Specific instructions to students

- Answer all of the questions in the spaces provided.
- Show all workings in questions where more than one mark is available.
- An exact value must be provided in questions where a numerical answer is required, unless otherwise specified.

QUESTION 1

For the function $f(x) = e^x\sqrt{x}$, find $f'(x)$.

$f(x) = e^x\sqrt{x} = e^x x^{\frac{1}{2}}$

Using the product rule,

$f'(x) = x^{\frac{1}{2}}e^x + e^x \frac{1}{2}x^{-\frac{1}{2}}$

$= \sqrt{x}e^x + e^x\frac{1}{2\sqrt{x}} = e^x\left(\sqrt{x} + \frac{1}{2\sqrt{x}}\right)$

2 marks

QUESTION 2

a For the function $f(x) = e^x x^2$, find $f'(x)$.

$f(x) = e^x x^2$

Using the product rule,

$f'(x) = e^x x^2 + 2e^x x$

2 marks

b Hence find $f'(2)$.

$f'(x) = e^x x^2 + 2e^x x$

so $f'(2) = 4e^2 + 4e^2 = 8e^2$

1 mark

(Total: 3 marks)

QUESTION 3

a For the function $f(x) = \frac{\cos(2x)}{x+2}$, find $f'(x)$.

$f(x) = \frac{\cos(2x)}{x+2}$

Using the quotient rule,

$f'(x) = \frac{-2(x+2)\sin(2x) - \cos(2x)}{(x+2)^2}$

2 marks

b Hence find $f'(\pi)$.

$f'(x) = \frac{-2(x+2)\sin(2x) - \cos(2x)}{(x+2)^2}$

$= \frac{-2(\pi+2)\sin(2\pi) - \cos(2\pi)}{(\pi+2)^2}$

$= \frac{-1}{(\pi+2)^2}$

1 mark

(Total: 3 marks)

QUESTION 4

a For the function $y = \frac{\sin(2x)}{\left(x^2-2\right)^2}$, find $\frac{dy}{dx}$.

$y = \frac{\sin(2x)}{\left(x^2-2\right)^2}$

Using the quotient and chain rules,

$\frac{dy}{dx} = \frac{2\left(x^2-2\right)^2\cos(2x) - \sin(2x)\times 2\left(x^2-2\right)\times 2x}{\left(x^2-2\right)^4}$

$= \frac{2\left(x^2-2\right)\cos(2x) - 4x\sin(2x)}{\left(x^2-2\right)^3}$

3 marks

b Hence find $\frac{dy}{dx}$ at $x = \pi$

At $x = \pi$,

$\frac{dy}{dx} = \frac{2(\pi^2-2)\cos(2\pi) - 4\pi\sin(2\pi)}{(\pi^2-2)^3}$

$= \frac{2(\pi^2-2)}{(\pi^2-2)^3} = \frac{2}{(\pi^2-2)^2}$

1 mark

(Total: 4 marks)

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QUESTION 5

The volume $V(t)$ litres of liquid in a drum at time t minutes is described by the formula

$V(t) = 3t^3 + \frac{1}{t}, t > 0.$

a Find the average rate of change of liquid, in litres/min, from $t = 1$ to $t = 2$.

Average rate of change $= \frac{V(2) - V(1)}{2 - 1}$

$= 24.5 - 4$

$= 20.5$ litres/min

2 marks

b Find the rate of change of liquid, in litres/min, when $t = 1$.

$V(t) = 3t^3 + \frac{1}{t} = 3t^3 + t^{-1}$

$V'(t) = 9t^2 - t^{-2} = 9t^2 - \frac{1}{t^2}$

$V'(1) = 9 - 1 = 8$ litres/min

2 marks

c Find at what time, in mins, there is a minimum amount of liquid in the drum.

Let $V'(t) = 9t^2 - \frac{1}{t^2} = 0$ for minimum.

$9t^2 = \frac{1}{t^2}$

$t^4 = \frac{1}{9}$

Select the +ve solution. $t = \frac{1}{\sqrt{3}}$ mins (can see from the graph that this is a minimum)

2 marks
(Total: 6 marks)

QUESTION 6

The equation of a graph is $f(x) = 2x^3 + 1 - kx^2$, where k is a constant.
The tangent to the graph at $x = 1$ meets the x-axis at the point (2, 0). Find the value of k.

$f(x) = 2x^3 + 1 - kx^2$

$\Rightarrow f(1) = 3 - k$

$f'(x) = 6x^2 - 2kx$

$\Rightarrow f'(1) = 6 - 2k$

The equation of the tangent using $y - y_1 = m(x - x_1)$ is:

$y - (3 - k) = (6 - 2k)(x - 1)$

At the point (2, 0),

$0 - (3 - k) = (6 - 2k)(2 - 1)$

$\Rightarrow -3 + k = 6 - 2k$

This gives $k = 3$.

3 marks

QUESTION 7

a The graph of $y = \frac{x^3}{3} - \frac{x^2}{4} + ax + b$ has a stationary point at $\left(2, \frac{2}{3}\right)$. Find the values of a and b.

$y = \frac{x^3}{3} - \frac{x^2}{4} + ax + b$

Substituting $\left(2, \frac{2}{3}\right)$ gives

$\frac{2}{3} = \frac{8}{3} - \frac{4}{4} + 2a + b$

So $2a + b = -1$

Also, $\frac{dy}{dx} = x^2 - \frac{x}{2} + a$

$x^2 - \frac{x}{2} + a = 0$ at $x = 2$.

So $3 + a = 0$

$\therefore a = -3, b = 5$

3 marks

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b Hence, find the x-coordinate of the other stationary point.

$y = \frac{x^3}{3} - \frac{x^2}{4} - 3x + 5$

$\frac{dy}{dx} = x^2 - \frac{x}{2} - 3 = 0$ for stationary points

$2x^2 - x - 6 = 0$

$(2x + 3)(x - 2) = 0$ gives $x = 2$ and $x = -\frac{3}{2}$

Other stationary point: $x = -\frac{3}{2}$

2 marks
(Total: 5 marks)

QUESTION 8

a For the function $y = \frac{e^{3x}}{\sin(3x)}$, find an expression for $\frac{dy}{dx}$.

$y = \frac{e^{3x}}{\sin(3x)}$

Using the quotient rule,

$$\frac{dy}{dx} = \frac{\sin(3x) \times 3e^{3x} - e^{3x} \times 3\cos(3x)}{\sin^2(3x)}$$

$$= \frac{3\sin(3x)e^{3x} - 3\cos(3x)e^{3x}}{\sin^2(3x)}$$

2 marks

b Hence, find $\left\{x : \frac{dy}{dx} = 0\right\}$ for $x \in \left[0, \frac{\pi}{2}\right]$.

$$\frac{3\sin(3x)e^{3x} - 3\cos(3x)e^{3x}}{\sin^2(3x)} = 0$$

$\Rightarrow 3e^{3x}\sin(3x) - 3e^{3x}\cos(3x) = 0$

$\Rightarrow 3e^{3x}(\sin(3x) - \cos(3x)) = 0$

No solution for $3e^{3x} = 0$.

$\therefore \sin(3x) - \cos(3x) = 0$

$\sin(3x) = \cos(3x)$

$\tan(3x) = 1$

$3x = \frac{\pi}{4}, \pi + \frac{\pi}{4} = \frac{\pi}{4}, \frac{5\pi}{4}$

$x = \frac{\pi}{12}, \frac{5\pi}{12}$

2 marks
(Total: 4 marks)

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