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Student Name……………………………………

MATHEMATICAL METHODS (CAS) UNITS 3 & 4

TRIAL EXAMINATION 1

2015

Reading Time: 15 minutes
Writing time: 1 hour

Instructions to students

This exam consists of 10 questions.
All questions should be answered in the spaces provided.
There is a total of 40 marks available.
The marks allocated to each of the questions are indicated throughout.
Students may **not** bring any calculators or notes into the exam.
Where an exact answer is required a decimal approximation will not be accepted.
Where more than one mark is allocated to a question, appropriate working must be shown.
Diagrams in this trial exam are not drawn to scale.
A formula sheet can be found on page 12 of this exam.

Question 1 (5 marks)

a. Differentiate $(4x^3 - x)^6$ with respect to x. 2 marks

b. If $g(x) = \dfrac{\log_e(2x)}{1+2x}$, find $g'\left(\dfrac{1}{2}\right)$. 3 marks

Question 2 (2 marks)

Find $\displaystyle\int \frac{2}{(3x-5)^4}\,dx$

Question 3 (2 marks)

Solve $3^{2x} - 8 \times 3^{x} = 9$ for x.

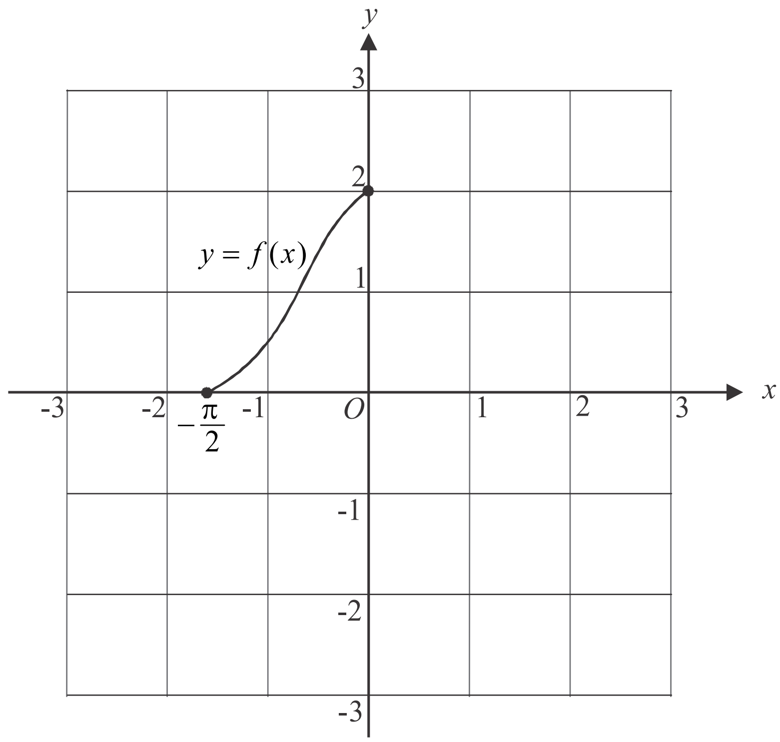
Question 4 (2 marks)

Solve $\log_5(x^3) + 2\log_5(x) = 15$ for x given $x > 0$.

Question 5 (5 marks)

Let $f:\left[-\frac{\pi}{2},0\right]\to R,\ f(x)=(x+2)\cos(x)$.

The graph of $y=f(x)$ is shown below.



a. Find the gradient of the graph of $y=f(x)$ at the point where $x=-\frac{\pi}{6}$. 3 marks

b. Sketch the graph of the inverse function f^{-1} on the set of axes above.
Indicate clearly the coordinates of the endpoints of the graph. 2 marks

Question 6 (3 marks)

The transformation $T : R^2 \to R^2$ is defined by $T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} \frac{1}{2} & 0 \\ 0 & -3 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} -2 \\ 1 \end{bmatrix}$.

Under this transformation the image of the curve $y = e^{\frac{x-4}{2}} - 1$ has equation $y = a + be^{x}$.
Find the values of a and b.

Question 7 (5 marks)

Consider the functions $f:[0,9] \to R, f(x) = 2\sin\left(\frac{\pi x}{9}\right)$ and $g:[0,9] \to R, g(x) = \sqrt{3}$.

a. Find the coordinates of the points of intersection of the graphs of $y = f(x)$ and $y = g(x)$. 2 marks

b. Find the area enclosed by the graphs of $y = f(x)$ and $y = g(x)$. 3 marks

Question 8 (4 marks)

A continuous random variable X has a probability density function given by

$$f(x) = \begin{cases} \dfrac{1}{x+1} & 0 \le x \le a \\ 0 & \text{otherwise} \end{cases}$$

where a is a positive constant.

a. Find the value of a. 2 marks

b. The median of X is m where $m = \sqrt{e} - 1$ and $m < 1$.
Find $\Pr(X < m | X < 1)$. 2 marks

Question 9 (6 marks)

Geoff owns a lawn mowing business. If there is no rain during a week then the probability that he completes all his jobs is $\frac{9}{10}$, but if there is rain, then the probability that he completes all his jobs is $\frac{2}{5}$.
The probability that there is rain one week is assumed to be independent of there being rain during any other week.

a. Find the probability that during the last fortnight, when there was no rain, Geoff completed all of his jobs in one week but not the other. 2 marks

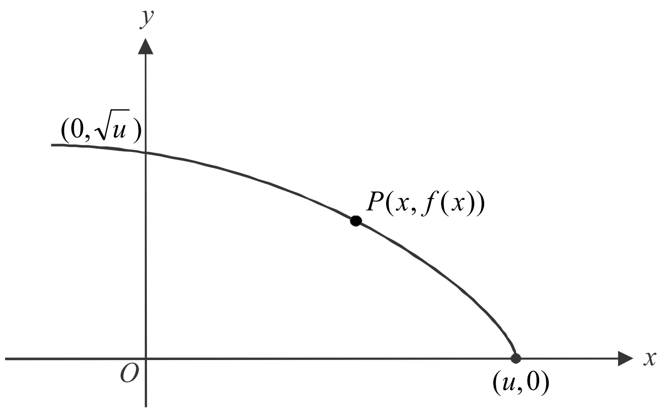
During winter, the probability of there being rain during a week is $\frac{3}{4}$.

b. **i.** Find the probability that during the first week of winter, Geoff completed all of his jobs. 2 marks

ii. Find the probability that during a different week in winter there was rain, given that Geoff didn't complete all of his jobs that week. 2 marks

Question 10 (6 marks)

The graph of $f(x)=\sqrt{u-x}$ is shown below where u is a positive real number.



The point $P(x,f(x))$ lies on the graph.

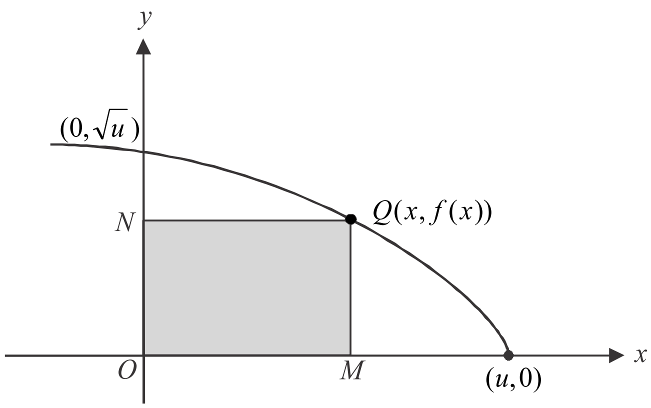
a. When the gradient of the tangent to the graph at P equals -1, find

i. the y-coordinate of P. 2 marks

ii. the horizontal distance between P and the x-intercept of this tangent. 1 mark

b. The point $Q(x, f(x))$ also lies on the graph of *f*.

The rectangle *OMQN* is shaded in the diagram below.



Find the maximum area of this rectangle. Express your answer in terms of *u*. 3 marks

Mathematical Methods (CAS) Formulas

Mensuration

area of a trapezium:	$\frac{1}{2}(a+b)h$	volume of a pyramid:	$\frac{1}{3}Ah$
curved surface area of a cylinder:	$2\pi rh$	volume of a sphere:	$\frac{4}{3}\pi r^3$
volume of a cylinder:	$\pi r^2 h$	area of a triangle:	$\frac{1}{2}bc\sin A$
volume of a cone:	$\frac{1}{3}\pi r^2 h$		

Calculus

$\frac{d}{dx}\left(x^n\right) = nx^{n-1}$

$\frac{d}{dx}\left(e^{ax}\right) = ae^{ax}$

$\frac{d}{dx}\left(\log_e(x)\right) = \frac{1}{x}$

$\frac{d}{dx}\left(\sin(ax)\right) = a\cos(ax)$

$\frac{d}{dx}\left(\cos(ax)\right) = -a\sin(ax)$

$\frac{d}{dx}\left(\tan(ax)\right) = \frac{a}{\cos^2(ax)} = a\sec^2(ax)$

$\int x^n\,dx = \frac{1}{n+1}x^{n+1} + c,\ n \neq -1$

$\int e^{ax}\,dx = \frac{1}{a}e^{ax} + c$

$\int \frac{1}{x}\,dx = \log_e|x| + c$

$\int \sin(ax)\,dx = -\frac{1}{a}\cos(ax) + c$

$\int \cos\ (ax)\,dx = \frac{1}{a}\sin(ax) + c$

product rule: $\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$

quotient rule: $\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$

chain rule: $\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx}$

approximation: $f(x+h) \approx f(x) + hf'(x)$

Probability

$\Pr(A) = 1 - \Pr(A')$

$\Pr(A \cup B) = \Pr(A) + \Pr(B) - \Pr(A \cap B)$

$\Pr(A|B) = \frac{\Pr(A \cap B)}{\Pr(B)}$

transition matrices: $S_n = T^n \times S_0$

mean: $\mu = \mathrm{E}(X)$

variance: $\mathrm{var}(X) = \sigma^2 = \mathrm{E}((X-\mu)^2) = \mathrm{E}(X^2) - \mu^2$

probability distribution		mean	variance
discrete	$\Pr(X = x) = p(x)$	$\mu = \Sigma x\, p(x)$	$\sigma^2 = \Sigma(x-\mu)^2 p(x)$
continuous	$\Pr(a < X < b) = \int_a^b f(x)dx$	$\mu = \int_{-\infty}^{\infty} x\, f(x)dx$	$\sigma^2 = \int_{-\infty}^{\infty}(x-\mu)^2 f(x)\,dx$