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Free Exam
for 2006-15 VCE study design

Engage Education Foundation

Units 3 and 4 Maths Methods (CAS): Exam 2

Technology-enabled Practice Exam Question and Answer Booklet

Duration: 15 minutes reading time, 2 hours writing time

Structure of book:

Section	Number of questions	Number of questions to be answered	Number of marks
A	22	22	22
B	4	4	58
		Total	80

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, rulers, a protractor, set-squares, aids for curve sketching, one bound reference, one approved CAS calculator (memory does not need to be cleared) and, if desired, one scientific calculator.
- Students are not permitted to bring into the examination room: blank sheets of paper and/or white out liquid/tape.

Materials supplied:
- This question and answer booklet of 23 pages, including a detachable formula sheet.

Instructions:
- You must complete all questions of the examination.
- Write all your answers in the spaces provided in this booklet.

Section A – Multiple-choice questions

Instructions

Answer all questions by circling your choice.
Choose the response that is correct or that best answers the question.
A correct answer scores 1, an incorrect answer scores 0.
Marks will not be deducted for incorrect answers.
No marks will be given if more than one answer is completed for any question.

Questions

Question 1

The graph of a cubic function has a local maximum at $(m, 2)$ and a local minimum at $(n, -2)$. For what values of c does $f(x) + c = 0$ have 3 solutions?

A. $c > -2$
B. $c < -2$ or $c > 2$
C. $c < -2$
D. $-2 < c < 2$
E. $c < 2$

Question 2

$f(x) = \ln(x) + 2$. The average rate of change over the interval [1, 4] is:

A. $\frac{1}{4}$
B. $2\ln(2)$
C. 1
D. $\frac{2\ln(2)}{3}$
E. $\frac{3}{4}$

Question 3

$x(-2k+9) - 6y = -2k - 4$
$6x + y(-2k-3) = -7$

The simultaneous linear equations above will have an infinite number of solutions when k equals:

A. $\frac{3}{2}$
B. $\frac{2}{3}$
C. 1
D. -2
E. $\frac{-2}{3}$

Question 4

f is a function. The derivative of $\sin(f(x))$ is:

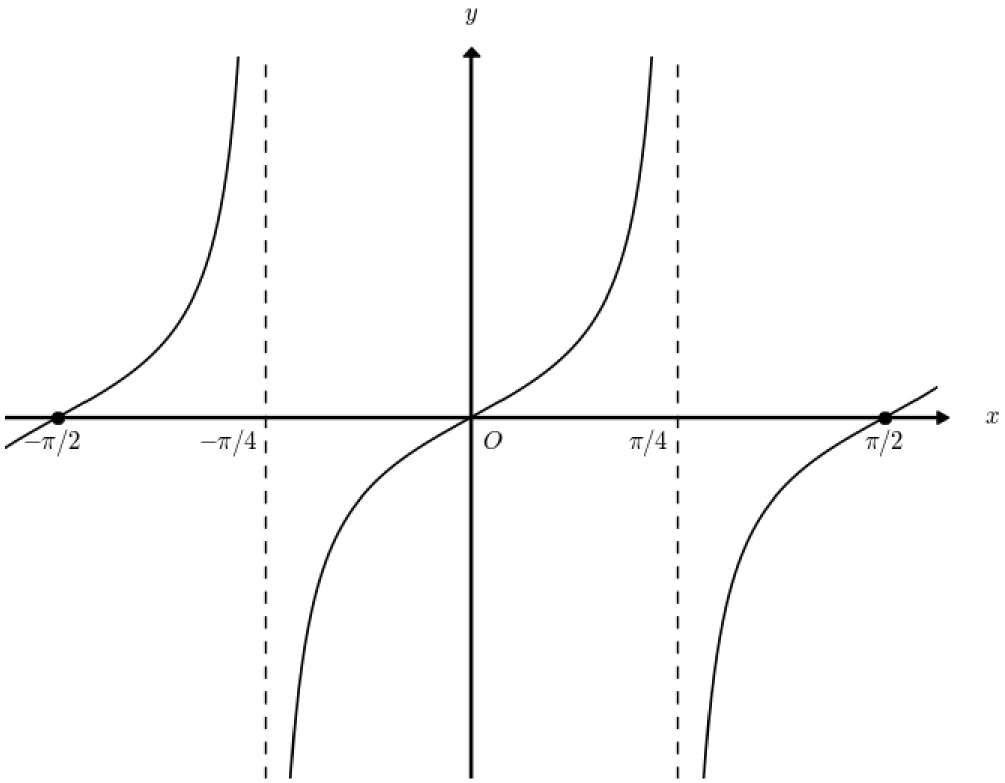
A. $f'(x)\sin(x)$
B. $f'(x)\sin(f(x))$
C. $f(x)\cos(f(x))$
D. $f'(x)\cos(f(x))$
E. $f'(x)\cos(x)$

Question 5

$f(x) = \ln(|x-3|-2)+3$. The domain of f is:

A. (1, 5)
B. $(-\infty,1) \cup (5,\infty)$
C. $(-\infty, 1] \cup [5,\infty)$
D. $(-\infty, 1) \cup (3,\infty)$
E. $(-3, 3)$

Question 6



Which of the following is the equation of the graph above?

A. $\tan(x)$
B. $\tan\left(2\left(x-\frac{\pi}{2}\right)\right)$
C. $\tan\left(x-\frac{\pi}{2}\right)$
D. $\tan\left(\frac{1}{2}x\right)$
E. $\tan\left(\frac{1}{2}\left(x-\frac{\pi}{2}\right)\right)$

Question 7
The voltage in a wire at hour x after midnight is given by $f(x) = 13 - 7\sin\frac{\pi}{12(x-3)}$. What is the closest average voltage between 9am and 9pm?

A. 7
B. 13
C. 6
D. 20
E. 15

Question 8
A function is monotonically increasing if its gradient is always greater than or equal to 0. Which of the following polynomials is monotonically increasing?

A. $\int(x+2)(x-2)dx$
B. $\int(x-1)(x-2)\,dx$
C. $\int(3(x+1)^2)\,dx$
D. $\int(-3(x+1)^2)\,dx$
E. $\int(-(x+1)(x+3))\,dx$

Question 9
The normal of the graph $y = e^{kx}$ has gradient of 2 when $x = 0$. Find k.

A. $k = -\frac{1}{2}$
B. $k = 2$
C. $k = \frac{1}{2}$
D. $k = -2$
E. $k = -1$

Question 10
The average value of $f(x) = \frac{3}{x}$ over [1, 10] is:

A. $\frac{1}{3}(\ln(10) - 1)$
B. $\frac{27}{10}$
C. $3\ln(10)$
D. $\frac{1}{3}\ln(10)$
E. $\frac{3}{10}$

Question 11
The weight of baskets is normally distributed with mean 900g and standard deviation 17g. The manufacturer says 10% are less than x grams. x is equal to:

A. 878.21
B. 860.45
C. 832
D. 872.04
E. 939.55

Question 12

The person in the upstairs apartment has dropped a large jar of honey on the ground, smashing a hole in the floor. The honey drips into your apartment floor forming a puddle of size $S = \pi r^2$. If $\frac{dS}{dt} = 10cm^2/min$, $\frac{dr}{dt}$ is equal to:

A. $\frac{5}{\pi r}$
B. $\frac{10}{\pi r}$
C. $2\pi r$
D. $20\pi r$
E. $\frac{\pi r}{5}$

Question 13

The table below gives incomplete probabilities for the events A and B.

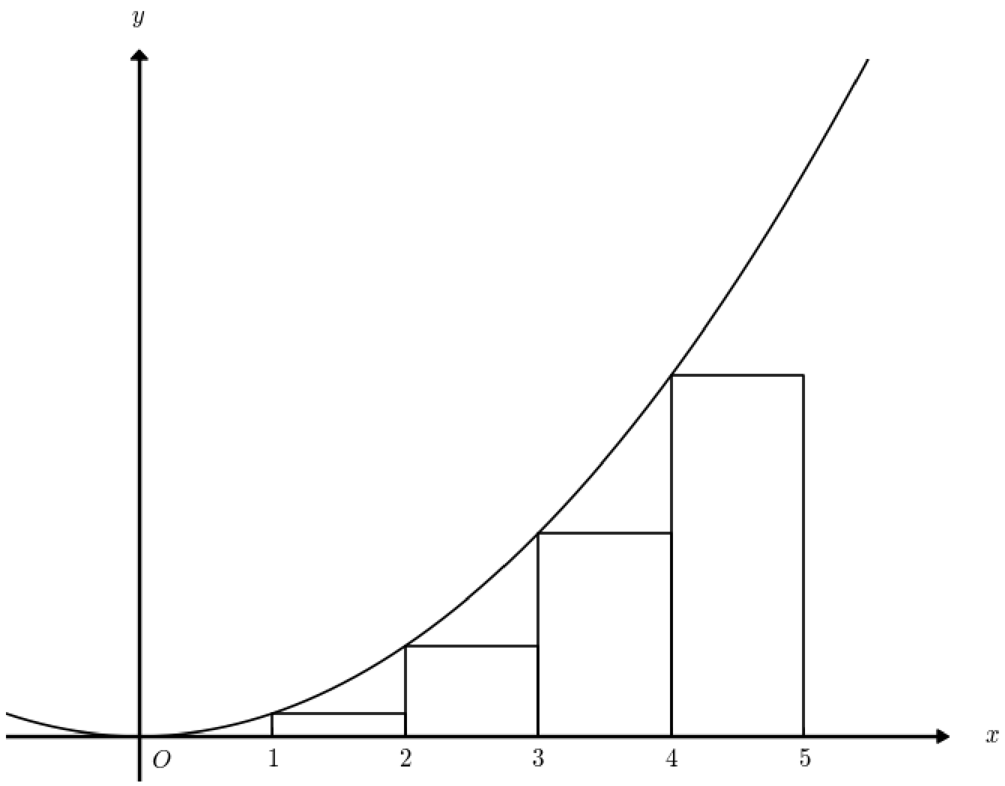
	A	A'	
B	1/7		1/6
B'			
	2/3		1

Hence, $Pr(A \text{ or } B')$ is:

A. $\frac{6}{7}$
B. $\frac{41}{42}$
C. $\frac{11}{21}$
D. $\frac{10}{21}$
E. $\frac{13}{42}$

Question 14

The graph of $f(x) = x^2$ is shown below. In order to find an approximation to the area of the region bounded by the graph, the y-axis and the line $x = 5$, there have been 4 rectangles drawn as shown. Calculate the value of this approximation.

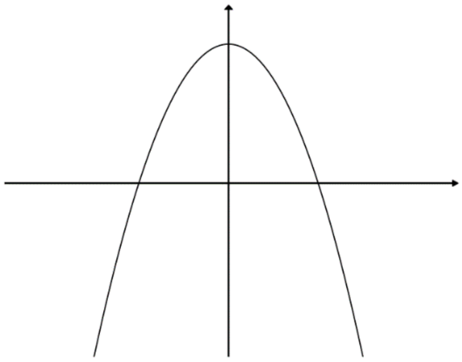


A. 30
B. 55
C. 29
D. $\frac{125}{3}$
E. 25

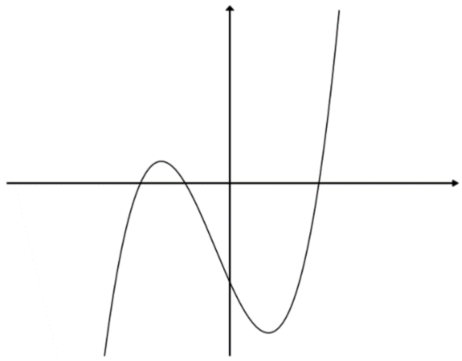
Question 15

$f'(x) = -x^2 + 4$. $f(x)$ could be:

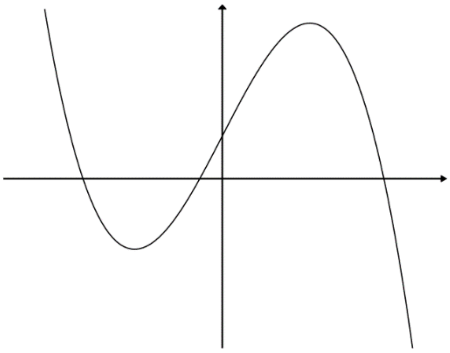
A.



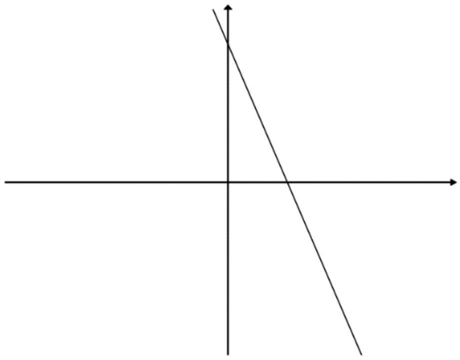
B.



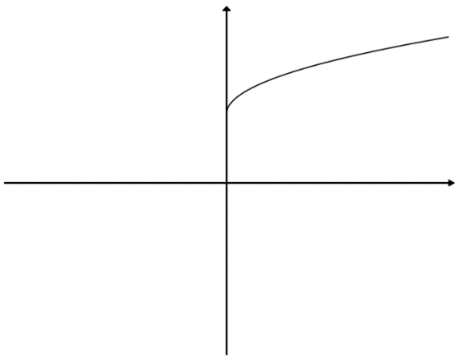
C.



D.



E.



Question 16

The function $\frac{3 \tan 2\pi x}{2} + 5$ has period:

A. $\frac{1}{2}$
B. 1
C. 2π
D. $\frac{\pi}{2}$
E. 3

Question 17

The range of $f(x) = x^2 - 8x + 12$ over [2, 8) is:

A. [-4,0]
B. [-4,-12]
C. [0,12)
D. [0,12]
E. [-4,12)

Question 18

Given that $f(x) = \sin(x)$ and $g(x) = \frac{1}{2}\sin(2x) - 3$ which of the following describes the true set of transformations from $f(x)$ to $g(x)$?

A. Dilation from the y-axis by factor 2, dilation from the x-axis by factor 2, translation in the negative y-direction by 3
B. Dilation from the y-axis by factor 1/2, dilation from the x-axis by factor 2, translation in the negative y-direction by 3
C. Dilation from the y-axis by factor 2, dilation from the x-axis by factor 1/2, translation in the positive y-direction by 3
D. Dilation from the y-axis by factor 1/2, dilation from the x-axis by factor 2, translation in the positive y-direction by 3
E. Dilation from the y-axis by factor 1/2, dilation from the x-axis by factor 1/2, translation in the negative y-direction by 3

Question 19

A function f has the property that for all real values of x: $f(x) = f(-x)$. The function could be:

A. $\sin(x)$
B. $\cos(x)$
C. $\tan(x)$
D. $\frac{x^3}{4}$
E. $\frac{x}{2}$

Question 20

A discrete random variable X has the probability function $\Pr(X = k) = \frac{2^k e^{-2}}{k!}, k = 0,1,2 \ldots$
What is $\Pr(X \geq 1)$?

A. $1 - e^{-2}$
B. e^{-2}
C. $2e^{-2}$
D. $3e^{-2}$
E. $1 - 2e^{-2}$

Question 21

The probability of a strawberry plant flowering next year given that it flowered this year is equal to 0.9. The probability that it flowers next year given that it didn't flower this year is equal to 0.3. What is the long term probability of a plant flowering (to 2 decimal places)?

A. 0.27
B. 0.25
C. 0.33
D. 0.66
E. 0.75

Question 22

The graph of the differentiable function f has a local minimum at (a, b) where $a > 0$ and $b < 0$, and a local maximum at (c, d) where $c > 0$ and $d > 0$. The graph $y = -|f(-x)|$ has:

A. a local minimum at $(a, -b)$ and a local maximum at $(c, -d)$
B. a local maximum at $(-a, b)$ and a local minimum at $(-c, d)$
C. a local minimum at $(-a, -b)$ and a local maximum at $(-c, -d)$
D. a local minimum at $(-a, b)$ and a local minimum at $(-c, -d)$
E. a local maximum at $(a, -b)$ and a local minimum at $(c, -d)$

Section B – Short-answer questions

Instructions

Answer all questions in the spaces provided.
In all questions where a numerical answer is required an exact value must be given unless otherwise specified.
In questions where more than one mark is available, appropriate working must be shown.
Unless otherwise indicated, the diagrams in this book are not drawn to scale.

Questions

Question 1

$f(x) = \frac{1}{2}\ln(2x + 3) + 3$

a. Sketch the graph of $f(x)$.

3 marks

b. Find:

i. the range and domain of f

1 mark

ii. $f'(x)$

1 mark

iii. Show that $f^{-1}(x) = \frac{e^{2(x-3)}-3}{2}$

2 marks

c. (a, b) is a point of intersection between $f(x)$ and $f^{-1}(x)$, where $a > 0$. Find a and b correct to four decimal places.

2 marks

d. For every point (p, q) on $f(x)$, its inverse point is (q, p) on $f^{-1}(x)$.

i. Find q in terms of p.

1 mark

ii. Show that the equation of the line passing through (p, q) and (q, p) is $y = \frac{(p-q)}{(q-p)}(x-p)+q$.

2 marks

iii. Find the length of the line segment between (p, q) and (q, p) in terms of p.

2 marks

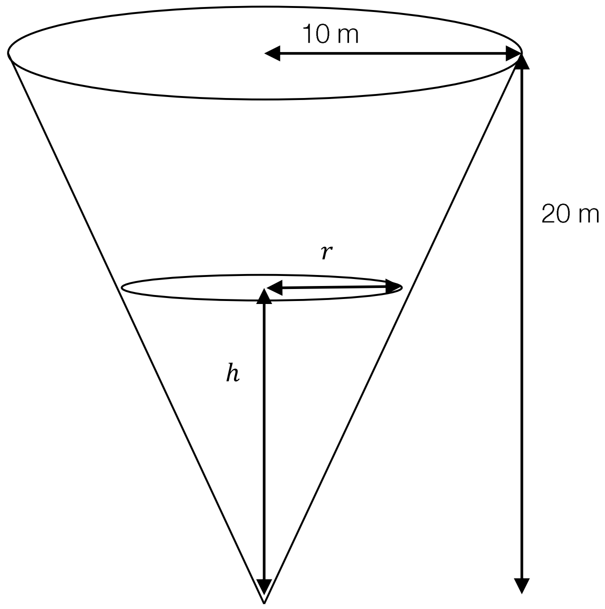
e. Given that $-1.4999 < \mathrm{p} < 4.2184$ find the value of p that maximises this length, correct to four decimal places (there is no need to find the length itself).

2 marks

Total: 16 marks

Question 2

There is an upside-down cone shape with dimensions (radius 10 m, height 20 m) housing a certain level of water inside. Let h be the height of the water level inside the cone, and r be the radius of the surface of the water.



a.

i. Find r in terms of h.

1 mark

ii. The volume of water in the cone is defined by a function of h. Find this function.

1 mark

b. There is a tap at the base of the cylinder. It is open and water is flowing out of it at a rate $\frac{dv}{dt} = 0.1m^3/min$. Find $\frac{dh}{dt}$.

2 marks

c. If $h = 4$, find the rate at which the depth is decreasing.

1 mark

d. When the volume of water falls below 1 m^3, the tank must be refilled to the top. Given that the cone starts full - from the moment water starts flowing, how long will it take until the tank needs to be refilled? Give your answer correct to 2 decimal places.

2 marks

e. The tap at the bottom is closed so that no more water flows out until the tank is filled. Water is added to the tank such that the volume at time t hours is equal to $t^3 - t^2 + t + 1$ metres cubed.

i. Find the time t, correct to three decimal places, at which the tank will be full again.

1 mark

ii. Find the rate of volume increase, correct to three decimal places, when the volume is increasing the slowest.

4 marks

f. 'Down-time' is when the tank is being refilled. What percentage of time is being spent as 'down-time'? Give your answer correct to 2 decimal places.

1 mark

Total: 13 marks

Question 3

The amount of salt, S (in grams), on a dish in a restaurant is distributed normally with a mean of one gram and standard deviation of 0.05 grams.

a.

i. Find $\Pr(S \geq 1.10)$ correct to three decimal places.

1 mark

ii. The amount of pepper P (in grams) is distributed normally with mean a and standard deviation b. In this restaurant, $\Pr(P \geq 1.00) = \Pr(S \geq 1.10)$ and $\Pr(P \geq 1.10) = \Pr(S \geq 1.30)$. Find a and b.

4 marks

b. When the amount of salt in a dish is below 1.10g, the dish is liked by all customers. However, when the amount of salt is 1.10g or above 40% of customers do not like the dish. We refer to dishes with 1.10g or more salt as being 'salty dishes'. Let X be the random variable representing the number of people who do not like the dish.

i. Given on one night there have been 10 'salty dishes' served, what is the probability, correct to four decimal places, that all 10 customers do not like the dish?

1 mark

ii. What is the probability, correct to four decimal places, that at least 2 customers do not like the dish?

2 marks

iii. What is the mean and variance of X on a night where 10 dishes are served?

2 marks

c. A regular customer is one that comes back to the restaurant each week. If a person paid \$20 or more for their meal one week, the probability that they pay \$20 or more the following week is 0.7. If a person paid less than \$20 one week, the probability that they pay less than \$20 the following week is 0.4.

i. What is the probability that, if a person pays more than \$20 one week, that two weeks later they pay less than \$20?

2 marks

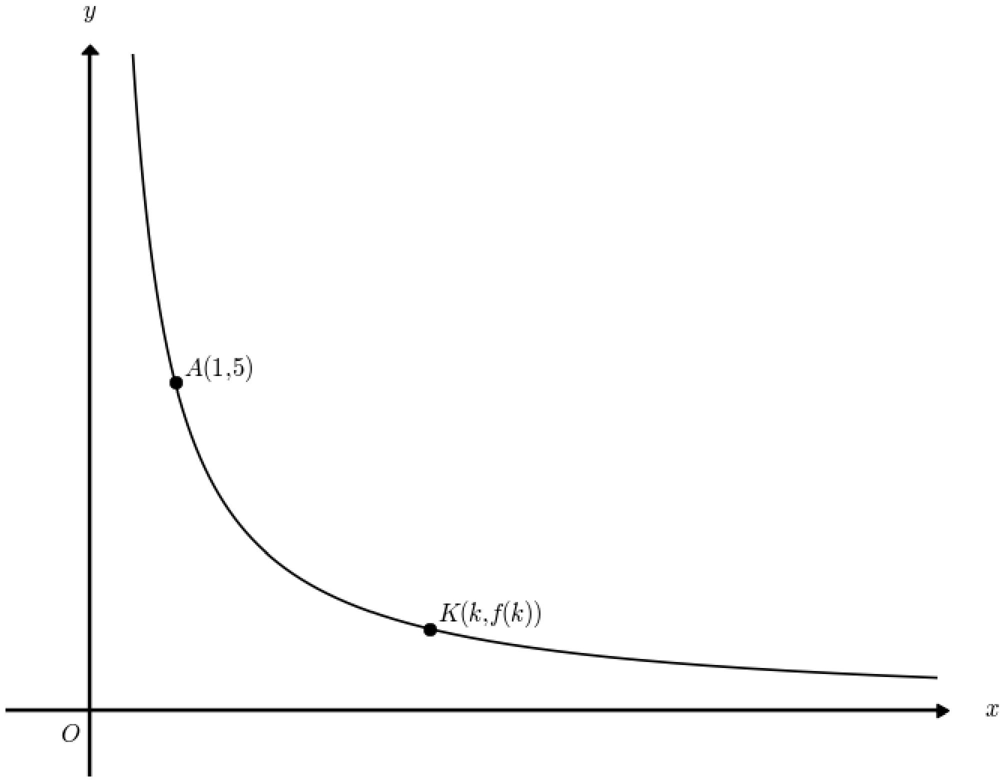
ii. What is the long term probability that they pay \$20 or over for their meal? Answer in exact form.

1 marks

Total: 13 marks

Question 4

The following is a graph of $= \frac{5}{x}$. Consider the first quadrant ($x > 0, y > 0$) only.



a.

i. Calculate the gradient between the two points in terms of k.

1 mark

ii. At what value of x between 1 and k does the tangent to the graph of f have the same gradient as AK?

2 marks

b.

i. Calculate the definite integral of f, between 1 and e^2.

1 mark

ii. Let a be a real number less than 1. Find the value of a such that the value of the definite integral of f between a and 1 is 10.

2 marks

c.

i. Find the area bounded by the line segment AK, the lines $x = 1$, $x = k$, and the x-axis, in terms of k.

2 marks

ii. For what value of k does this area equal 10?

2 marks

iii. Using the value for k determined in part ii, explain in words, without evaluating the integral, why the definite integral of $f(x)$ between 1 and k is less than 10. Hence, use this result to explain why $k < e^2$.

1 mark

d. Find the exact values of p and q such that $\int_1^{pq} f(x) = 6$ and $\int_1^{\frac{p}{q}} f(x) = 4$.

2 marks

e.

i. The line $y = n$ intersects $f(x)$. What is the equation of the tangent at this point?

2 marks

ii. For what value of n are the x and y-intercepts of the tangent the same value?

1 marks

Total: 16 marks

Formula sheet

Mensuration

area of a trapezium	$\frac{1}{2}(a+b)h$	volume of a pyramid	$\frac{1}{3}Ah$
curved surface area of a cylinder	$2\pi rh$	volume of a sphere	$\frac{4}{3}\pi r^3$
volume of a cylinder	$\pi r^2 h$	area of a triangle	$\frac{1}{2}bc\sin A$
volume of a cone	$\frac{1}{3}\pi r^2 h$		

Calculus

$\frac{d}{dx}(x^n) = nx^{n-1}$

$\int x^n dx = \frac{1}{n+1}x^{n+1} + c, n \neq -1$

$\frac{d}{dx}(e^{ax}) = ae^{ax}$

$\int e^{ax} dx = \frac{1}{a}e^{ax} + c$

$\frac{d}{dx}(\log_e x) = \frac{1}{x}$

$\int \frac{1}{x} dx = \log_e|x| + c$

$\frac{d}{dx}(\sin(ax)) = a\cos(ax)$

$\int \sin(ax)\, dx = -\frac{1}{a}\cos(ax) + c$

$\frac{d}{dx}(\cos(ax)) = -a\sin(ax)$

$\int \cos(ax)\, dx = \frac{1}{a}\sin(ax) + c$

$\frac{d}{dx}(\tan(ax)) = \frac{a}{\cos^2(ax)} = a\sec^2(ax)$

product rule $\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$

quotient rule $\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{\left(v\frac{du}{dx} - u\frac{dv}{dx}\right)}{v^2}$

chain rule $\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx}$

approximation $f(x+h) = f(x) + hf'(x)$

Probability

$\Pr(A) = 1 - \Pr(A')$

$\Pr(A \cup B) = \Pr(A) + \Pr(B) - \Pr(A \cap B)$

$\Pr(A|B) = \frac{\Pr(A \cap B)}{\Pr(B)}$

transition matrices $S_n = T^n \times S_0$

mean $\mu = E(X)$

variance $var(X) = \sigma^2 = E((X-\mu)^2) = E(X^2) - \mu^2$

probability distribution		mean	variance
discrete	$\Pr(X = x) = p(x)$	$\mu = \Sigma xp(x)$	$\sigma^2 = \Sigma(x-\mu)^2 p(x)$
continuous	$\Pr(a < X < b) = \int_a^b f(x)dx$	$\mu = \int_{-\infty}^{\infty} xf(x)dx$	$\sigma^2 = \int_{-\infty}^{\infty} (x-\mu)^2 f(x)dx$

End of Booklet

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