E

Free Exam
for 2006-15 VCE study design

Engage
Education
Foundation

Units 3 and 4 Maths Methods (CAS): Exam 2

Practice Exam Solutions

Stop!

Don't look at these solutions until you have attempted the exam.

Any questions?

Check the Engage website for updated solutions, then email practiceexams@ee.org.au.

Section A – Multiple-choice questions

Question 1
The correct answer is C.

Question 2
The correct answer is D.

Question 3
The correct answer is A.

Question 4
The correct answer is B.

Question 5
The correct answer is C.

Question 6
The correct answer is A.

Question 7
The correct answer is A.

Question 8
The correct answer is D.

Question 9
The correct answer is D.

Question 10
The correct answer is B.

Question 11
The correct answer is D.

Question 12
The correct answer is E.

Question 13
The correct answer is E.

Question 14
The correct answer is C.

Question 15
The correct answer is B.

Question 16
The correct answer is E.

Question 17
The correct answer is B.

Question 18
The correct answer is A.

Question 19
The correct answer is C.

Question 20
The correct answer is A.

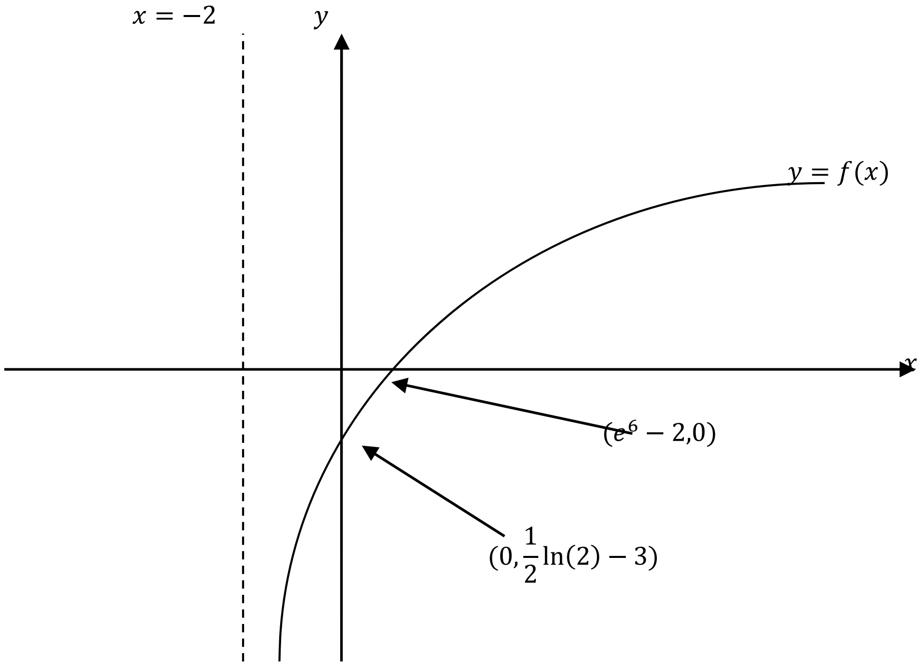
Question 21
The correct answer is D.

Question 22
The correct answer is B.

Section B – Short-answer questions

Marks allocated are indicated by a number in square brackets, for example, [1] indicates that the line is worth one mark.

Question 1a



Requires a sketched graph with:

- Vertical asymptote at $x = -\frac{3}{2}$ [1 mark]
- Correct shape including intercepts on the correct side of the origin [1 mark]
- y-intercept at $(0, \frac{1}{2}\ln(2) - 3)$ and x-intercept at $(e^6 - 2, 0)$ [1 mark]

Question 1b i

Domain: $(-2, \infty)$, Range: $(-\infty, \infty)$ [1 mark]

Question 1b ii

$f'(x) = \frac{1}{2(x+2)}$ [1 mark]

Question 1c

$y_0 = \frac{1}{2}(x + 2) - 3$

Reflection in the y-axis ⇒ $y_1 = \frac{1}{2}(2 - x) - 3$ [1 mark]

Dilation of factor 3 from the x-axis ⇒ $y_2 = \frac{3}{2}(2 - x) - 9$ [1 mark]

Translation 2 units in the negative direction of the x-axis and 7 units in the positive direction of the y-axis ⇒ $y_3 = \frac{3}{2}(-x) - 2$ [1 mark]

Therefore ⇒ $g(x) = \frac{3}{2}(-x) - 2$

Question 1d i
$h'(x) = 2 + 4$

Therefore minimum occurs at $2x + 4 = 0 \Rightarrow x = -2$ [1 mark]

At $x = -2$ we have $y = (-2)^2 + 4(-2) + c = -4 + c$

For $f(h(x))$ to exist we need $Range\ of\ h(x) \subseteq Domain\ of\ f(x) = (-2, \infty)$

So we need $\min h(x) > -2 \Rightarrow -4 + c > -2$ [1 mark]

So $c > 2$ [1 mark-do not get this mark if ≥ used]

Question 1d ii
$f\big(h(x)\big) = \frac{1}{2}\ln(x^2 + 4x + 5) - 3$ [1 mark]

Minima occurs at $x = -2$, so $f\big(h(-2)\big) = \frac{1}{2}\ln(4 - 8 + 5) - 3 = \frac{1}{2}\ln(1) - 3 = -3$

Maxima is ∞

So range is $[-3, \infty)$ [1 mark]

Question 1d iii
$Range\ f(x) = R$ and $Domain\ h(x) = R$

So $h\big(f(x)\big)$ exist as $Range\ f(x) \subseteq Domain\ h(x)$ [1 mark]

Question 2
This question demands a strong knowledge of calculator use, it is advised that all questions are done using a calculator and that the function is defined at the beginning of the question. Make sure you use exact values stored on calculator in all calculations and only round at the end. As there are multiple methods of solving these questions on the calculator full marks if the correct answer is stated to any numerical question.

Question 2a i
Solve $h(x) = e^{\frac{-(6-x)^2}{40}}(-3x^3 + 36x^2) = 0 \Rightarrow x = 0, 12$

Therefore $b = 12$ [1 mark]

Question 2a ii
$h'(x) = \frac{1}{20}e^{\frac{-(x-6)^2}{40}}(3x^4 - 54x^3 + 36x^2 + 1440x)$ [1 mark]

Question 2a iii
Solve $h'(x) = \frac{1}{20}e^{\frac{-(x-6)^2}{40}}(3x^4 - 54x^3 + 36x^2 + 1440x) = 0$ [1 mark]

In our domain we get $x = 0, 7.27$ and so we have maximum at $x = 7.27$

So $h(7.27) = 720.34$ so maximum height is 720.34m [1 mark]

Question 2b i
Maximum gradient occurs when $h''(x) = 0$ (no need to write or find h''(x)) [1 mark]

This yields $x = 4.32, 10.65$ but we want max which occurs at $x = 4.32$

So maximum gradient is $h'(4.32)$ =166.99

So the gradient of h(x) exceeds 150m/km and so the catapults cannot be transported from town A to town B [1 mark]

(Note: Showing that the gradient exceeds 150 at any point is sufficient to gain full marks on this question)

Question 2bii
$\boldsymbol{Average\ rate\ of\ change} = \frac{\boldsymbol{h(a)-h(0)}}{\boldsymbol{a-0}} = \boldsymbol{100}$ [1 mark]
Solving the above equation gives $a = 4.80, 7.20$

but we need first solution $a = 4.80$ [1 mark]

(This is because path to second point intersects first point and so passes through the mountain)

Hence $h(4.80) = 479.89$

So coordinates are $(4.80, 479.89)$ [1 mark]

Question 2c i
Solve $h(x) = e^{\frac{-(6-x)^2}{40}}(-3x^3 + 36x^2) = 600$ [1 mark]

This give x = 5.60, 8.88 [1 mark]

Question 2c ii
$Area = \int_{5.60}^{8.88} h(x) - 600dx$ [1 mark]

Question 2d
Solve $h(x) = h(x + 2)$ to find position of tunnel length 2km [1 mark]

Hence $x = 6.26$

$h(6.26) = 673.71$ therefore minimum height is 673.71m [1 mark]

Question 3a i

$$T = \begin{array}{c} \begin{matrix} B & W \end{matrix} \\ \begin{bmatrix} 0.8 & 0.4 \\ 0.2 & 0.6 \end{bmatrix} \end{array} \begin{matrix} B_{next} \\ W_{next} \end{matrix}$$

Question 3a ii
$\Pr = 0.8 \times 0.8 \times 0.2 + 0.8 \times 0.2 \times 0.4 + 0.2 \times 0.4 \times 0.8$ [1 mark]

$\Pr = 0.256$ [1 mark]

Question 3b i
$S_4 = T^4 \times S_0 = \begin{bmatrix} 0.8 & 0.4 \\ 0.2 & 0.6 \end{bmatrix}^4 \begin{bmatrix} 140 \\ 400 \end{bmatrix} = \begin{bmatrix} 354.368 \\ 185.632 \end{bmatrix}$ [1 mark]

Therefore the number of people shopping at Wendy's Supermarket is 186 people. [1 mark]

Question 3b ii

In long run number of people at Wendy's = $\frac{0.2}{0.4+0.2} \times 540 = 180$ [1 mark]

Question 3c i

From calculator $\Pr = \ 0.1798$ [1 mark]

Question 3c ii

$0.9 = \Pr(X > 2) = 1 - \Pr(X < 2)$

$0.9 = 1 - \binom{n}{0} p^0 (1-p)^n - \binom{n}{1} p^1 (1-p)^{n-1}$

$0.9 = 1 - (0.85)^n - n(0.15)(0.85)^{n-1}$ [1 mark]-any of the last three steps

This solves to give $n = 24.45$ [1 mark]

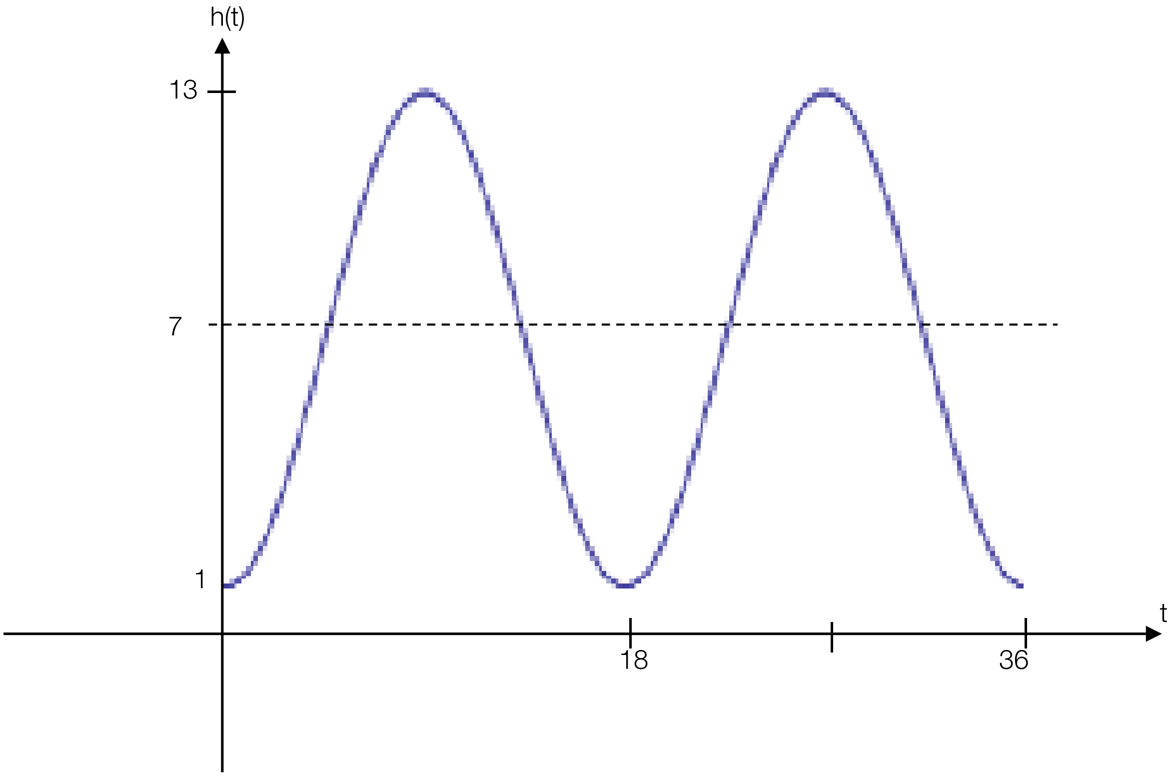
So die must be rolled at least 25 times [1 mark]

Question 3c iii

The mean is, $E(X) = np = 20 \times 0.15 = 3$ [1 mark]

The variance is $Var(X) = np(1-p) = 20 \times 0.15 \times 0.85 = 2.55$ [1 mark]

Question 4a



- correct shape including two periods [1 mark]
- correct maxima at h(t)=13 and minima at h(t)=1 [1 mark]
- everything else including correct position of both endpoints [1 mark]

Question 4b i

Period = 18 [1 mark]

Question 4b ii

Max=13m and average=7m [1 mark]

Question 4c i

Solve $\mathbf{11 = 7 - 6\cos\left(\frac{\pi t}{9}\right)}$ [1 mark]

To get $t = 6.59, 11.41, 24.59, 29.41$ [1 mark]

Question 4c ii

Time above 11m is $2 \times (11.41 - 6.59) = 9.64$ [1 mark]

Therefore proportion is $\frac{9.64}{36} = 0.27$ [1 mark]

Question 4d

- dilation of $\frac{2}{3}$ from the y-axis [1 mark]
- dilation of $\frac{3}{2}$ from the x-axis [1 mark]
- translation of $\frac{1}{2}$ half a unit in the negative direction of the y-axis [1 mark]

Question 4e i

$$V = \frac{1}{3}\pi r^2 h$$

$h = \frac{3V}{\pi r^2} = \frac{3\times 500}{\pi \times 36} = \frac{125}{3\pi}$ [1 mark]

Question 4e ii

$V = \frac{125}{3\pi} - 10t$ [1 mark]

$$V = 0 = \frac{125}{3\pi} - 10t \Rightarrow 10t = \frac{125}{3\pi}$$

So all water is drained when $t = \frac{25}{6\pi}$ [1 mark]

Question 4e iii

$\frac{dV}{dt} = \frac{dV}{dh} \times \frac{dh}{dt}$ [1 mark]

$\frac{dV}{dh} = \frac{1}{3}\pi r^2 = \frac{1}{3}\pi(3)^2 = 3\pi$ [1 mark]

$$\frac{dV}{dt} = -10$$

$\frac{dh}{dt} = \frac{dV}{dt} \div \frac{dV}{dh} = \frac{-10}{3\pi}\ m/second$ [1 mark]