$ on the axes below.

1 mark



QUESTION 3 – continued

c. The function $f(x)$ undergoes a transformation, defined by the matrix

$$\begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}.$$

State the new equation.

1 mark

TURN OVER

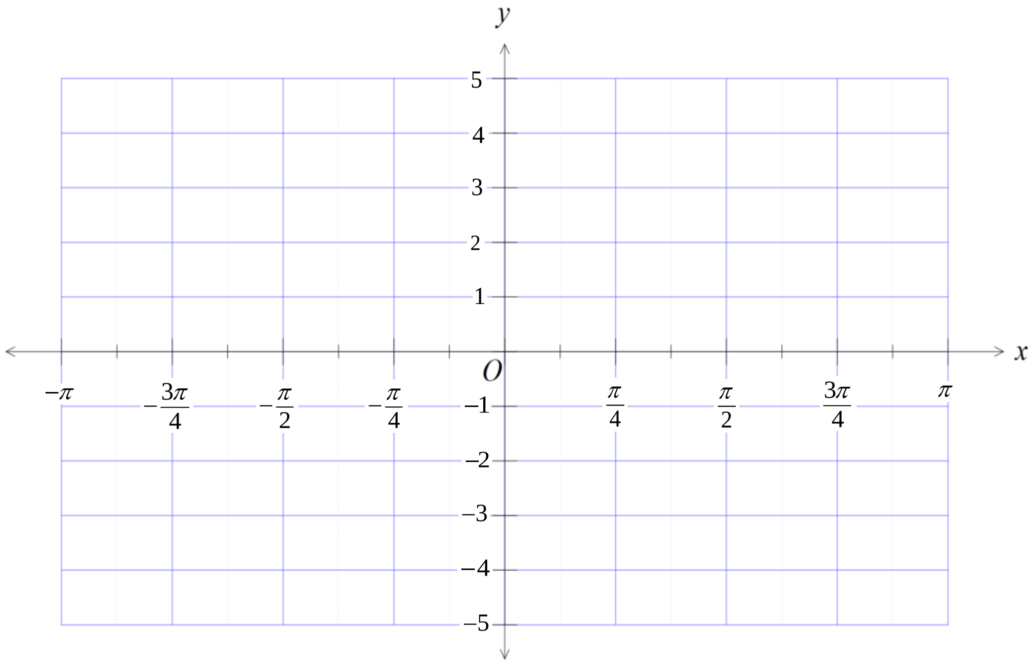
Question 4 (4 marks)

The graph of $y = \sin(x)$ undergoes the following transformations:

- a dilation of factor 2 from the x-axis
- a translation of $+3$ units up

a. Sketch the transformed graph over the domain $[-\pi, \pi]$ on the axes below.

2 marks



b. Find the equation of the normal to the transformed graph in part **a** when $x = 0$.

2 marks

Question 5 (3 marks)

Max drinks either cola or lemonade at morning tea break. If he has cola one morning, the probability he has cola the next is 0.3. If he has lemonade one morning, the probability he has lemonade the next is 0.4 and this is represented in the following transition matrix.

$$\begin{array}{c c} & \begin{array}{cc} C_i & L_i \end{array} \\ \begin{array}{c} C_{i+1} \\ L_{i+1} \end{array} & \begin{bmatrix} 0.3 & 0.6 \\ 0.7 & 0.4 \end{bmatrix} \end{array}$$

a. Suppose Max has cola on a Monday morning. What is the probability that he does not have cola again until Thursday morning?

2 marks

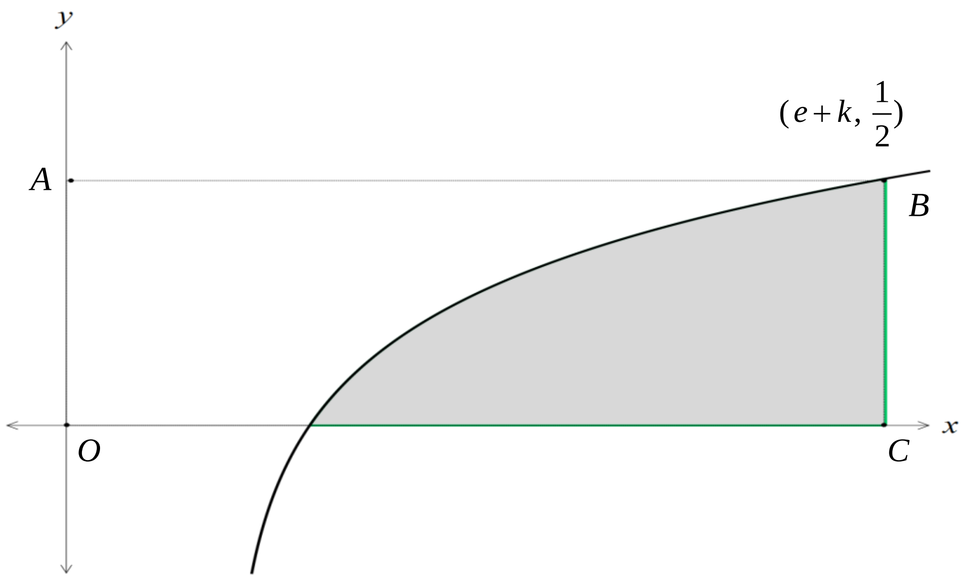
b. What is the probability that he drinks lemonade in the long term?

1 mark

TURN OVER

Question 6 (5 marks)

Consider the graph of $y = \frac{1}{2}\log_e(x-k)$. $OABC$ is a rectangle, as shown.



a. Show that the inverse of $y = \frac{1}{2}\log_e(x-k)$ is $y = e^{2x} + k$.

1 mark

b. Hence, find k when the shaded and unshaded regions of the rectangle are equal in area.

4 marks

TURN OVER

Question 7 (4 marks)

a. Find the general solution of $\sqrt{2}\sin(3x) = -1$.

2 marks

b. Find the average rate of change of the function $y = 2\sin(3x)$ over the interval $\left[\frac{\pi}{4}, \frac{\pi}{3}\right]$.

2 marks

Question 8 (5 marks)

Solve the following for *x*.

a. $2\log_8(x+1)+\log_8 4=1$

2 marks

b. $e^{2x}-8e^{x}+7=0$

3 marks

TURN OVER

Question 9 (3 marks)

The random variable X is normally distributed with mean 60 and standard deviation 15.
The random variable Z is normally distributed with mean 0 and standard deviation 1.

If $\Pr(Z < -2) = 0.0228$, find

1 mark

a. $\Pr(X < 30)$

b. $\Pr(X < 60 \mid X > 30)$

2 marks

Question 10 (3 marks)

Consider the function $f(x) = (x - a)(x - b)^3$, where a and b are positive constants with $a > b$.

Find the values of a and b if the stationary points occur when $x = 3$ and $x = 4$.

END OF QUESTION AND ANSWER BOOK