2014 Mathematical Methods (CAS) GA 3: Examination 2

GENERAL COMMENTS

In the 2014 Mathematical Methods (CAS) examination 2, students achieved scores across the whole range of available marks. As in 2013, a small number of students sat the computer-based version of this examination. Responses showed that the examination was accessible and that it provided an opportunity for students to demonstrate their knowledge.

In Section 1, most students answered Questions 1 and 7 correctly, whereas few students answered Question 21 correctly. Less than half of the students obtained the correct answers for Questions 13, 14, 15, 16, 20, 21 and 22.

In Section 2, students need to take care with providing answers in the correct form. Exact answers are required unless otherwise stated. This caused problems in Questions 2 and 5 in Section 2. Some students did not work to the required number of decimal places. This was evident in Question 3cii. There were a number of rounding errors in Questions 3 and 4.

Students need to include units in answers as applicable. Students often used incorrect units in Questions 3 and 4. Some students incorrectly converted centimetres to millimetres in Question 4.

Brackets were often missing from equations, leading to incorrect answers. This occurred in Questions 4 and 5. 'Equal to' signs were often used incorrectly in Question 4. There was poor use of the inequality sign in Questions 4 and 5. Some students did not have their technology set for radians, as was needed for Questions 1 and 3.

Some students used inefficient methods when answering questions; for example, using a tree diagram when a transition matrix could have been used, using *z* scores when they were not required and converting hours to minutes when it was not necessary.

Students need to check their responses to make sure they have answered all parts of a question and have put their answers in the correct form. They need to reread questions to make sure they are answering the question asked. Adequate working must be shown for questions worth more than one mark.

SPECIFIC INFORMATION

This report provides sample answers or an indication of what answers may have included. Unless otherwise stated, these are not intended to be exemplary or complete responses.

The statistics in this report may be subject to rounding errors resulting in a total less than 100 per cent.

Section 1

The table below indicates the percentage of students who chose each option. The correct answer is indicated by the shading.

Question	% A	% B	% C	% D	% E	% No Answer	Comments
1	89	1	1	1	7	0	
2	10	2	5	80	2	0	
3	8	12	5	60	14	0	
4	9	20	4	2	65	0	
5	12	11	58	8	10	0	
6	20	55	12	9	3	1	
7	1	1	3	6	89	0	
8	59	11	13	9	7	1	
9	76	2	3	4	15	0	
10	6	10	11	6	66	0	
11	10	10	8	10	62	0	
12	10	12	51	16	11	1	

Question	% A	% B	% C	% D	% E	% No Answer	Comments
13	13	13	42	20	11	1	$h(x)=\cos(\log_a(x))$ $\cos(\log_a(1))=\cos(0)=1$ $\cos\left(\log_a(a^{\frac{\pi}{2}})\right)=\cos\left(\frac{\pi}{2}\right)=0$ The domain could be $\left[1,\ a^{\frac{\pi}{2}}\right]$.
14	11	18	10	14	45	1	If X is a continuous random variable, then $\Pr(X<5\mid X<8)=\frac{\Pr(X<5\cap X<8)}{\Pr(X<8)}$ $=\frac{\Pr(X<5)}{\Pr(X<8)}=\frac{1-a}{1-b}=\frac{a-1}{b-1}$
15	7	44	16	13	19	1	$V(x)=x(6-2x)(8-2x)$, solve $V'(x)=0$, $x=1.13...$ as $V(x)>0$ x is closest to 1.1
16	14	17	46	18	4	1	$\mathrm{E}(X^2)=\int_{-\infty}^{\infty}x^2p(x)dx$ $\mathrm{Var}(X)=\mathrm{E}(X^2)-(\mathrm{E}(X))^2$ $5=\mathrm{E}(X^2)-4$ $\mathrm{E}(X^2)=9$
17	15	50	17	10	8	1	
18	5	53	17	20	4	1	
19	5	20	61	10	3	1	
20	7	12	30	44	6	1	From inspection, the average value of h is either 6 or 7. $(11-1)\times$ average value = area under the curve = area of the rectangle + area of the triangle Hence, $10\times$ average value = $10\times4+5\times6$ average value = 7
21	12	18	27	28	15	1	Area of a trapezium = $\frac{h(a+b)}{2}$ $=\frac{p\sin(x)(p+p+2\cos(x))}{2}$ Maximum area is when $x=\frac{\pi}{3}$.
22	17	15	15	15	37	1	$\Pr(R\geq1):\Pr(J\geq1)$ $1-\Pr(R=0):1-\Pr(J=0)$ $1-\left(\frac{1}{2}\right)^2:1-\left(\frac{3}{4}\right)^4$ 192:175

Section 2

Question 1
1a.

Marks	0	1	2	Average
%	4	13	83	1.8

$n(t) = 1200 + 400\cos\left(\frac{\pi t}{3}\right)$, amplitude is 400, period = $\frac{2\pi}{\frac{\pi}{3}} = 2\pi \times \frac{3}{\pi} = 6$ months

This question was answered well. However, some students did not answer both parts of the question. Most had the correct period but some expressed the amplitude as $[800, 1600]$.

1b.

Marks	0	1	2	Average
%	9	5	86	1.8

The minimum is $1200 - 400 = 800$ wombats and the maximum is $1200 + 400 = 1600$ wombats.

This question was answered well. Some students used calculus, which was not necessary. Some gave their answers as coordinate pairs, such as (0, 1600) and (3, 800), but this was incorrect.

1c.

Marks	0	1	Average
%	9	91	0.9

$n(10) = 1000$ wombats

Some students used their technology in degrees instead of radians and gave the answer 1593 wombats.

1d.

Marks	0	1	2	Average
%	33	38	29	1

Solve $n(t) < 1000$, $0 \le t \le 12$, $4 - 2 + 10 - 8 = 4$ months, fraction of time: $\frac{4}{12} = \frac{1}{3}$ year

Many students obtained 4 months but did not find the fraction of time. Others did not find all the *t* values. Some wrote their answers in terms of dates.

Question 2
2a.

Marks	0	1	2	Average
%	15	10	75	1.6

$V = \pi r^2 h$, $216 = \pi\left(\frac{d}{2}\right)^2 h$, $h = \frac{864}{\pi d^2}$

This question was quite well answered. Some students used the formula for the volume of a cone instead of a cylinder. Some used poor notation, omitting brackets and writing $\frac{d^2}{2} = \frac{d^2}{4}$. Many left their answer in the form $h = \frac{216}{\pi\left(\frac{d}{2}\right)^2}$, which was accepted; however, it is preferable to write in simplified form.

2b.

Marks	0	1	Average
%	43	57	**0.6**

$$S=\pi dh+\pi\left(\frac{d}{2}\right)^2=\pi d\left(\frac{864}{\pi d^2}\right)+\frac{\pi d^2}{4}=\frac{\pi d^2}{4}+\frac{864}{d}$$

This was a 'show that' question and some students showed sufficient working. Some included the area of the base of the cylinder. Others did not include the area of the top of the cylinder and only considered the curved surface area.

2c.

Marks	0	1	2	Average
%	25	25	50	**1.3**

Solve $\frac{dS}{dd}=0$ for $d=\frac{12}{\sqrt[3]{\pi}}$ m, $S=108\sqrt[3]{\pi}$ m^2

Some students answered only part of the question, finding the correct value for *d* but not attempting to find *S*. Exact answers were required. Answers such as *d* = 8.19… and *S* = 158.17… were often given.

2d.

Marks	0	1	Average
%	58	42	**0.4**

$$h=\frac{864}{\pi\left(\frac{12}{\sqrt[3]{\pi}}\right)^2}=\frac{6}{\sqrt[3]{\pi}}\text{ m}$$

Some students did not square $\frac{12}{\sqrt[3]{\pi}}$, using $h=\frac{864}{\pi\left(\frac{12}{\sqrt[3]{\pi}}\right)}$ to get $\frac{72}{\pi^{\frac{2}{3}}}$. An exact answer was required, not a decimal expression such as 4.09, as was given by some students. Some substituted $d=\frac{12}{\sqrt[3]{\pi}}$ into $S=\frac{\pi d^2}{4}+\frac{864}{d}$.

2e.

Marks	0	1	Average
%	41	59	**0.6**

$$d=2h,\ V=\pi\left(\frac{d}{2}\right)^2h=\pi\left(\frac{2h}{2}\right)^2h=\pi h^3$$

Some students used an incorrect formula, such as $V=\pi(2h)^2h=4\pi h^3$ or $V=2\pi rh=2\pi h^2$.

2f.

Marks	0	1	2	3	Average
%	27	12	38	22	**1.6**

$$\frac{dh}{dt}=\frac{dh}{dV}\times\frac{dV}{dt},\ \frac{dV}{dh}=3\pi h^2,\ \frac{dV}{dt}=-10,\ \frac{dh}{dt}=\frac{1}{3\pi h^2}\times-10=-\frac{10}{3\pi h^2}\ \text{m/year}$$

Many students were able to set up the related rates equation and find $\frac{dV}{dh}$. Some students did not find the reciprocal before substituting into $\frac{dh}{dV}$. Many used $\frac{dV}{dt}=10$ m^3/year.

2g.

Marks	0	1	Average
%	60	40	**0.4**

$\frac{dh}{dt}=-\frac{10}{3\pi(1)^2}=-\frac{10}{3\pi}$ m/year, decreasing at $\frac{10}{3\pi}$ m/year

Students who answered Question 2f. correctly tended to also answer this question correctly. An exact answer was required. Some students gave incorrect units.

2h.

Marks	0	1	2	Average
%	86	4	9	**0.3**

$V=\pi h^3=\pi(1)^3=\pi$, $t=\frac{216-\pi}{10}=21.2858...$, 2010 + 21.2… = 2031.2… or $\frac{dh}{dt}=-\frac{10}{3\pi h^2}$

$t=\int_1^{\frac{6}{\sqrt[3]{\pi}}}\frac{3\pi h^2}{10}dh=21.2858...$, during 2031

This question was not answered well. A number of different approaches could have been used. Some students gave 2032 as their final answer. Some did not subtract π from 216 and used $t=\frac{216}{10}$. Incorrect terminals were often used or if evaluating $t=\int\left(-\frac{3\pi h^2}{10}\right)dh$, a constant of integration was often missing. Some used $t=\int\left(-\frac{10}{3\pi h^2}\right)dh$.

Question 3

3a.

Marks	0	1	Average
%	25	75	**0.8**

$c(t)=\frac{5}{2}te^{-\frac{3t}{2}}$, Solve $c'(t)=0$ for t, $t=\frac{2}{3}$, $c\left(\frac{2}{3}\right)=0.61$ mg/L, correct to two decimal places

This question was answered well. Some students had incorrect units, such as mm for milligrams. Some left their answers in exact form. Some found *t* correct to two decimal places and left their answer as 0.67.

3bi.

Marks	0	1	Average
%	19	81	**0.8**

Solve $c(t)=0.5$ for t, $t=0.33$ h, correct to two decimal places

This question was answered well. Some students gave two answers, 0.33 and 1.19, instead of only the first one, as specified in the question. Some students rounded incorrectly and gave 0.32 as their answer.

3bii.

Marks	0	1	2	Average
%	19	9	72	**1.6**

$1.187558...-0.326268...=0.86$ h, correct to two decimal places

Students should always work to suitable accuracy in intermediate calculations to support rounding the answer to the required accuracy. Some students wrote down the two values but did not find the difference for the length of time. Some added the two values. Some students incorrectly converted the time to minutes.

3ci.

Marks	0	1	2	Average
%	27	21	52	1.3

Average rate of change $= \dfrac{c(3) - c\left(\frac{2}{3}\right)}{3 - \frac{2}{3}} = -0.23$ mg/L/h, correct to two decimal places

Some students worked out the average value of the function. Some used $\dfrac{3 - \frac{2}{3}}{c(3) - c\left(\frac{2}{3}\right)}$. Others had incorrect units.

Some changed their answer to 0.23 mg/L/h.

3cii.

Marks	0	1	2	Average
%	51	23	26	0.8

Solve $c'(t) = -0.227...$ for t, $t = 0.90$ or $t = 2.12$ h, correct to two decimal places

Some rounded their answers incorrectly. Others did not work to the required number of decimal places.

3d.

Marks	0	1	2	3	Average
%	31	27	12	29	1.4

$n(t) = Ate^{-kt}$, solve $n(0.5) = 0.74$ and $n'(0.5) = 0$ for A and k, A = 4, correct to the nearest integer

Many students were able to set up at least one of the equations. $n'(0.5) = 0.74$ was often used. Some students differentiated by hand incorrectly. Some students gave the value of k, not A. Others gave an exact answer.

Question 4

4a.

Marks	0	1	Average
%	57	43	0.5

$X \sim N\left(14, 4^2\right)$, $\Pr\left(X > x\right) = 0.1$, $x = 19.1$ cm or 191 mm, correct to the nearest millimetre

Many students thought 100 mm = 1 cm, giving their final answer as 1913 mm. Others had incorrect units, such as 19.1 mm. Some entered the incorrect probability into their technology.

4b.

Marks	0	1	2	Average
%	40	13	47	1.1

$\Pr\left(X < 9\right) = 0.10565...$, $0.10565... \times 2000 = 211$ basil plants

Some students had incorrect working, such as $\Pr\left(X < 9\right) = 0.10565... = 0.10565... \times 2000 = 211$ basil plants. Some students used $\Pr\left(X < 8.9\right)$ or $\Pr\left(X < 8\right)$. Some rounded incorrectly. Some used technology syntax in their working. Correct mathematical notation was required. Other students complicated the question by using z values. Many of these attempts were unsuccessful.

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4c.

Marks	0	1	Average
%	27	73	**0.8**

$$E(X) = \int_0^{50} \left(x \times \frac{\pi}{100} \sin\left(\frac{\pi x}{50}\right)\right) dx = 25 \text{ cm}$$

This question was answered well. Some students used the median formula. Others used their technology in degrees rather than radians.

4d.

Marks	0	1	2	Average
%	60	15	25	**0.7**

Solve $\int_0^{a} \left(\frac{\pi}{100} \sin\left(\frac{\pi x}{50}\right)\right) dx = 0.15$ for a, $a = 12.7$ cm or 127 mm

Some students attempted to use the normal distribution to answer this question. Some had incorrect units or conversions.

4e.

Marks	0	1	2	Average
%	63	14	23	**0.6**

$Y \sim \text{Bi}(n, 0.2)$, $\Pr(Y \geq 1) > 0.95$, solve $0.8^n < 0.05$ for n, $n = 14$ tomato plants

Many students did not know to use the binomial distribution and others used the inequality sign incorrectly. Many different approaches could have been used. Many different approaches were used, including trial and error.

4fi.

Marks	0	1	2	Average
%	43	7	50	**1.1**

$$0.7 \times 0.7 + 0.3(1-p) = 0.79 - 0.3p \text{ or } \begin{bmatrix} 0.7 & 1-p \\ 0.3 & p \end{bmatrix}^2 \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0.79 - 0.3p \\ 0.21 + 0.3p \end{bmatrix}$$

Probability that the third pot is smooth = $0.79 - 0.3p$

Students who used a tree diagram often gave the correct answer. $\begin{bmatrix} 0.7 & 1-p \\ 0.3 & p \end{bmatrix}^3 \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ or $\begin{bmatrix} 0.7 & p \\ 0.3 & 1-p \end{bmatrix}^2 \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ were common incorrect formulations. Brackets were sometimes omitted, giving the incorrect answer $0.7 \times 0.7 + 0.3 \times 1 - p = 0.79 - p$

4fii.

Marks	0	1	2	Average
%	34	18	48	**1.2**

Solve $0.79 - 0.3p = 0.61$, $p = 0.6$

4g.

Marks	0	1	2	Average
%	49	8	43	**1**

$T^4 \times S_0 = \begin{bmatrix} 0.4375 \\ 0.5625 \end{bmatrix}$, probability that the fifth pot is smooth = 0.4375 or $\frac{7}{16}$

Some students used a tree diagram but were unsuccessful as there were too many branches. $T^5 \times S_0$ was a common incorrect formulation. Some students gave incorrect working, such as $T^4 \times S_0 = \begin{bmatrix} 0.4375 \\ 0.5625 \end{bmatrix} = 0.4375$.

Question 5

5a.

Marks	0	1	2	Average
%	33	19	48	**1.2**

$x^4 - 8x = x(x-2)(x^2+2x+4) = x(x-2)((x+1)^2+3)$

Some students attempted to factorise by hand and obtained two correct linear factors but an incorrect quadratic factor such as $x(x-2)(x^2-2x+4)$. Others used their technology to factorise appropriately.

5b.

Marks	0	1	Average
%	63	37	**0.4**

Translation of 1 unit in the negative direction of the *x*-axis

Some students did not make the connection between Question 5a. and Question 5b. Others described the translation as one unit in the positive direction of the *x*-axis.

5ci.

Marks	0	1	Average
%	93	7	**0.1**

Translate the graph of f, 1 unit to the left and there will be one positive *x*-intercept.
If the graph is translated 3 units to the left, there will be no positive intercepts. $[1,\ 3)$ or $1 \le d < 3$

This question was not answered well. $(1, 3)$ and $1 < d < 3$ were common incorrect answers.

5cii.

Marks	0	1	Average
%	81	19	**0.1**

For two positive *x*-intercepts $d < 1$ or $(-\infty, 1)$

Some students used incorrect notation such as $(1, -\infty)$.

5d.

Marks	0	1	Average
%	83	17	**0.2**

$x^4 - 8x = n$, solve $g'(x) = 0$ for x, $x = 2^{\frac{1}{3}}$, $g\left(2^{\frac{1}{3}}\right) = -6 \times 2^{\frac{1}{3}}$

An exact answer was required. Some students gave only the *x* value, and not the required value of *n*.

5ei.

Marks	0	1	2	Average
%	68	8	24	**0.6**

$g'(u) = m, g'(v) = -m$, $4u^3 - 8 = m$ ….(1), $4v^3 - 8 = -m$ …(2)

Add (1) and (2), $4u^3 + 4v^3 = 16$, $u^3 + v^3 = 4$

A number of different approaches could be used to answer this question. Many students were unable to set up the two equations. Most students who were able to set up the two equations were able to answer the question correctly.

5eii.

Marks	0	1	Average
%	90	10	0.1

Solve $u^3 + v^3 = 4$ and $u + v = 1$ simultaneously, $u = \frac{\sqrt{5}+1}{2}$ and $v = \frac{-\sqrt{5}+1}{2}$ as $m > 0$

Many students gave two solutions when only one was required as $m > 0$. Some did not give exact answers.

5fi.

Marks	0	1	Average
%	78	22	0.2

$y - g(p) = g'(p)(x - p)$, $y - (p^4 - 8p) = (4p^3 - 8)(x - p)$, $y = (4p^3 - 8)x - 3p^4$

Some students omitted the brackets. $y - (p^4 - 8p) = 4p^3 - 8(x - p)$ and $y - p^4 - 8p = 4p^3 - 8(x - p)$ were often given.

5fii.

Marks	0	1	2	3	Average
%	80	6	2	11	0.5

Solve $-12 = (4p^3 - 8) \times \frac{3}{2} - 3p^4$, $p = 0$ or $p = 2$, when $p = 0$, $y = -8x$, when $p = 2$, $y = 24x - 48$

Many students did not read the question carefully and tried to find the equation of the tangent to $g(x)$ at $x = \frac{3}{2}$.

$\left(\frac{3}{2}, -12\right)$ was not a point on $g(x)$. Some did not realise that they could use their answer from the previous question. Others obtained $p = 0$ by incorrect working. The correct answers had to be obtained by correct working.