TSSM
Creating VCE Success

THIS BOX IS FOR ILLUSTRATIVE PURPOSES ONLY

'2016 Examination Package' -
Trial Examination 4 of 5

STUDENT NUMBER

									Letter
Figures									
Words									

MATHEMATICAL METHODS

Units 3 & 4 – Written examination 1

*(*TSSM's *2014 trial exam updated for the current study design)*

Reading time: 15 minutes
Writing time: 1 hour

QUESTION & ANSWER BOOK

Structure of book

Number of questions	*Number of questions to be answered*	*Number of marks*
8	8	40
		Total 40

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners and rulers.
- Students are NOT permitted to bring into the examination room: notes of any kind, blank sheets of paper, white out liquid/tape or a calculator of any type.

Materials supplied

- Question and answer book of 9 pages.
- Working space is provided throughout the book.

Instructions

- Print your name in the space provided on the top of this page.
- All written responses must be in English.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic communication devices into the examination room.

Instructions

Answer **all** questions in the spaces provided.
A decimal approximation will not be accepted if an exact answer is required to a question.
In questions where more than one mark is available, appropriate working must be shown.
Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Question 1 (6 marks)

a. If $f(x) = 2x^2 log_e(3x)$, find $f'(x)$. 2 marks

b. Find $f'\left(\frac{1}{3}\right)$. 1 mark

c. Find the value(s) of x for which $f'(x) = 0$. 3 marks

Question 2 (6 marks)

a. Find the derivative of $(2x - 3)\sin(x)$. 2 marks

b. Hence, find an anti-derivative of $x\cos(x)$. 4 marks

TURN OVER

Question 3 (2 marks)

Solve the equation $2\cos\left(\frac{x}{3}\right) + 1 = 0, \quad x \in [3\pi, 6\pi]$

Question 4 (9 marks)

a. Find the positive value of k for which $4e^{2x} - ke^{x} + 9 = 0$ has a unique solution.

2 marks

b. Find the unique solution for the equation $4e^{2x} - ke^{x} + 9 = 0$ for the positive value of k found in part **a**.

3 marks

Question 4 – continued

Let $g(x) = 2e^x - 3$

c. Find the rule for the inverse $g^{-1}(x)$. 2 marks

d. State the domain and range of $g^{-1}(x)$. 2 marks

TURN OVER

Question 5 (4 marks)
Let $f: R \to R,\ f(x) = 4x + 9$.

The average value of f on the interval $[0, a]$ is $\frac{1}{a}$, where a is a real constant.
Find all possible values of a.

Question 6 (6 marks)

The probability distribution of a discrete random variable, *X*, is given by the table below.

x	0	1	2	3	4
$\Pr(X = x)$	$\frac{1}{2k}$	$\frac{k}{10}$	$\frac{k}{10}$	$\frac{3}{10k}$	$\frac{2}{5k}$

a. Show that $k = 3$ or $k = 2$. 3 marks

Let $k = 3$.

b. Calculate E(X). 2 marks

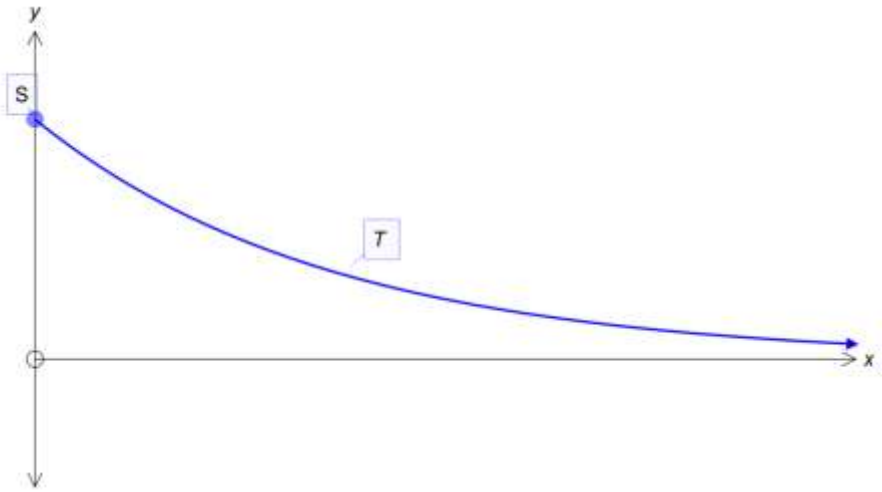
c. Find $\Pr(x \leq E(X))$. 1 mark

TURN OVER

Question 7 (4 marks)

Let $f: [0,\ \infty) \to R,\ f(x) = 2e^{-\frac{x}{3}}$

Let *S* be the point on the graph of *f* on the *y*-axis and let *T* be the point on the graph of *f* with the *y*-coordinate $\frac{1}{2}$.



Find the area of the region bounded by the graph of *f* and the line segment *ST*.

Question 8 (3 marks)
A continuous random variable, X, has a probability density function

$$f(x) = \begin{cases} \frac{1}{2}\sin(x), & 0 \leq x \leq \pi \\ 0, & elsewhere \end{cases}$$

a. Find the median of X. 2 marks

b. Find the mode of X. 1 mark

END OF QUESTION AND ANSWER BOOK