Quality Assessment Tasks

NAME: ____________________________

VCE MATHEMATICAL METHODS (CAS)

Practice written examination 1

Reading time: 15 minutes **Writing time: 1 hour**

QUESTION AND ANSWER BOOK

Structure of book

Number of questions	*Number of questions to be answered*	*Number of marks*
10	10	40

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers.
- Students are **not** permitted to bring into the examination room: notes of any kind, a calculator of any type, blank sheets of paper and/or white out liquid/tape.

Materials supplied

- Question and answer book of 10 pages.
- Formula booklet PROVIDED BY YOUR TEACHER.
- Working space is provided throughout the book.

Instructions

- All written responses must be in English.
- Write your student name in the space provided above on this page.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

Instructions

Answer **all** questions in the spaces provided.
In all questions where a numerical answer is required an exact value must be given unless otherwise specified.
In questions where more than one mark is available, appropriate working **must** be shown.
Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Question 1

A function $f(x) = \sqrt{4 - x^2}$ undergoes a reflection in the x-axis and then translation by 2 units in the negative x-direction. Determine the equation of the transformed function, *g(x)*.

2 marks

Question 2

a) If $y = (x + 2x^4)^2$ then find $\frac{dy}{dx}$.

1 mark

b) If $g(x) = \frac{\cos(2x)}{x}$ then find $g'\left(\frac{\pi}{2}\right)$.

2 marks

Question 3

a) Find an antiderivative of $\frac{3}{(3-3x)^2}$ with respect to x.

2 marks

b) Determine the average value of the function $h(x) = 4x - \frac{x^3}{4}$ between x = 0 and x = 4.

3 marks

Question 4

Solve the equation $e^x + \frac{25}{e^x} = 10$ for x.

3 marks

Question 5

The equation for the function g is $g(x) = \frac{10}{(2x^2 + 5)}$.

a) Find an equation for the inverse of g.

2 marks

b) Determine a maximal domain for *g(x)* that will make the inverse also a function.

3 marks

Question 6

Consider the function $h: R \rightarrow R, h(x) = 2 - 2\left|sin\left(\frac{\pi x}{2}\right)\right|$

a) State the range of *h(x)*.

1 mark

b) Determine the period of *h(x)*.

1 mark

c) Solve *h(x) = 0* for all $x \in [-2, 2]$.

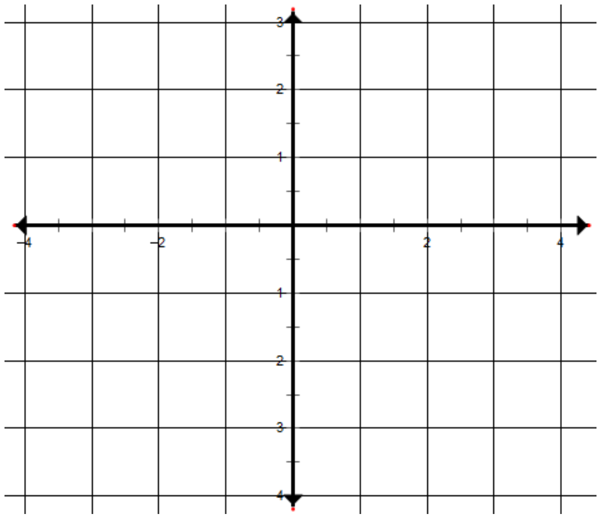
1 mark

d) For $0 \leq x \leq 2$ *h(x)* is equivalent to $2 - 2sin\left(\frac{\pi x}{2}\right)$. Use this information and calculus to determine the area bounded by *h(x)*, the x-axis, x = - 2 and x = 2.

2 marks

Question 7

a) Sketch the graph of $f(x) = log_e((x-1)^2)$ on the axes provided.



3 marks

b) Differentiate $f(x) = log_e((x-1)^2)$.

1 mark

c) Determine the equation of the normal through the point where x = 2.

2 marks

Question 8

A weighted six sided die has a distribution as given:

X	1	2	3	4	5	6
Pr(X = x)	k	k – 0.2	0.1	0.2	0.1	2k

a) Determine the value of k in the distribution.

2 marks

b) Find the mean of the distribution.

1 mark

c) What is the probability that the die will land face up on an even number?

1 mark

Question 9

a) A binomial distribution has a mean of 20 and a variance of 12
Determine the values of n and p.

2 marks

b) Hence determine the value of Pr(X = 0). Leave your answer in exact form.

1 mark

Question 10

A continuous random variable is defined by the hybrid function

$$p(x) = \begin{cases} \dfrac{x}{16} & 0 \leq x \leq 4 \\ \dfrac{(x-1)(7-x)}{36} & 4 < x \leq 7 \end{cases}$$

a) What is the *Pr(X ≤ 4)*?

2 marks

b) Hence or otherwise determine the value of the 25th percentile.

2 marks

END OF QUESTION AND ANSWER BOOKLET

Solution Pathway

Question 1

Reflection in the x-axis gives $-\sqrt{4-x^2}$	(1 mark)
Translation in the negative x-axis gives $-\sqrt{4-(x+2)^2}$	
So $g(x) = -\sqrt{4-(x+2)^2}$	(1 mark)

Question 2

a)

Use chain rule. Let $u = x + 2x^4$ and then $y = u^2$	
$\frac{du}{dx} = 1 + 8x^3$ and $\frac{dy}{du} = 2u$	
Then $\frac{dy}{dx} = 2(x+2x^4) \times (1+8x^3)$	(1 mark)

b)

Use quotient rule: Let $u = \cos(2x)$ and $v = x$	
$\frac{du}{dx} = -2\sin(2x)$ and $\frac{dv}{dx} = 1$	
Then $\frac{dy}{dx} = \frac{x(-2\sin(2x)) - \cos(2x)}{x^2}$	(1 mark)
$\therefore g'\left(\frac{\pi}{2}\right) = \frac{1}{\frac{\pi^2}{4}} = \frac{4}{\pi^2}$	(1 mark)

Question 3

a)

$\int \frac{3}{(3-3x)^2}\,dx = 3\int (3-3x)^{-2}\,dx$	
$= 3\left(\frac{(3-3x)^{-2+1}}{-3\times -1}\right) + c$	(1 M)
$= (3-3x)^{-1} + c$	
$= \frac{1}{3-3x} + c$	(1 A)

b)

	$\frac{1}{4-0}\int_0^4 4x - \frac{x^3}{4}\,dx = \frac{1}{4}\left[\frac{4x^2}{2} - \frac{x^4}{16}\right]_0^4$	(1 M)
	$= \frac{1}{4}\left(\frac{4\times16}{2} - \frac{16\times16}{16} - 0\right)$	(1 M)
	$= \frac{1}{4}(4\times8-16)$	
	$= \frac{1}{4}\times16 = 4$	(1 A)

Question 4

Multiply through by e^x:	$e^{2x} + 25 = 10e^x$	
	$e^{2x} - 10e^x + 25 = 0$	(1 M)
Let A = e^x	$A^2 - 10A + 25 = 0$	
Solve for A	$(A-5)^2 = 0$	
	$(A-5) = 0$	
	A = 5	(1 M)
Now solve for x	$e^x = 5$	
	$x = log_e(5)$	(1 A)

Question 5

a)

	Let $g(x) = y$	
	$y = \frac{10}{(2x^2+5)}$.	
Swap x and y	$x = \frac{10}{(2y^2+5)}$.	(1 mark)
Rearrange	$2y^2 + 5 = \frac{10}{x}$	
	$y^2 = \frac{5}{x} - 2.5$	
	$y = \pm\sqrt{\frac{5}{x} - \frac{5}{2}}$	(1 mark)

b)

Domain restrictions must make the original function 1 to 1. Therefore it is necessary to find turning points of the original function.

	$g'(x) = \frac{-40x}{(2x^2+5)^2}$	(1 mark)
Let $g'(x) = 0$	$0 = \frac{-40x}{(2x^2+5)^2}$	
Hence	0 = -40x	
and	x = 0	(1 mark)
So maximal domain is	$0 \le x < \infty$ or $-\infty < x \le 0$	(1 mark for either)

Question 6

a)

All sine functions have a range of $-1 \le x \le 1$

The modulus converts this to a range $0 \le x \le 1$

$2 - 2 = 0$ and $2 - 0 = 2$.

Hence Ran = [0, 2] (1 mark)

b)

The $sin\left(\frac{\pi x}{2}\right)$ component of the function has a period of $2\pi \times \frac{2}{\pi} = 4$

The modulus converts this to half the original period so Period = 2. (1 mark)

c)

$$0 = 2 - 2\left|sin\left(\frac{\pi x}{2}\right)\right|$$

$$2 = 2\left|sin\left(\frac{\pi x}{2}\right)\right|$$

$$1 = \left|sin\left(\frac{\pi x}{2}\right)\right|$$

$$\pm 1 = sin\left(\frac{\pi x}{2}\right)$$

$$\pm\frac{\pi}{2} = \frac{\pi x}{2}$$

$x = \pm 1$ (1 mark)

d)

$$\int_{-2}^{2} h(x)dx = 2\int_{0}^{2} 2 - 2sin\left(\frac{\pi x}{2}\right)dx$$

$$= 2\left[(2x + \frac{4}{\pi}cos\left(\frac{\pi x}{2}\right)\right]_0^2$$ (1 mark)

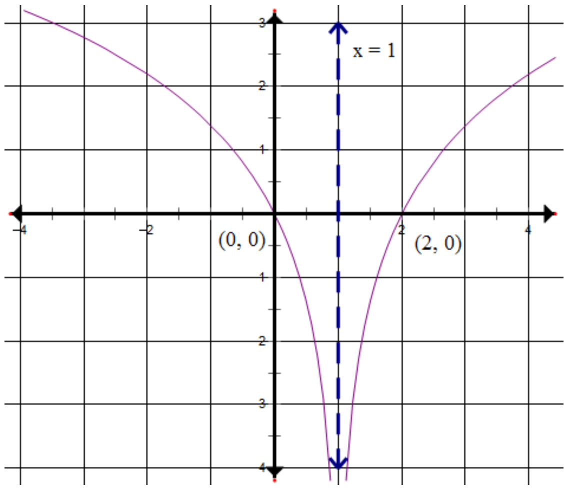
$$= 2\left[(4 + \frac{4}{\pi}cos(\pi) - (0 + \frac{4}{\pi}cos(0)\right]$$

$$= 2\left[(4 - \frac{4}{\pi} - \frac{4}{\pi}\right]$$

$= 8 - \frac{16}{\pi}$ units (1 mark)

Question 7

a)



Shape – 1, intercepts – 1, asymptote – 1 (3 marks)

b)

$f'(x) = \frac{2(x-1)}{(x-1)^2}$

$f'(x) = \frac{2}{x-1}$ (1 mark)

c)

$m_T = f'(2) = \frac{2}{2-1} = 2$

$m_N = \frac{-1}{2}$ (1 mark)

$y - 0 = \frac{-1}{2}(x-2)$

$y = \frac{-1}{2}x + 1$ (1 mark)

Question 8

a)

Sum = 4k + 0.2

1 = 4k + 0.2 (1 mark)

4k = 0.8

k = 0.2 (1 mark)

b)

0.2 + 0 + 0.3 + 0.8 + 0.5 + 2.4 = 4.2 (1 mark)

c)

0 + 0.2 + 2 x 0.2 = 0.6 (1 mark)

Question 9

a)

$np = 20$

$npq = 12$

$q = \frac{12}{20}$

Therefore $p = 1 - \frac{12}{20} = \frac{8}{20}$ (1 mark)

And $n = \frac{20 \times 20}{8} = 50$ (1 mark)

b)

$\Pr(X = 0) = {}^{50}C_0 \left(\frac{2}{5}\right)^0 \left(\frac{3}{5}\right)^{50} = 1 \times 1 \times \left(\frac{3}{5}\right)^{50} = \left(\frac{3}{5}\right)^{50}$ (1 mark)

Question 10

a)

$\int_0^4 \frac{x}{16}\, dx = \left[\frac{x^2}{32}\right]_0^4$ (1 mark)

$= \frac{16}{32} - 0 = \frac{1}{2}$ (1 mark)

b)

$\frac{1}{4} = \int_0^q \frac{x}{16}\, dx = \left[\frac{x^2}{32}\right]_0^q$ (1 mark)

$\frac{1}{4} = \frac{q^2}{32}$, Hence, $q^2 = 8$, so $q = \sqrt{8}$ (1 mark)

END OF SOLUTIONS