The Mathematical Association of Victoria

Trial Exam 2014

MATHEMATICAL METHODS (CAS)

WRITTEN EXAMINATION 1

STUDENT NAME ____

Reading time: 15 minutes
Writing time: 1 hour

QUESTION AND ANSWER BOOK

Structure of book

Number of questions	*Number of questions to be answered*	*Number of marks*
10	10	40

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers.
- Students are NOT permitted to bring into the examination room: notes of any kind, blank sheets of paper, white out liquid/tape or a calculator of any type.

Materials supplied
- Question and answer book of 9 pages, a detachable sheet of miscellaneous formulas in the centrefold.
- Working space is provided throughout the book.

Instructions
- Detach the formula sheet from the centre of this book during reading time.
- Write your **name** in the space provided above on this page.
- All written responses must be in English.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

Question 1 (3 marks)
For what value(s) of m is/are there no real solutions to the following system of equations?

$$mx + 2y = 6$$
$$x + (m-1)y = -3$$

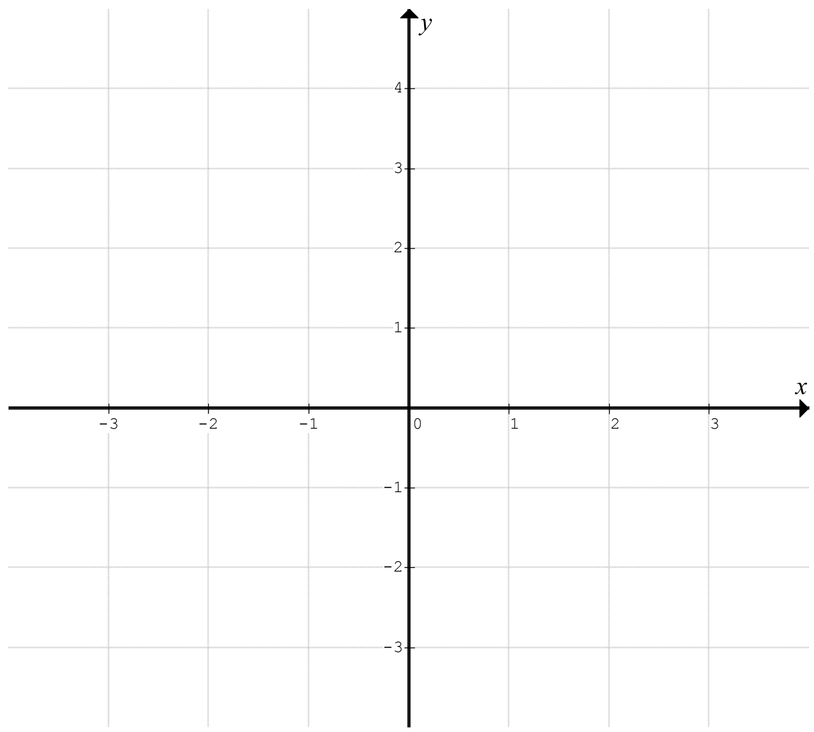
Question 2 (3 marks)
Solve the equation $2\sin^2(2x) - 1 = 0$ for x where $x \in [0, \pi]$.

Question 3 (3 marks)
Solve $2\log_2(2x+1)+\log_2(3)=3$ for *x*.

Question 4 (4 marks)

a. Sketch the graphs of $f:[-2,2)\to R, f(x)=|x-1|$ and $g:(-2,2]\to R, g(x)=|x+1|$ on the set of axes below. 2 marks

b. Hence sketch $h(x)=f(x)+g(x)$ over its maximal domain on the same set of axes. 2 marks



Question 5 (5 marks)

Consider the function $f(x) = kx^2 \tan(2x)$, where *k* is a positive real constant.

a. Find $f'(x)$. 2 marks

b. Show that $f'\left(\frac{\pi}{8}\right) = \frac{k\pi(4+\pi)}{16}$. 1 mark

c. If $f'\left(\frac{\pi}{8}\right) = 0.75$, find the value of *k*. 2 marks

Question 6 (4 marks)

Let $f:(-\infty, A] \rightarrow R, f(x)=x^2+\frac{2}{3}x+3$.

a. Find the maximum value of A if f^{-1} exists. 1 mark

b. Define f^{-1}. 3 marks

Question 7 (4 marks)

a. Show that the equation of the **normal** to the curve with equation $f(x) = e^{3x+2}$ at $x = 0$ is $y = -\frac{1}{3e^2}x + e^2$. 1 mark

b. Hence, find the area bounded by *f*, the normal line and the line $x = -2$. 3 marks

Question 8 (4 marks)

For events A and B, $\Pr(A) = \frac{1}{4}$ and $\Pr\left(A \cap B\right) = \frac{1}{5}$.

a. If $\Pr(A' \cap B) = \Pr(A)$ evaluate $\Pr(B)$. 2 marks

Working Space

b. If events A and B are independent evaluate $\Pr(A \cup B)$. 2 marks

Working Space

Question 9 (6 marks)

The probability density function for the random variable, *X*, is given by

$$f(x) = \begin{cases} \dfrac{k}{2x} & 1 \le x \le e^3 \\ 0 & \text{elsewhere} \end{cases}.$$

a. Show that $k = \dfrac{2}{3}$. 2 marks

b. Find $\Pr\left(X < 10 \mid X \ge 1\right)$. 2 marks

c. Find $E(X)$ the mean of *X*. 2 marks

Question 10 (4 marks)

a. Find $\frac{d}{dx}\left(\frac{5}{2x^2-1}\right)$. 1 mark

b. **Hence** find the value of a if $\int_1^a \left(\frac{20x}{\left(2x^2-1\right)^2}+1\right)dx=\frac{37}{7}$. 3 marks

END OF QUESTION AND ANSWER BOOKLET