VICTORIAN CURRICULUM AND ASSESSMENT AUTHORITY

Victorian Certificate of Education 2014

SUPERVISOR TO ATTACH PROCESSING LABEL HERE

STUDENT NUMBER | | | | | | | | | Letter | |

MATHEMATICAL METHODS (CAS)

Written examination 1

Wednesday 5 November 2014

Reading time: 9.00 am to 9.15 am (15 minutes)
Writing time: 9.15 am to 10.15 am (1 hour)

QUESTION AND ANSWER BOOK

Structure of book

Number of questions	*Number of questions to be answered*	*Number of marks*
10	10	40

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers.
- Students are NOT permitted to bring into the examination room: notes of any kind, blank sheets of paper, white out liquid/tape or a calculator of any type.

Materials supplied
- Question and answer book of 12 pages, with a detachable sheet of miscellaneous formulas in the centrefold.
- Working space is provided throughout the book.

Instructions
- Detach the formula sheet from the centre of this book during reading time.
- Write your **student number** in the space provided above on this page.
- All written responses must be in English.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

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Instructions

Answer **all** questions in the spaces provided.

In all questions where a numerical answer is required, an exact value must be given unless otherwise specified.

In questions where more than one mark is available, appropriate working **must** be shown.

Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Question 1 (5 marks)

a. If $y = x^2 \sin(x)$, find $\frac{dy}{dx}$. 2 marks

b. If $f(x) = \sqrt{x^2 + 3}$, find $f'(1)$. 3 marks

Question 2 (2 marks)

Let $\int_4^5 \frac{2}{2x-1}\,dx = \log_e(b)$.

Find the value of b.

Question 3 (2 marks)

Solve $2\cos(2x) = -\sqrt{3}$ for x, where $0 \le x \le \pi$.

Question 4 (2 marks)

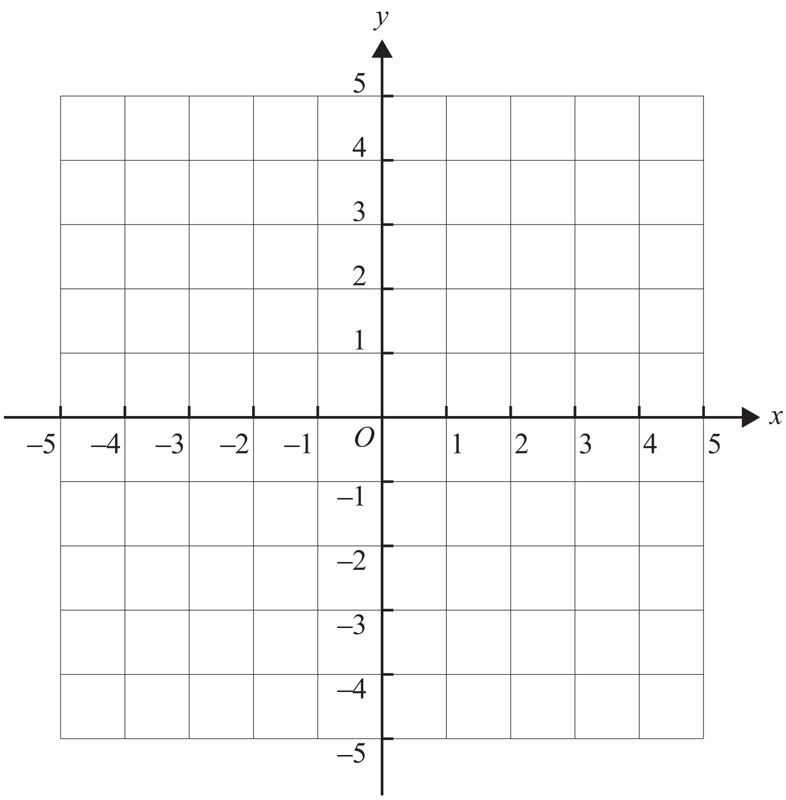
Solve the equation $2^{3x-3} = 8^{2-x}$ for x.

Question 5 (7 marks)

Consider the function $f:[-1,3]\to R,\ f(x)=3x^2-x^3$.

a. Find the coordinates of the stationary points of the function. 2 marks

b. On the axes below, sketch the graph of f.
Label any end points with their coordinates. 2 marks



c. Find the area enclosed by the graph of the function and the horizontal line given by $y = 4$. 3 marks

Question 6 (2 marks)

Solve $\log_e(x) - 3 = \log_e(\sqrt{x})$ for x, where $x > 0$.

Question 7 (3 marks)

If $f'(x) = 2\cos(x) - \sin(2x)$ and $f\left(\frac{\pi}{2}\right) = \frac{1}{2}$, find $f(x)$.

Question 8 (4 marks)

A continuous random variable, X, has a probability density function given by

$$f(x)=\begin{cases}\frac{1}{5}e^{-\frac{x}{5}} & x\geq 0\\ 0 & x<0\end{cases}$$

The median of X is m.

a. Determine the value of m. 2 marks

b. The value of m is a number greater than 1.

Find $\Pr(X<1|X\leq m)$. 2 marks

Question 9 (6 marks)

Sally aims to walk her dog, Mack, most mornings. If the weather is pleasant, the probability that she will walk Mack is $\frac{3}{4}$, and if the weather is unpleasant, the probability that she will walk Mack is $\frac{1}{3}$.

Assume that pleasant weather on any morning is independent of pleasant weather on any other morning.

a. In a particular week, the weather was pleasant on Monday morning and unpleasant on Tuesday morning.

Find the probability that Sally walked Mack on at least one of these two mornings. 2 marks

Question 9 – continued

b. In the month of April, the probability of pleasant weather in the morning was $\frac{5}{8}$.

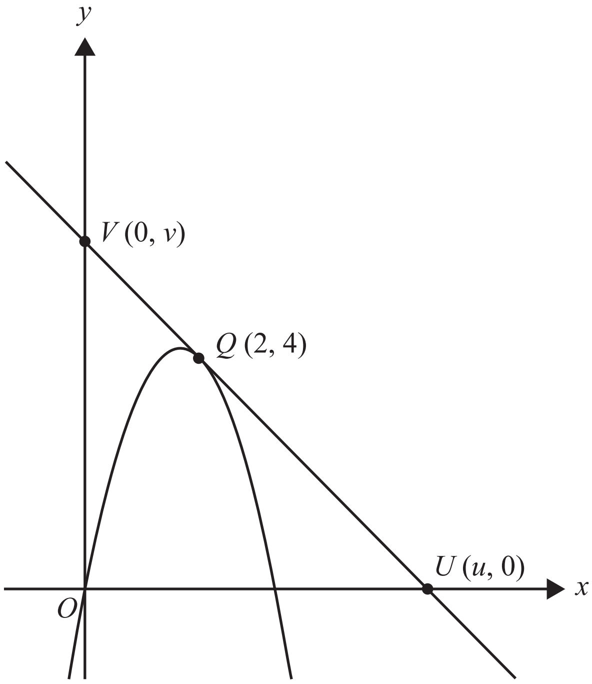
i. Find the probability that on a particular morning in April, Sally walked Mack. 2 marks

ii. Using your answer from **part b.i.**, or otherwise, find the probability that on a particular morning in April, the weather was pleasant, given that Sally walked Mack that morning. 2 marks

Question 10 (7 marks)

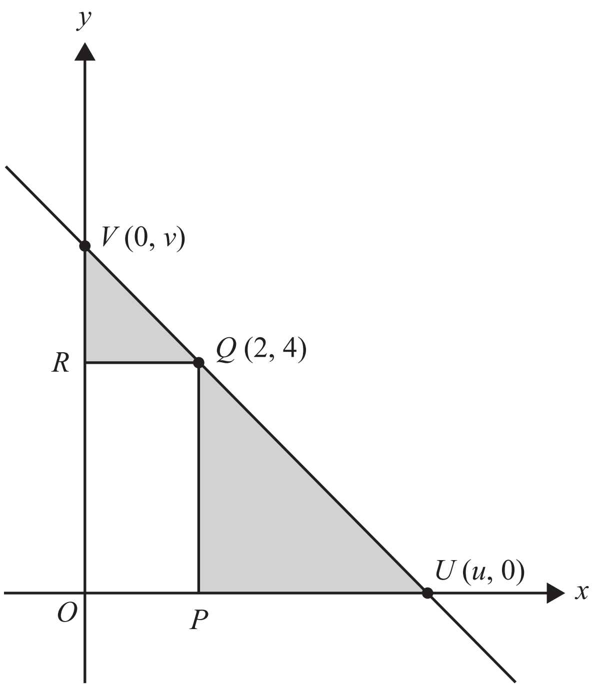
A line intersects the coordinate axes at the points U and V with coordinates $(u, 0)$ and $(0, v)$, respectively, where u and v are positive real numbers and $\frac{5}{2} \le u \le 6$.

a. When $u = 6$, the line is a tangent to the graph of $y = ax^2 + bx$ at the point Q with coordinates $(2, 4)$, as shown.



If a and b are non-zero real numbers, find the values of a and b. 3 marks

b. The rectangle *OPQR* has a vertex at *Q* on the line. The coordinates of *Q* are (2, 4), as shown.



i. Find an expression for v in terms of u. 1 mark

ii. Find the **minimum** total shaded area and the value of u for which the area is a minimum. 2 marks

iii. Find the **maximum** total shaded area and the value of u for which the area is a maximum. 1 mark

END OF QUESTION AND ANSWER BOOK

MATHEMATICAL METHODS (CAS)

Written examinations 1 and 2

FORMULA SHEET

Directions to students

Detach this formula sheet during reading time.
This formula sheet is provided for your reference.

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Mathematical Methods (CAS)
Formulas

Mensuration

area of a trapezium: $\frac{1}{2}(a+b)h$

curved surface area of a cylinder: $2\pi rh$

volume of a cylinder: $\pi r^2 h$

volume of a cone: $\frac{1}{3}\pi r^2 h$

volume of a pyramid: $\frac{1}{3}Ah$

volume of a sphere: $\frac{4}{3}\pi r^3$

area of a triangle: $\frac{1}{2}bc\sin A$

Calculus

$\frac{d}{dx}\left(x^n\right) = nx^{n-1}$

$\frac{d}{dx}\left(e^{ax}\right) = ae^{ax}$

$\frac{d}{dx}\left(\log_e(x)\right) = \frac{1}{x}$

$\frac{d}{dx}\left(\sin(ax)\right) = a\ \cos(ax)$

$\frac{d}{dx}\left(\cos(ax)\right) = -a\ \sin(ax)$

$\frac{d}{dx}\left(\tan(ax)\right) = \frac{a}{\cos^2(ax)} = a\ \sec^2(ax)$

$\int x^n dx = \frac{1}{n+1} x^{n+1} + c, n \neq -1$

$\int e^{ax} dx = \frac{1}{a} e^{ax} + c$

$\int \frac{1}{x} dx = \log_e |x| + c$

$\int \sin(ax)dx = -\frac{1}{a}\cos(ax) + c$

$\int \cos(ax)dx = \frac{1}{a}\sin(ax) + c$

product rule: $\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$

quotient rule: $\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$

chain rule: $\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx}$

approximation: $f(x+h) \approx f(x) + hf'(x)$

Probability

$\Pr(A) = 1 - \Pr(A')$

$\Pr(A \cup B) = \Pr(A) + \Pr(B) - \Pr(A \cap B)$

$\Pr(A|B) = \frac{\Pr(A \cap B)}{\Pr(B)}$

transition matrices: $S_n = T^n \times S_0$

mean: $\mu = E(X)$

variance: $\mathrm{var}(X) = \sigma^2 = E((X-\mu)^2) = E(X^2) - \mu^2$

Probability distribution		Mean	Variance
discrete	$\Pr(X = x) = p(x)$	$\mu = \sum x\, p(x)$	$\sigma^2 = \sum (x-\mu)^2\, p(x)$
continuous	$\Pr(a < X < b) = \int_a^b f(x)dx$	$\mu = \int_{-\infty}^{\infty} x\, f(x)dx$	$\sigma^2 = \int_{-\infty}^{\infty} (x-\mu)^2 f(x)dx$

END OF FORMULA SHEET