

insightTM
YEAR 12 *Trial Exam Paper*

2014

MATHEMATICAL METHODS (CAS)

Written examination 2

Worked solutions

This book presents:

- worked solutions, giving you a series of points to show you how to work through the questions
- mark allocations
- tips on how to approach the questions

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SECTION 1

Question 1

Answer is D

Worked solution

The range is given by $[5 - 2, 5 + 2] = [3, 7]$.

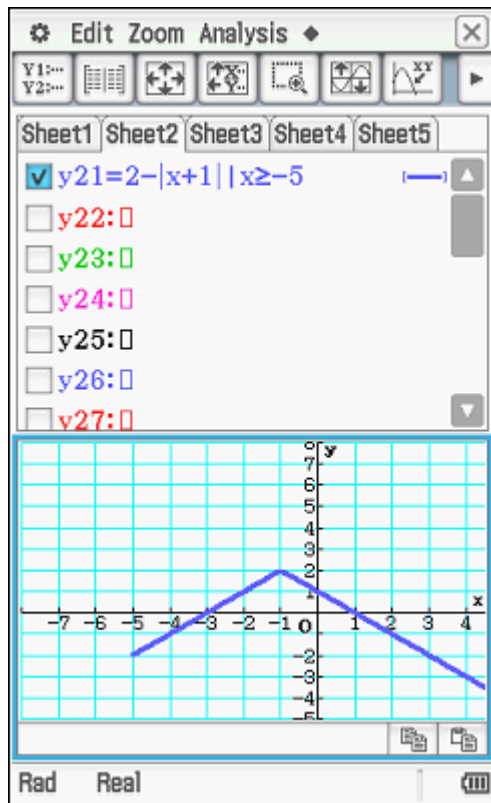
The period is given by $\frac{2\pi}{4\pi} = \frac{1}{2}$.

Question 2

Answer is D.

Worked solution

Using CAS, the range is $(-\infty, 2]$.



Tip

- *To find the range always draw the graph.*

Question 3*Answer is E.***Worked solution**

For $f(x)$ to be defined, $x > k$, so the domain is (k, ∞) .

And $\sqrt{x-k} > 0$, so the range is R^+ .

**Tip**

- *If it helps, an arbitrary value of k can be chosen and then a graph produced.*

Question 4*Answer is D.***Worked solution**

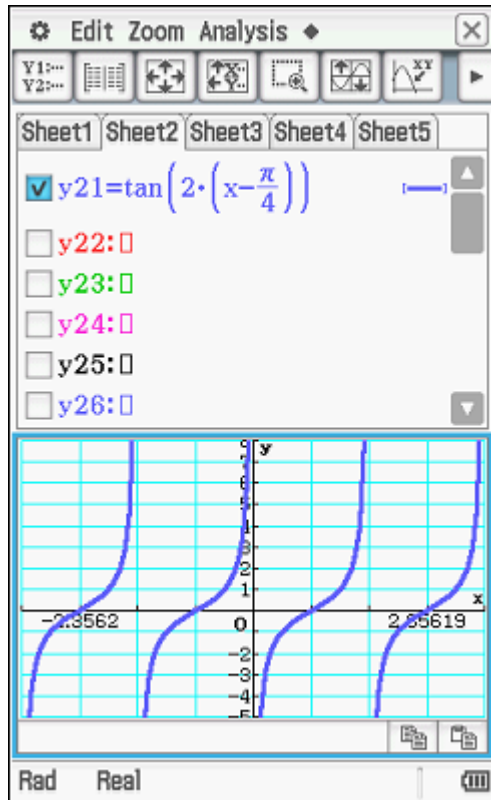
Using the chain rule for $y = g(f(x))$, $\frac{dy}{dx} = g'(f(x)) \times f'(x)$.

Now $f(x) = \sin(5x)$ and $f'(x) = 5 \cos(5x)$, so $\frac{d}{dx}[g(\sin(5x))] = g'(\sin(5x)) \times 5 \cos(5x)$.

Question 5*Answer is B.***Worked solution**

The graph is a tan graph with a period of $\frac{\pi}{2}$ that has been shifted to the left $\frac{\pi}{4}$ units.

Checking using CAS gives



Question 6*Answer is A.***Worked solution**

$$f(3) = 27 - 27 = 0$$

$$f(1) = 1 - 3 = -2$$

$$\text{Average rate of change} = \frac{f(3) - f(1)}{3 - 1} = \frac{0 - (-2)}{2} = 1$$

$$\text{Average value of the function is } \frac{1}{3-1} \int_1^3 x^3 - 3x^2 dx = -3 \text{ (found using CAS).}$$

The screenshot shows a CAS calculator interface with the following elements:

- Header:** Edit Action Interactive
- Toolbar:** Includes buttons for fraction conversion (0.5 1/2), derivative (d/dx), integral (∫), simplify (Simp), and other mathematical functions.
- Input Area:** The expression $\frac{1}{2} \int_1^3 x^3 - 3x^2 dx$ is entered. The result -3 is displayed on the right side of the input area.
- Function Palette:** A grid of mathematical symbols and functions categorized by Math1, Math2, Math3, Trig, Var, and abc.
- Bottom Bar:** Shows the current mode (Alg, Standard, Real, Rad) and a calculator icon.

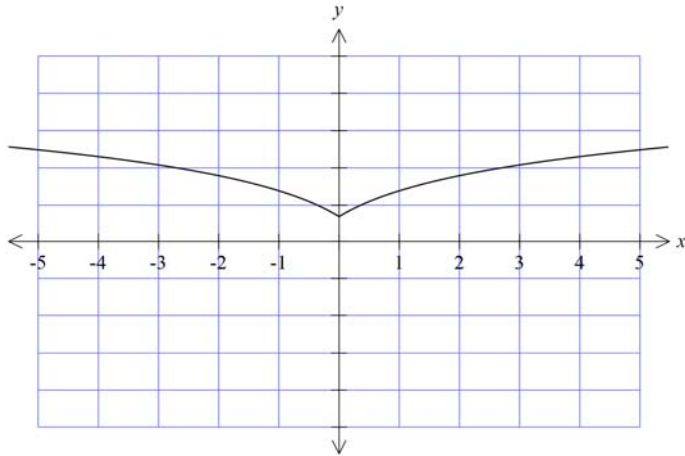
**Tip**

- *Be careful! Option E is the answer to the **average value** of the function—this is easily and readily confused.*

Question 7*Answer is A.***Worked solution**

For an inverse function to exist, the function $f(x)$ must be one-to-one; i.e. for each x value there is exactly one y value.

The graph of $f(x)$ is



To be one-to-one $x \in [0, \infty)$.

**Tip**

- *For the graph to be one-to-one it must pass the horizontal line test.*

Question 8

Answer is D.

Worked solution

Expanding the matrix gives

The screenshot shows a TI-84 Plus calculator interface. The main display area shows the matrix multiplication:

$$\begin{pmatrix} 0 & 1 & 1 \\ -1 & 0 & 1 \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

The result of the multiplication is shown as a column vector:

$$\begin{bmatrix} y+z \\ -x+z \\ z \end{bmatrix}$$

The calculator interface includes a toolbar with various mathematical functions and a keypad with categories like Math1, Math2, Math3, Trig, Var, and abc.



Tip

- *Ensure the equations are first re-written in the form*

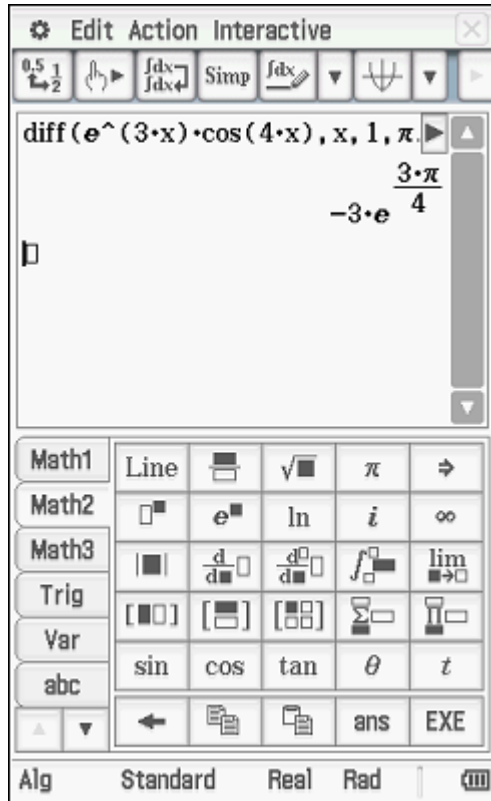
$$0x + 0y + z = 1$$

$$-1x + 0y + z = 2$$

$$0x + y + z = 5$$

Question 9*Answer is C.***Worked solution**

Using CAS gives

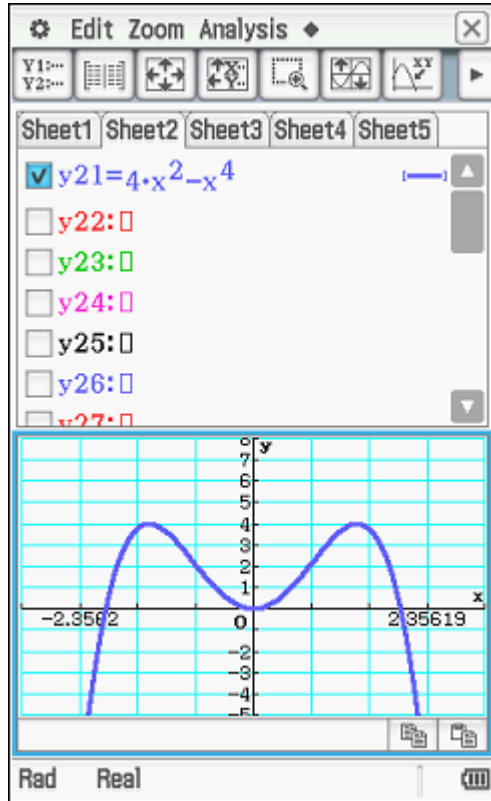


The screenshot shows a CAS interface with the following elements:

- Header:** Edit Action Interactive
- Toolbar:** Includes buttons for fraction conversion (0.5 to 1/2), differentiation (d/dx), simplification (Simp), and other mathematical functions.
- Input Field:** Contains the expression $\text{diff}(e^{(3 \cdot x)} \cdot \cos(4 \cdot x), x, 1, \pi)$.
- Output Field:** Displays the result $-3 \cdot e^{\frac{3 \cdot \pi}{4}}$.
- Mathematical Palette:** A grid of buttons for mathematical symbols and functions, categorized by Math1, Math2, Math3, Trig, Var, and abc.
- Bottom Bar:** Shows the current mode (Alg) and settings (Standard, Real, Rad).

Question 10*Answer is E.***Worked solution**

A suitable way to do this question is to choose an arbitrary value for a ; e.g. $a = 4$.
A sample graph would be



Owing to the condition that $u < t$, the gradient is positive for $x \in (-\infty, u) \cup (0, t)$.

Question 11*Answer is A.***Worked solution**

For this approximation $h = 0.2$, $x = 9$ and $f'(x) = \frac{3}{2}\sqrt{x}$.

So $f(9+h) \approx f(9) + hf'(9)$

$$\begin{aligned} &= \left(9^{\frac{1}{2}}\right)^3 + 0.2 \times \frac{3}{2}\sqrt{9} \\ &= 3^3 + 0.2 \times \frac{9}{2} = 27.9 \end{aligned}$$

Question 12*Answer is A.***Worked solution**

Rearranging the equation $y = 1 - 2\sin(3x + \pi)$ gives $\frac{y-1}{-2} = \sin\left(3\left(x + \frac{\pi}{3}\right)\right)$.

So $y = \frac{y'-1}{-2}$ and $x = 3\left(x' + \frac{\pi}{3}\right)$.

Therefore, $y' = -2y + 1$ and $x' = \frac{x}{3} - \frac{\pi}{3}$.

The expansion of the matrix in option A; i.e. $T \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \frac{1}{3} & 0 \\ 0 & -2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} -\frac{\pi}{3} \\ 1 \end{bmatrix}$, gives

$x' = \frac{x}{3} - \frac{\pi}{3}$ and $y' = -2y + 1$, as required.

Question 13

Answer is E.

Worked solution

$$X \sim \text{Bi}$$

$$E(X) = np = 20$$

$$\text{var}(X) = npq = 12 = 20q$$

$$\text{So } q = \frac{12}{20} = 0.6 \Rightarrow p = 0.4$$

$$\therefore n = \frac{20}{0.4} = 50$$

Question 14

Answer is A.

Worked solution

$$\sum \Pr(X = x) = 1 \Rightarrow 10a = 1, \therefore a = 0.1$$

$$E(X) = \sum x \Pr(X = x)$$

$$= 1 \times 0.2 + 2 \times 0.3 + 3 \times 0.1$$

$$= 1.1$$

Question 15*Answer is D.***Worked solution**

The transformation represents a reflection in the y-axis and a translation of 2 units up.
 Operating this transformation on the tangent line gives

$$y = 2(-x) - 5 + 2$$

$$y = -2x - 3$$

Question 16*Answer is B.***Worked solution**

Using the given information

$$\int_{-4}^5 f(x) dx = 4$$

$$\Rightarrow F(5) - F(-4) = 4 \quad (1)$$

$$\begin{aligned} \text{So } \int_{-1}^2 (f(3x-1)) dx &= \frac{1}{3} (F(3(2)-1) - F(3(-1)-1)) \\ &= \frac{1}{3} (F(5) - F(-4)) \\ &= \frac{1}{3} (4) \quad (\text{from equation 1}) \\ &= \frac{4}{3} \end{aligned}$$

Question 17*Answer is C.***Worked solution**

$$\Pr(G' | G) = 0.65 \text{ and } \Pr(G' | B) = 0.3$$

So the matrix is set up as $\begin{bmatrix} \Pr(G' | G) & \Pr(G' | B) \\ \Pr(B' | G) & \Pr(B' | B) \end{bmatrix} = \begin{bmatrix} 0.65 & 0.3 \\ 0.35 & 0.7 \end{bmatrix}$.

Question 18*Answer is C.***Worked solution**

$$\begin{aligned} [f(u)]^2 - 2 &= (e^x + e^{-x})^2 - 2 \\ &= e^{2x} + 2 + e^{-2x} - 2 \\ &= e^{2x} + e^{-2x} \\ &= f(2u) \end{aligned}$$

Question 19*Answer is E.***Worked solution**

$$\Pr(X > 71) = \Pr\left(Z > \frac{71-50}{7}\right) = \Pr(Z > 3)$$

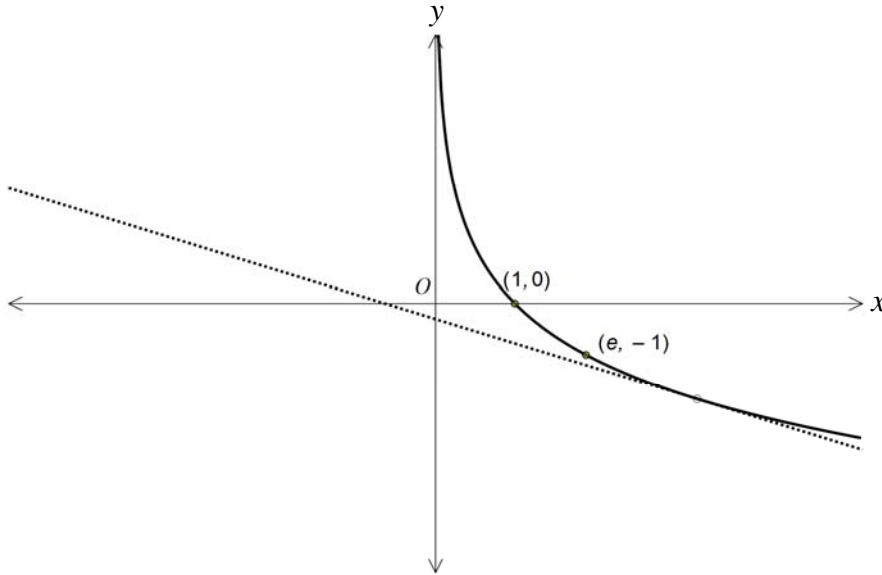
Using symmetry, $\Pr(Z > 3) = \Pr(Z < -3)$.

Question 21

Answer is E.

Worked solution

The graph of $y = -\log_e(x)$ is shown below.



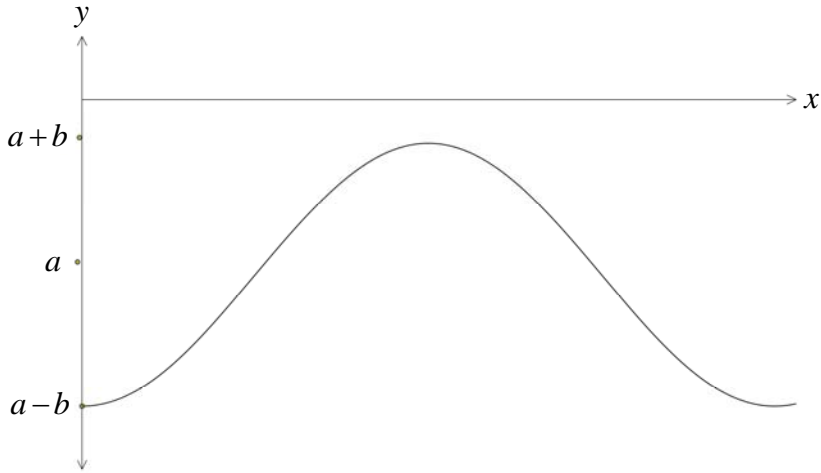
By choosing a value greater than e the x -intercept of the tangent line to the graph is negative, which means that option E is false.

Question 22

Answer is E.

Worked solution

A graph of the general curve looks like



So for $f(x) < 0$, then $a + b < 0$; i.e. $a < -b$.

**Tip**

- *If it helps, choose arbitrary values for a and b to produce a sketch.*

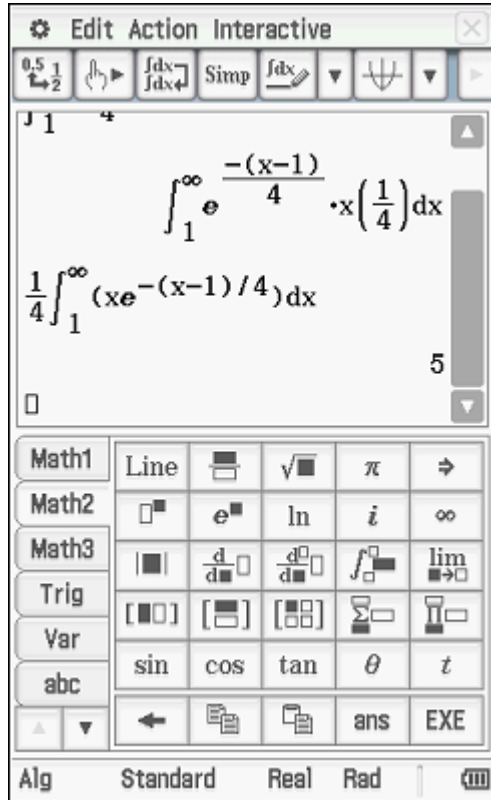
SECTION 2

Question 1a.

Worked solution

$$\text{Mean} = \int_1^{\infty} \left(x \times \frac{1}{4} e^{-\frac{(x-1)}{4}} \right) dx$$

Using CAS, we get



This gives the mean of 5 years.

Mark allocation: 2 marks

- 1 mark for writing: $\text{Mean} = \int_1^{\infty} \left(x \times \frac{1}{4} e^{-\frac{(x-1)}{4}} \right) dx.$
- 1 mark for answer of 5 years.

Question 1b.**Worked solution**

Median = m , such that $\int_1^m \frac{1}{4} e^{\frac{-(x-1)}{4}} dx = 0.5$.

Using CAS to solve for m gives $m = 3.77$ years.

The screenshot shows a CAS interface with the following content:

0.5 1
2 4

$\int_1^m \frac{1}{4} e^{\frac{-(x-1)}{4}} dx = 0.5$

Solve

$\left\{ m = 4 \cdot \ln(2) + 1 \right\}$

$\int_1^m \frac{1}{4} e^{\frac{-(x-1)}{4}} dx = 0.5$

$\left\{ m = 3.772588722 \right\}$

Math1: Line, $\frac{\square}{\square}$, $\sqrt{\square}$, π , \Rightarrow

Math2: \square^\square , e^\square , \ln , i , ∞

Math3: \square^\square , $\frac{d}{d\square}$, $\frac{d}{d\square}$, $\int \square$, $\lim_{\square \rightarrow \square}$

Trig: $[\square]$, $[\square]$, $[\square]$, $\Sigma \square$, $\int \square$

Var: \sin , \cos , \tan , θ , t

abc: \leftarrow , \square , \square , ans , EXE

Alg | Decimal | Real | Rad | \square

Mark allocation: 2 marks

- 1 mark for writing $\int_1^m \frac{1}{4} e^{\frac{-(x-1)}{4}} dx = 0.5$.
- 1 mark for answer of $m = 3.77$ years.

Question 1c.**Worked solution**

$$\begin{aligned} \text{var}(X) &= E(X^2) - (E(X))^2 \\ &= \int_1^{\infty} x^2 f(x) dx - \left(\int_1^{\infty} x f(x) dx \right)^2 \\ &= \int_1^{\infty} x^2 \frac{1}{4} e^{-\frac{(x-1)}{4}} dx - \left(\int_1^{\infty} x \frac{1}{4} e^{-\frac{(x-1)}{4}} dx \right)^2 \end{aligned}$$

Using CAS, this gives $\text{var}(X) = 16$ years.

The screenshot shows a CAS interface with the following content:

- Toolbar: $\frac{0.5}{2}$, \int , $\frac{dx}{dx}$, $\frac{dx}{dx^4}$, Simp , $\frac{dx}{dx}$, $\frac{dx}{dx}$, $\frac{dx}{dx}$, $\frac{dx}{dx}$
- Input: $4 \int_1^{\infty} (x^2 e^{-\frac{(x-1)}{4}}) dx$
- Output: 5
- Equation: $\text{solve}\left(\frac{1}{4} \cdot \int_1^m e^{-\frac{(x-1)}{4}} dx = 0.5, \right)$
- Result: $\{m = 4 \cdot \ln(2) + 1\}$
- Equation: $\text{solve}\left(\frac{1}{4} \cdot \int_1^m e^{-\frac{(x-1)}{4}} dx = 0.5, \right)$
- Result: $\{m = 3.772588722\}$
- Equation: $\frac{1}{4} \int_1^{\infty} (x^2 e^{-\frac{(x-1)}{4}}) dx$
- Output: 41
- Equation: $41 - 25$
- Output: 16
- Bottom bar: Alg, Decimal, Real, Rad, $\frac{dx}{dx}$

Mark allocation: 2 marks

- 1 mark for stating $\text{var}(X) = \int_1^{\infty} x^2 \frac{1}{4} e^{-\frac{(x-1)}{4}} dx - \left(\int_1^{\infty} x \frac{1}{4} e^{-\frac{(x-1)}{4}} dx \right)^2$.
- 1 mark for answer $\text{var}(X) = 16$ years.

Question 1d.**Worked solution**

$$\begin{aligned}\Pr(X > 5) &= \int_5^{\infty} \frac{1}{4} e^{-\frac{(x-1)}{4}} dx \\ &= \frac{1}{e} \quad (\text{Found using CAS.})\end{aligned}$$

Using CAS gives

The screenshot shows a CAS calculator window titled "Edit Action Interactive". The input is $\text{solve}\left(\frac{1}{4} \int_1^m e^{-x/4} dx = 0.5, m\right)$. The output shows the value $\{m=3.772588722\}$. Below this, the calculator shows the integral $\frac{1}{4} \int_1^{\infty} (x^2 e^{-(x-1)/4}) dx$ with a result of 41. Then it shows $41 - 25$ with a result of 16. Next, it shows $\frac{1}{4} \int_5^{\infty} (e^{-(x-1)/4}) dx$ with a result of 0.3678794412. Finally, it shows $\frac{1}{4} \int_5^{\infty} (e^{-(x-1)/4}) dx$ with a result of e^{-1} . The calculator is in "Standard" mode.

Mark allocation: 2 marks

- 1 mark for stating $\Pr(X > 5) = \int_5^{\infty} \frac{1}{4} e^{-\frac{(x-1)}{4}} dx$.
- 1 mark for answer $\frac{1}{e}$.

**Tip**

- *The answer requires an exact answer, so be sure to have your CAS calculator in exact/standard mode. A decimal answer, no matter how accurate, will not be accepted in this instance.*

Question 1e.**Worked solution**

$$Y \sim \text{Bi}\left(n = 4, p = \Pr(X > 5) = \frac{1}{e}\right)$$

$$E(Y) = n \times p = 4 \times \frac{1}{e} = \frac{4}{e}$$

Mark allocation: 2 marks

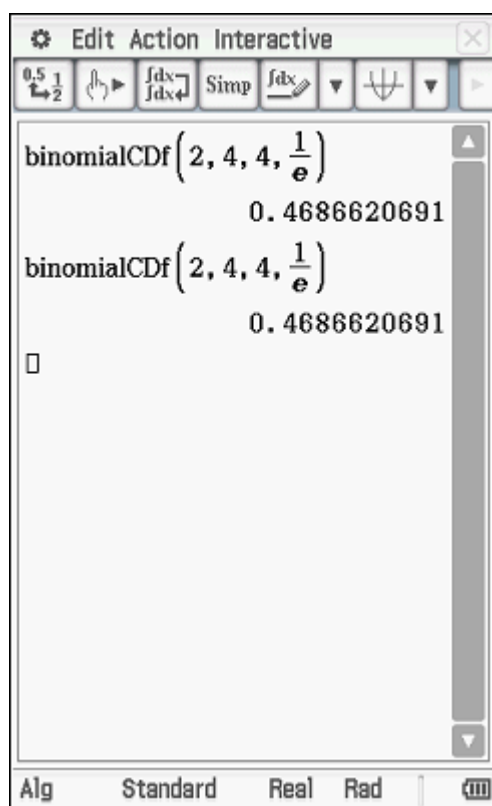
- 1 mark for identifying the binomial and the parameters.
- 1 mark for answer $E(Y) = \frac{4}{e}$.

**Tip**

- *Although not explicitly asked for, this question also requires an exact answer.*

Question 1f.**Worked solution**

To be operational, the television requires at least two switches to be working; i.e. $\Pr(Y \geq 2)$.
Using CAS, this gives $\Pr(Y \geq 2) = 0.4687$.

**Mark allocation: 2 marks**

- 1 mark for stating $\Pr(Y \geq 2)$.
- 1 mark for answer 0.4687.

Question 1g.**Worked solution**

$$N \sim (n = 12, p = 0.468662)$$

$$\Pr(N = 7) = 0.1666 \quad (\text{Found using CAS.})$$

Using CAS gives

The screenshot shows a CAS interface with the following content:

$\text{binomialCDF}\left(2, 4, 4, \frac{1}{e}\right)$	0.4686620691
$\text{binomialCDF}\left(2, 4, 4, \frac{1}{e}\right)$	0.4686620691
$\text{binomialPDF}(7, 12, 0.468662)$	0.1665703721

At the bottom of the interface, there are tabs for 'Alg', 'Standard', 'Real', and 'Rad', along with a calculator icon.

Mark allocation: 2 marks

- 1 mark for recognising the binomial with $n = 12$ and $p = 0.4688$.
- 1 mark for answer 0.1666.

**Tip**

- *To answer correct to 4 decimal places requires that your calculations be carried out to at least 5 decimal places.*

Question 2a.**Worked solution**

Let $4a\sqrt{x} - x = 0$.

So $4a\sqrt{x} = x$

$$16a^2x = x^2$$

$$16a^2x - x^2 = 0$$

$$x(16a^2 - x) = 0$$

$$x = 0 \text{ or } x = 16a^2$$

Since $c \neq 0$, $c = 16a^2$.

Mark allocation: 2 marks

- 1 mark for setting $4a\sqrt{x} - x = 0$.
- 1 mark for $c = 16a^2$.

Question 2b.**Worked solution**

f_a is strictly decreasing for $f'_a \leq 0$.

Using CAS, we get $f'(x) = 0$ for $x = 4a^2$.

So, it is strictly decreasing for $x \in [4a^2, \infty)$.

The screenshot shows a CAS interface with the following content:

- Input: $\frac{d}{dx} (4 \cdot a \cdot \sqrt{x} - x)$
- Result: $\frac{2 \cdot a - \sqrt{x}}{\sqrt{x}}$
- Input: $\text{solve}\left(\frac{2 \cdot a - \sqrt{x}}{\sqrt{x}} = 0, x\right)$
- Result: $\{x = 4 \cdot a^2\}$

The interface includes a toolbar with icons for differentiation, simplification, and solving, and a keypad with mathematical symbols and functions.

Mark allocation: 2 marks

- 1 mark for finding $x = 4a^2$.
- 1 mark for $x \in [4a^2, \infty)$.

**Tip**

- For strictly increasing/decreasing conditions, turning points must be included.

Question 2c.**Worked solution**

At $x = c$, $x = 16a^2$ and $y = 0$ (as determined from part a).

Using CAS, we get $f'(x) = \frac{2a - \sqrt{x}}{\sqrt{x}}$.

$$\begin{aligned} \text{At } x = 16a^2, f'(16a^2) &= \frac{-(4|a| - 2a)}{4|a|} \\ &= -\frac{1}{2}, \text{ since } a > 0. \end{aligned}$$

$$\text{So, } m = -\frac{1}{2}.$$

The image shows two screenshots of a CAS interface. The left screenshot shows the equation $\text{solve}\left(\frac{2 \cdot a - \sqrt{x}}{\sqrt{x}} = 0, x\right)$ being solved, resulting in $\{x = 4 \cdot a^2\}$. Below this, the expression $\frac{2 \cdot a - \sqrt{x}}{\sqrt{x}} |_{x=16a^2}$ is evaluated to $\frac{-(4 \cdot |a| - 2 \cdot a)}{4 \cdot |a|}$. The right screenshot shows the same expression evaluated at $x = 16a^2$, resulting in $\frac{-(4 \cdot |a| - 2 \cdot a)}{4 \cdot |a|}$, which simplifies to $-(4a - 2a) / (4a)$ and finally to $-\frac{1}{2}$. Both screenshots include a toolbar with various mathematical symbols and a keypad with buttons for Math1, Math2, Math3, Trig, Var, abc, and a numeric keypad.

So equation of the tangent line is

$$y - y_1 = m(x - x_1)$$

$$y - 0 = -\frac{1}{2}(x - 16a^2)$$

$$y = -\frac{1}{2}(x - 16a^2)$$

Mark allocation: 3 marks

- 1 mark for finding $m = -\frac{1}{2}$.
- 1 mark for finding y-intercept.
- 1 mark for finding tangent line equation.

**Tip**

- *The question specifically stated ‘show that ...’, therefore your working must show step-by-step how you obtained your answer.*

Question 2d.**Worked solution**

$$\begin{aligned}y &= -\frac{1}{2}(x - 16a^2) \\ &= -\frac{1}{2}x + 8a^2\end{aligned}$$

The tangent line drawn to $f_a(x)$ at $x = c$ passes through the point $(0, 8a^2)$.

So in order for the tangent drawn to $g_a(x)$ to pass through the origin, the graph tangent line must be translated down by $8a^2$, so $b = 8a^2$.

Mark allocation: 2 marks

- 1 mark for finding y -intercept $(0, 8a^2)$.
- 1 mark for finding $b = 8a^2$.

Question 2e.i.**Worked solution**

The area under the curve is given by $\int_0^{16a^2} 4a\sqrt{x} - x \, dx$.

Evaluating this using CAS gives the area is equal to $\frac{128a^4}{3}$ square units, since $a > 0$.

The image shows two screenshots of a CAS interface. The left screenshot shows the integral $\int_0^{16a^2} 4a\sqrt{x} - x \, dx$ being evaluated, with intermediate steps showing $-128a^4 + \frac{512a^3 \cdot a}{3}$ and $\frac{128a^4}{3}$. The right screenshot shows the same integral being evaluated, with intermediate steps showing $-128a^4 + \frac{512a^3 \cdot |a|}{3}$ and $\frac{128a^4}{3}$. Both screenshots show a toolbar with various mathematical functions and a keypad with mathematical symbols and variables.

Mark allocation: 2 marks

- 1 mark for setting up the integral.
- 1 mark for the answer $\frac{128a^4}{3}$ square units.

**Tip**

- *Make use of the dilation factor and understand what effect this has on the graph and the resulting area.*

Question 2e.ii.**Worked solution**

$f\left(\frac{x}{2}\right)$ represents a dilation of factor of 2 parallel to the x -axis. This means the area under the curve is doubled, so the area equals $\frac{256a^4}{3}$ square units.

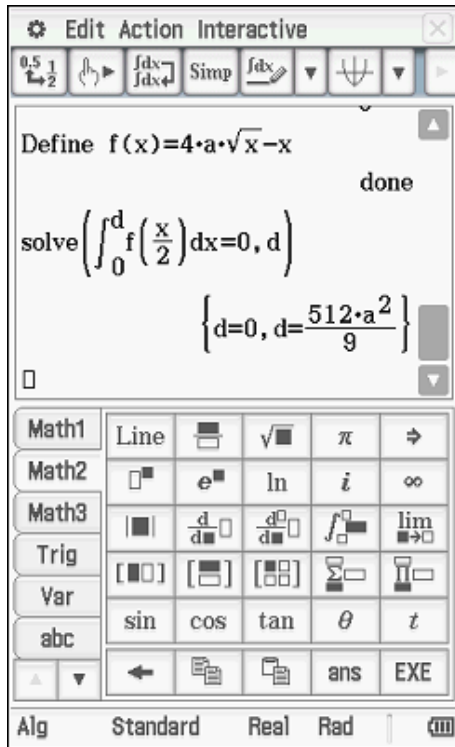
Mark allocation: 2 marks

- 1 mark for identifying the transformation correctly.
- 1 mark for the answer $\frac{256a^4}{3}$ square units.

Question 2f.**Worked solution**

Find d such that $\frac{1}{d} \int_0^d h_a(x) dx = 0$; i.e. $\int_0^d h_a(x) dx = 0$, since $d \neq 0$.

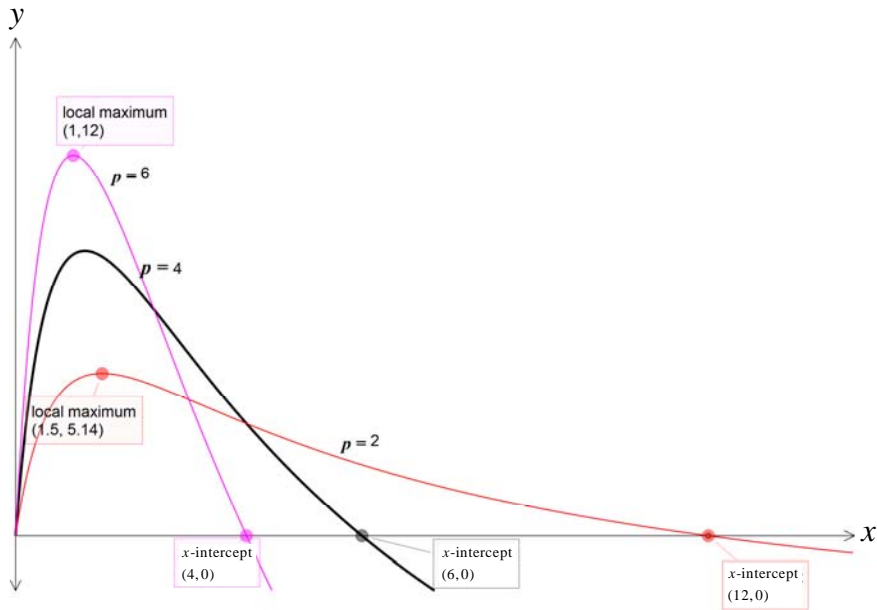
Using CAS to solve, gives



So $d = \frac{512a^2}{9}$.

Mark allocation: 2 marks

- 1 mark for setting up $\frac{1}{d} \int_0^d h_a(x) dx = 0$.
- 1 mark for the answer $d = \frac{512a^2}{9}$.

Question 3a.**Worked solution****Mark allocation: 2 marks**

- 1 mark for correct shapes and labelling x -intercept correctly.
- 1 mark for labelling turning points correctly.

Question 3b.**Worked solution**

Firework is extinguished when $F = 0$.

When $F = 0$, then

$$\Rightarrow -pt(pt - 24) = 0$$

$$\Rightarrow t = 0 \text{ or } pt = 24$$

$$\text{So } t = \frac{24}{p}.$$

Mark allocation: 2 marks

- 1 mark for setting $F = 0$.
- 1 mark for the answer $t = \frac{24}{p}$.

Question 3c.i.**Worked solution**

Maximum brightness is achieved when $F'(x) = 0$.

Using CAS gives

The screenshot shows a CAS interface with the following content:

Define $f(x) = \frac{-px(px-24)}{(x+2)^2}$

done

$\frac{d}{dx}(f(x))$

$$\frac{-(4 \cdot p^2 \cdot x + 24 \cdot p \cdot x - 48 \cdot p)}{(x+2)^3}$$

Below the main display is a toolbar with various mathematical functions and symbols, including Math1, Math2, Math3, Trig, Var, abc, and a bottom row with Alg, Standard, Real, Rad, and a calculator icon.

So the maximum brightness will occur when $\frac{-(4p^2t + 24pt - 48p)}{(t+2)^3} = 0$.

Mark allocation: 2 marks

- 1 mark for finding the derivative.
- 1 mark for setting it equal to zero.

Question 3c.ii.

The image shows two screenshots of a TI-84 Plus calculator interface. The left screenshot shows the derivative of $f(x) = \frac{-(4 \cdot p^2 \cdot x + 24 \cdot p \cdot x - 48 \cdot p)}{(x+2)^3}$ with respect to x . The user has entered the derivative and then used the 'solve' function to find the value of x that makes the derivative zero. The result is $x = \frac{48 \cdot p}{4 \cdot p^2 + 24 \cdot p}$. The right screenshot shows the user simplifying this expression. The simplified result is $\frac{12}{p+6}$.

So the maximum occurs when $t = \frac{12}{p+6}$.

Mark allocation: 2 marks

- 1 mark for finding t in terms of p .
- 1 mark for simplifying to get $t = \frac{12}{p+6}$.

Question 3d.**Worked solution**

Maximum brightness is at $x = \frac{12}{p+6}$ and $F\left(\frac{12}{p+6}\right) = \frac{36p}{p+12}$, which was found using CAS.

The screenshot shows a CAS interface with the following content:

Input: $(p+6) \cdot \left(\frac{12}{p+6} + 2\right)^2$

Operation: `simplify`

Intermediate expression: $\frac{-12 \cdot p \cdot \left(\frac{12 \cdot p}{p+6} - 24\right)}{(p+6) \cdot \left(\frac{12}{p+6} + 2\right)^2}$

Output: $\frac{36 \cdot p}{p+12}$

The interface includes a toolbar with icons for differentiation, simplification, and other mathematical operations, and a keypad with various mathematical symbols and functions.

Mark allocation: 2 marks

- 1 mark for finding $F\left(\frac{12}{p+6}\right)$.
- 1 mark for the answer $\frac{36p}{p+12}$.

Question 3e.**Worked solution**

For $F \leq 11$, equation becomes $\frac{36p}{p+12} \leq 11$.

Using CAS to find this value of p gives

The screenshot shows a CAS interface with the following elements:

- Top bar: Edit Action Interactive
- Toolbar: $\frac{0.5}{2}$, $\frac{1}{2}$, $\frac{dx}{dx}$, $\frac{dx^4}{dx^4}$, $\frac{dx}{dx}$, $\frac{dx}{dx}$, $\frac{dx}{dx}$, $\frac{dx}{dx}$
- Input area: $(p+6) \cdot \left(\frac{12}{p+6} + 2\right)^4$
- Output area: $\frac{36 \cdot p}{p+12}$
- Input area: $\text{solve}\left(\frac{36 \cdot p}{p+12} \leq 11, p\right)$
- Output area: $\left\{-12 < p \leq \frac{132}{25}\right\}$
- Bottom panel: Math1 (Line, $\frac{1}{x}$, $\sqrt{\quad}$, π , \rightarrow), Math2 (Define, f, g, i, ∞), Math3 (solve(, dSlv, ', $\left\{\frac{1}{x}, \frac{1}{y}\right\}$, |), Trig (<, >, (, {, [,]), Var (\leq , \geq , =, \neq , \angle), abc (\leftarrow , \rightarrow , $\frac{1}{x}$, ans, EXE), Alg (Standard, Real, Rad, $\frac{1}{x}$)

So the value of K is $\frac{132}{25} = 5.28$.

Mark allocation: 2 marks

- 1 mark for setting $\frac{36p}{p+12} \leq 11$.
- 1 mark for the answer $\frac{132}{25}$ or 5.28.

Question 3f.**Worked solution**

Maximum brightness occurs at $t = \frac{12}{p+6}$.

We want $t \leq 1.5$, so equation becomes $\frac{12}{p+6} \leq 1.5$.

The screenshot shows a TI-84 Plus calculator interface. The main display area contains the following text:

$$\text{solve}\left(\frac{36 \cdot p}{p+12} \leq 11, p\right)$$

$$\{-12 < p \leq \frac{132}{25}\}$$

$$\text{solve}\left(\frac{12}{p+6} \leq 1.5, p\right)$$

$$\{p < -6, 2 \leq p\}$$

Below the display is a keypad with various mathematical functions and symbols. The keypad includes rows for Math1, Math2, Math3, Trig, Var, abc, and a bottom row with Alg, Standard, Real, Rad, and a calculator icon.

So the values for p that satisfy the conditions are $2 \leq p \leq 5.28$.

Mark allocation: 2 marks

- 1 mark for finding $p \geq 2$.
- 1 mark for $2 \leq p \leq 5.28$.

Question 4a.**Worked solution**

The x -intercepts of the graph are at $(0, 0)$ and $(2, 0)$. These are found when $y = 0$.

Setting $y = 0$ gives

$$\frac{\pi x}{4} \cos(nx) = 0$$

$$\Rightarrow \frac{\pi x}{4} = 0 \text{ or } \cos(nx) = 0$$

$$\Rightarrow x = 0 \text{ or } nx = \frac{\pi}{2}$$

$$\text{So when } x = 2 \Rightarrow 2n = \frac{\pi}{2}.$$

$$\therefore n = \frac{\pi}{4}$$

Mark allocation: 2 marks

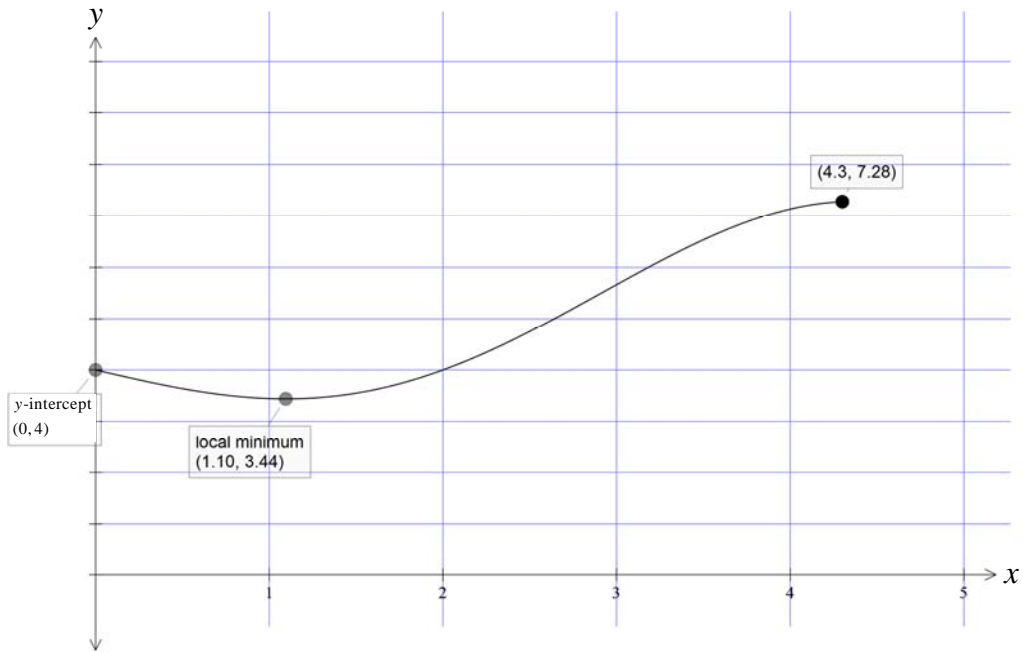
- 1 mark for setting $\frac{\pi x}{4} \cos(nx) = 0$.
- 1 mark for correctly simplifying to give $n = \frac{\pi}{4}$.

Question 4b.**Worked solution**

Using CAS, the second x -intercept occurs at $(6, 0)$; so $b = 6$.

Mark allocation: 1 mark

- 1 mark for the correct answer.

Question 4c.i.**Worked solution****Mark allocation: 3 marks**

- 1 mark for shape passing through (0, 4) and (2, 4).
- 1 mark for labelling endpoints correctly.
- 1 mark for labelling turning point correctly.

Question 4c.ii.**Worked solution**

Using CAS, at $x=1$, $y = \frac{-\sqrt{2}\pi}{8} + 4$.

The screenshot shows a CAS interface with the following content:

- Top bar: Edit Action Interactive
- Toolbar: $\frac{0.5}{2}$, $\frac{1}{2}$, $\frac{dx}{dx}$, $\frac{dx^4}{dx^4}$, Simp, $\frac{dx}{dx}$, $\frac{dx}{dx}$, $\frac{dx}{dx}$
- Main area: Define $f(x) = 4 - \frac{\pi x}{4} \cos\left(\frac{\pi x}{4}\right)$
- Input: $f(1)$
- Output: $\frac{-\sqrt{2} \cdot \pi}{8} + 4$
- Bottom panel:

Math1	Line	$\frac{dx}{dx}$	$\sqrt{\square}$	π	\Rightarrow
Math2	sin	cos	tan	i	∞
Math3	\sin^{-1}	\cos^{-1}	\tan^{-1}	θ	t
Trig	sinh	cosh	tanh	$^{\circ}$	r
Var	\sinh^{-1}	\cosh^{-1}	\tanh^{-1}	\square^{\square}	
abc					
	\leftarrow	$\frac{dx}{dx}$	$\frac{dx}{dx}$	ans	EXE
- Bottom bar: Alg Standard Real Rad $\frac{dx}{dx}$

Mark allocation: 1 mark

- 1 mark for the correct answer.

**Tip**

- *Note that an exact value is required here.*

Question 4c.iii.**Worked solution**

Use CAS to find when $f(x) = 6$.

Define $f(x) = 4 - 4 \cos\left(\frac{x}{4}\right)$

done

$f(1)$

$$\frac{-\sqrt{2} \cdot \pi}{8} + 4$$

solve($f(x) = 6, x$)

◀ 1.1579001, x=3.18151465, x ▶

Math1	Line	$\frac{\square}{\square}$	$\sqrt{\square}$	π	\rightarrow
Math2	sin	cos	tan	i	∞
Math3	\sin^{-1}	\cos^{-1}	\tan^{-1}	θ	t
Trig	sinh	cosh	tanh	$^{\circ}$	$^{\circ}$
Var	\sinh^{-1}	\cosh^{-1}	\tanh^{-1}	\square^{\square}	
abc					
	\leftarrow	$\frac{\square}{\square}$	$\frac{\square}{\square}$	ans	EXE

Alg Standard Real Rad $\frac{\square}{\square}$

So $f(x) \geq 6$ for $3.1815 \leq x \leq 4.3$, so for 1.119 km.

Mark allocation: 2 marks

- 1 mark for finding the points and intersection.
- 1 mark for the correct answer 1.119.

**Tips**

- *Be careful of the endpoint! The graph stops at $x = 4.3$, so the upper endpoint of the interval is 4.3.*
- *To give an answer that is correct to 3 decimal places requires that your working be done to at least 4 decimal places.*

Question 4d.i.**Worked solution**

Use CAS to find p for when $\frac{d}{dx}\left(4 - \frac{\pi x}{4}\cos(px)\right) = 0$ with $x = 1$.

2888, x=-1.095410747, x=

Define $f(x) = 4 - \frac{\pi x}{4}\cos(px)$

done

solve $\left(\frac{d}{dx}(f(x)) = 0, x\right)$

$\{p \cdot x \cdot \tan(p \cdot x) - 1 = 0\}$

Math1	a	b	c	d	e	f
Math2	g	h	i	j	k	l
Math3	m	n	o	p	q	r
Trig	s	t	u	v	w	x
Var	y	z				CAPS
abc						
	←	↵	↶	ans	EXE	

Alg Standard Real Rad

solve $\left(\frac{d}{dx}(f(x)) = 0, x\right)$

$\{p \cdot x \cdot \tan(p \cdot x) - 1 = 0\}$

$p \cdot x \cdot \tan(p \cdot x) - 1 = 0 \mid x = 1$

$p \cdot \tan(p) - 1 = 0$

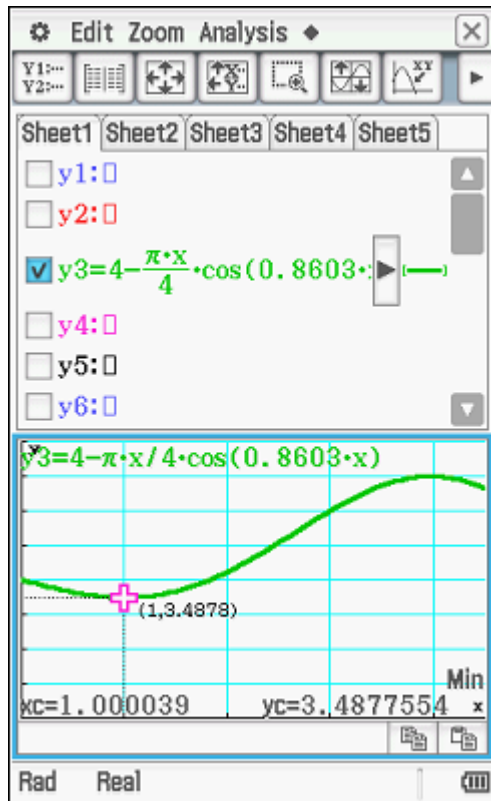
solve $(p \cdot \tan(p) - 1 = 0, p)$

0333589, p=0.860333589,

Math1	Line	$\frac{\square}{\square}$	$\sqrt{\square}$	π	\Rightarrow
Math2	Define	f	g	i	∞
Math3	solve(dSlv	'	$\left\{\frac{\square}{\square}\right\}$	
Trig	<	>	()	{ }	[]
Var	\leq	\geq	=	\neq	\angle
abc					
	←	↵	↶	ans	EXE

Alg Standard Real Rad

Although this gives a number of values for p , $p = 0.8603$ gives the correct outcome. This can be verified by checking the graph $y = 4 - \frac{\pi x}{4} \cos(px)$ with $p = 0.8603$.



Mark allocation: 2 marks

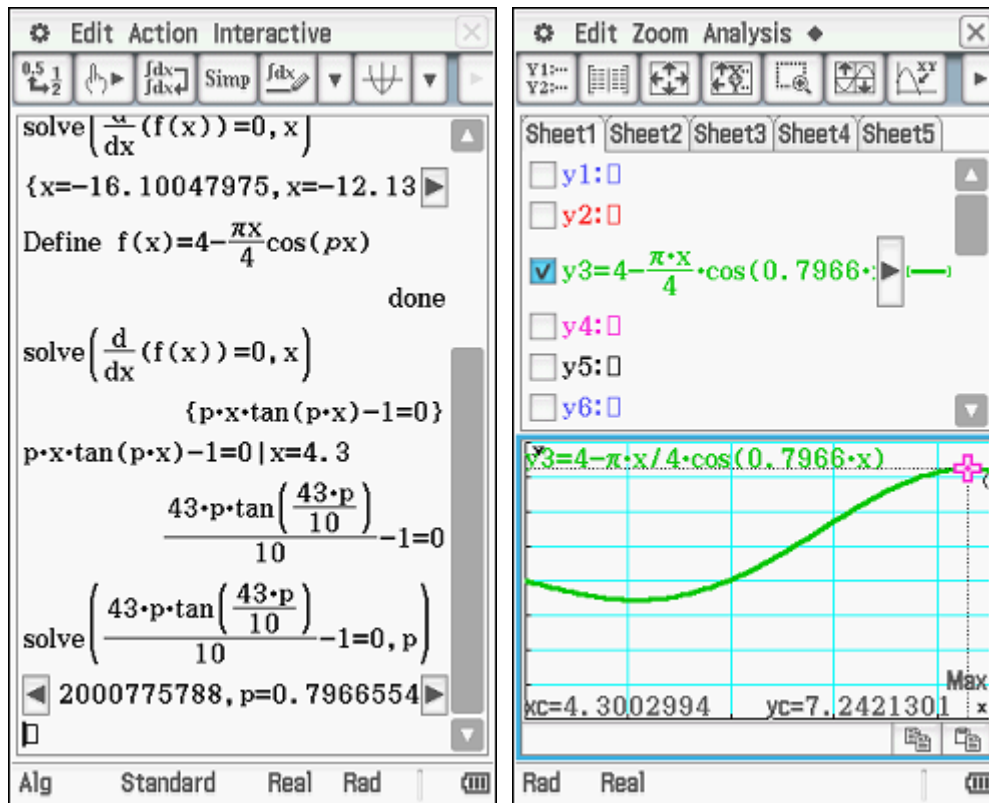
- 1 mark for setting $\frac{d}{dx} \left(4 - \frac{\pi x}{4} \cos(px) \right) = 0$ with $x = 1$.
- 1 mark for the answer $p = 0.8603$.

Question 4d.ii.**Worked solution**

For the maximum to occur at the endpoint, this requires that the maximum turning point occurs at $x \geq 4.3$ and that $f(4.3) > f(0) = 4$.

Use CAS to find p such that $\frac{d}{dx} \left(4 - \frac{\pi x}{4} \cos(px) \right) = 0$ with $x = 4.3$.

This gives a number of values for p . Verifying the correct value for p by checking the graph gives $p = 0.7966$. (Note: The value 0.7967 gives a turning point just before the endpoint.)



Using CAS to solve $f(4.3) > 4$ for p gives $p > 0.3653$.

So for the maximum to occur at $x = 4.3$, then $0.3653 < p < 0.7966$.

Mark allocation: 4 marks

- 1 mark for setting $\frac{d}{dx}\left(4 - \frac{\pi x}{4} \cos(px)\right) = 0$ with $x = 4.3$.
- 1 mark for $p < 0.7966$.
- 1 mark for $f(4.3) > 4$.
- 1 mark for $0.3653 < p < 0.7966$.



Tip

- *Don't assume that the normal rules for rounding apply. Always check your answer by sketching a graph and seeing whether it complies with the requirements. In this case, a 'rounded down' answer of 0.7966 was required.*

END OF WORKED SOLUTIONS BOOK

SECTION 2