# Engage
# Education
# Foundation

# *Units 3 and 4 Maths Methods (CAS): Exam 1*

*Practice Exam Question and Answer Booklet*

Duration: 15 minutes reading time, 1 hour writing time

Structure of book:

| Number of questions | Number of questions to be answered | Number of marks |
|---|---|---|
| 10 | 10 | 40 |

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers and rulers.
- Students are not permitted to bring into the examination room: blank sheets of paper and/or white out liquid/tape.
- No calculator is allowed in this examination.

Materials supplied:
- This question and answer booklet of 8 pages.

Instructions:
- You must complete all questions of the examination.
- Write all your answers in the spaces provided in this booklet.

## Instructions

Answer all questions in the spaces provided.

In all questions where a numerical answer is required an exact value must be given unless otherwise specified.

In questions where more than one mark is available, appropriate working must be shown.

Unless otherwise indicated, the diagrams in this book are not drawn to scale.

## Questions

### Question 1

a. Find an anti-derivative of $f(x) = 4(2x - 5)^3$ with respect to $x$.

2 marks

b. For $f(x) = \frac{2\sin(x)}{x}$, find $f'(\pi)$.

3 marks

Total: 5 marks

Question 2

Let X be a random variable with a normal distribution with mean 15 and standard deviation 4. The random variable Z has the normal standard distribution.

a.   Find Pr(X > 19)

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_____

_____

<div align="right">1 mark</div>

  b. Find c such that Pr(X < 10) = Pr(Z > c)

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<div align="right">2 marks</div>

<div align="right">Total: 3 marks</div>

## Question 3

The function $f(x) = \frac{1}{x}$ undergoes the following transformations.

- Dilation of factor 3 from $x$ –axis.
- Dilation of factor $\frac{1}{2}$ from $y$-axis.
- Reflection in the $x$ -axis.
- Translation of 1 unit in the positive $y$ direction.
- Translation of 2 units in the negative $x$ direction.

What is the equation of the resulting function?

4 marks

**Question 4**

Solve $2log_e(x) - log_e(x+4) = log_e(2)$ for $x$.

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_____

_____

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_____

<div align="right">4 marks</div>

**Question 5**

A Spinner is divided into 6 equal segments, 3 are coloured red, 2 are coloured blue and 1 is coloured green. The spinner is spun twice, what is the probability that two different colours are spun given that the first one was blue?

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<div align="right">3 marks</div>

## Question 6

a.  Solve $\sin(2x) = \sqrt{3}\cos(2x)$, for $x \in [-\frac{\pi}{2}, \frac{\pi}{2}]$.

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_____

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_____

2 marks

b.  Find the Range and End points of $f$, when $f(x) = 2\cos\left(x - \frac{\pi}{3}\right) + 1$, for $\in [0, 2\pi]$.

_____

_____

_____

_____

2 mark

**Total: 4 marks**

## Question 7

Find the value of $a$ such that the area bounded by the curve $y = ax - x^2$, the $x$-axis, the $y$-axis and the line $x = a$ is $^9/_2$.

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_____

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3 marks

### Question 8
A 5m ladder is leaning up against a wall with the upper end against a vertical wall and the lower end against the horizontal floor, the upper end of the ladder is sliding down the wall at a rate of 2 m/s. Find the rate at which the lower end of the ladder is moving the instant the upper end of the ladder is 3m up the wall.

4 marks

### Question 9
Sketch the function $f(x) = 4e^{x-2} + 1$, and its inverse $f^{-1}$ on the axes below, labelling asymptotes and any axial intercepts.
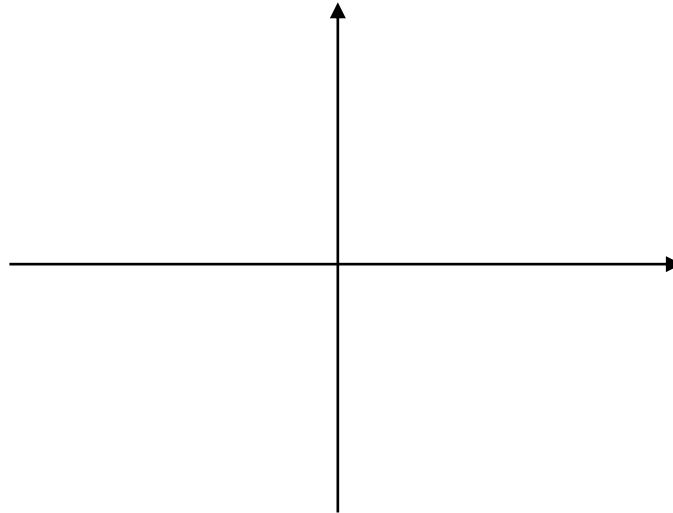


4 marks

## Question 10

a.    Find the normal to the equation $y = x^3 - x^2 - x + 1$ at the point (0, 1).

_____

_____

_____

_____

3 marks

b.    Hence, find the coordinates of where the normal and the function $= x^3 - x^2 - x + 1$ .

_____

_____

_____

3 mark

**Total: 6 marks**

## Formula sheet

### Mensuration

| | | | |
|---|---|---|---|
| area of a trapezium | $\frac{1}{2}(a+b)h$ | volume of a pyramid | $\frac{1}{3}Ah$ |
| curved surface area of a cylinder | $2\pi rh$ | volume of a sphere | $\frac{4}{3}\pi r^3$ |
| volume of a cylinder | $\pi r^2 h$ | area of a triangle | $\frac{1}{2}bc\sin A$ |
| volume of a cone | $\frac{1}{3}\pi r^2 h$ | | |

### Calculus

$$\frac{d}{dx}(x^n) = nx^{n-1}$$

$$\int x^n dx = \frac{1}{n+1}x^{n+1} + c, n \neq -1$$

$$\frac{d}{dx}(e^{ax}) = ae^{ax}$$

$$\int e^{ax} dx = \frac{1}{a}e^{ax} + c$$

$$\frac{d}{dx}(\log_e x) = \frac{1}{x}$$

$$\int \frac{1}{x} dx = \log_e |x| + c$$

$$\frac{d}{dx}(\sin(ax)) = a\cos(ax)$$

$$\int \sin(ax)\, dx = -\frac{1}{a}\cos(ax) + c$$

$$\frac{d}{dx}(\cos(ax)) = -a\sin(ax)$$

$$\int \cos(ax)\, dx = \frac{1}{a}\sin(ax) + c$$

$$\frac{d}{dx}(\tan(ax)) = \frac{a}{\cos^2(ax)} = a\sec^2(ax)$$

product rule $\quad \frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$

quotient rule $\quad \frac{d}{dx}\left(\frac{u}{v}\right) = \frac{\left(v\frac{du}{dx} - u\frac{dv}{dx}\right)}{v^2}$

chain rule $\quad \frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx}$

approximation $\quad f(x+h) = f(x) + hf'(x)$

### Probability

$$\Pr(A) = 1 - \Pr(A')$$

$$\Pr(A \cup B) = \Pr(A) + \Pr(B) - \Pr(A \cap B)$$

$$\Pr(A|B) = \frac{\Pr(A \cap B)}{\Pr(B)}$$

transition matrices $\quad S_n = T^n \times S_0$

mean $\quad \mu = E(X)$

variance $\quad var(X) = \sigma^2 = E((X-\mu)^2) = E(X^2) - \mu^2$

| | probability distribution | mean | variance |
|---|---|---|---|
| discrete | $\Pr(X = x) = p(x)$ | $\mu = \Sigma x p(x)$ | $\sigma^2 = \Sigma(x-\mu)^2 p(x)$ |
| continuous | $\Pr(a < X < b) = \int_a^b f(x)dx$ | $\mu = \int_{-\infty}^{\infty} xf(x)dx$ | $\sigma^2 = \int_{-\infty}^{\infty} (x-\mu)^2 f(x)dx$ |

End of Booklet

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