MATHEMATICAL METHODS (CAS)

Units 3 & 4 – Written examination 1

TSSM™
Creating VCE Success

2014 Trial Examination

SOLUTIONS

Question 1

a. $f(x) = 2x^2 log_e(3x)$

$f'(x) = 2x^2 \times \frac{1}{3x} \times 3 + log_e(3x) \times 4x$

$f'(x) = 2x + 4x log_e(3x)$

M1+A1
2 marks

b. $f'\left(\frac{1}{3}\right) = \frac{2}{3}$

A1
1 mark

c. $2x + 4x log_e(3x) = 0$

$2x\left(1 + 2log_e(3x)\right) = 0$

$x = 0\,,\ log_e(3x) = -\frac{1}{2}$

$x = 0\,,\ x = \frac{1}{3e^{\frac{1}{2}}}$

M1+A2
3 marks

Question 2

a. $\frac{d}{dx}\left((2x-3)\sin(x)\right) = (2x-3)\cos(x) + 2\sin(x)$

M1+A1
2 marks

b. $f'(x) = (2x-3)cos(x) + 2\sin(x)$
$f'(x) = 2xcos(x) - 3\cos(x) + 2\sin(x)$
Integrating both sides with respect to *x*
$\int f'(x)dx = \int 2xcos(x)dx - 3\int \cos(x)\,dx + 2\int \sin(x)\,dx$
$f(x) + c = 2\int xcos(x)dx - 3\sin(x) - 2\cos(x)$
$2\int xcos(x)dx = (2x-3)\sin(x) + 3\sin(x) + 2\cos(x) + c$
$\int xcos(x)dx = x\sin(x) + \cos(x) + c$

M3+A1
4 marks

Question 3

$2\cos\left(\frac{x}{3}\right) + 1 = 0, \quad \frac{x}{3} \in [\pi, 2\pi]$
$\cos\left(\frac{x}{3}\right) = -\frac{1}{2}, \quad \frac{x}{3} \in [\pi, 2\pi]$
$\frac{x}{3} = \frac{4\pi}{3}$
$x = 4\pi$

M1+A1
2 marks

Question 4

a. $Discriminant = 0$
$k^2 - 4 \times 4 \times 9 = 0$
$k^2 = 144$
$k = \pm 12$

M1+A1
2 marks

b. $4e^{2x} - 12e^x + 9 = 0$
$(2e^x - 3)^2 = 0$
$e^x = \frac{3}{2}$
$x = log_e\left(\frac{3}{2}\right)$

M2+A1
3 marks

c. $y = 2e^x - 3$
$let\ x = 2e^y - 3$
$e^y = \frac{x+3}{2}$
$y = log_e\left(\frac{x+3}{2}\right)$
$g^{-1}(x) = log_e\left(\frac{x+3}{2}\right)$

M1+A1
2 marks

d. $Domain\ of\ g^{-1}\ is\ (-3, \infty)$
$Range\ of\ g^{-1}\ is\ R$

A2
2 marks

Question 5

$\frac{1}{a}\int_0^a (4x + 9)dx = \frac{1}{a}$
$2a^2 + 9a - 1 = 0$
$a = \frac{-9 \pm \sqrt{89}}{4}$

M3+A1
4 marks

Question 6

a. $\frac{1}{2k} + \frac{k}{10} + \frac{k}{10} + \frac{3}{10k} + \frac{2}{5k} = 1$
$5 + k^2 + k^2 + 3 + 4 = 10k$
$2k^2 - 10k + 12 = 0$
$k^2 - 5k + 6 = 0$
$(k - 3)(k - 2) = 0$
$k = 3 \text{ or } k = 2$

M3
3 marks

b. $E(X) = \left(0 \times \frac{1}{6}\right) + \left(1 \times \frac{3}{10}\right) + \left(2 \times \frac{3}{10}\right) + \left(3 \times \frac{1}{10}\right) + \left(4 \times \frac{2}{15}\right)$
$E(X) = \frac{3}{10} + \frac{6}{10} + \frac{3}{10} + \frac{8}{15} = \frac{26}{15}$

M1+A1
2 marks

c. $\Pr\left(X \leq \frac{26}{15}\right) = \frac{1}{6} + \frac{3}{10} = \frac{7}{15}$

A1
1 mark

Question 7

$T \text{ has coordinates } \left(3ln4, \frac{1}{2}\right)$

$Area = \frac{1}{2} \times 3ln4 \times \left(2 + \frac{1}{2}\right) - \int_0^{3ln4} 2e^{-x/3} dx$

$Area = \frac{15}{4} ln4 + \left(6e^{-x/3}\right)_0^{3ln4}$

$Area = \frac{15}{4} ln4 + 6 \times \frac{1}{4} - 6 = \left(\frac{15}{4} ln4 - \frac{9}{2}\right) \text{ square units}$

M3+A1
4 marks

Question 8

a. $\int_0^m \frac{1}{2} \sin(x)\, dx = \frac{1}{2}$

$(-cosx)_0^m = 1$

$-\cos(m) + 1 = 1$

$\cos(m) = 0$

$m = \frac{\pi}{2}$

M1+A1
2 marks

b. $\text{Mode} = \frac{\pi}{2}$

A1
1 mark