The Mathematical Association of Victoria

Trial Exam 2013

MATHEMATICAL METHODS (CAS)

WRITTEN EXAMINATION 1

STUDENT NAME ____

Reading time: 15 minutes
Writing time: 1 hour

QUESTION AND ANSWER BOOK

Structure of book

Number of questions	*Number of questions to be answered*	*Number of marks*
11	11	40

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers.
- Students are NOT permitted to bring into the examination room: notes of any kind, blank sheets of paper, white out liquid/tape or a calculator of any type.

Materials supplied
- Question and answer book of 8 pages, with a detachable sheet of miscellaneous formulas at the back
- Working space is provided throughout the book.

Instructions
- Detach the formula sheet from the back of this book during reading time.
- Write your name in the space provided above on this page.
- All written responses must be in English.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

Question 1 (2 marks)

Find an antiderivative of $\dfrac{1}{(1-2x)^3}$.

Question 2 (2 marks)

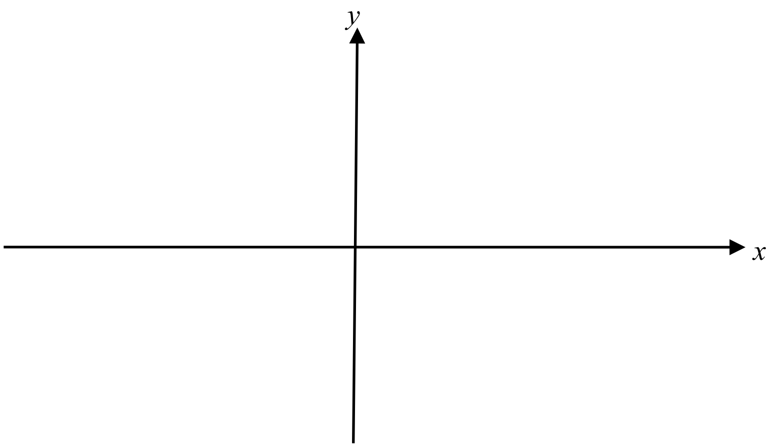
Consider $f:(2,\ 3] \to R,\ f(x)=(x-1)^{\frac{4}{5}}$ and $g:\left(-\dfrac{1}{4},\ 5\right] \to R,\ g(x)=\sqrt[5]{(x-1)}$.

Find a function h where $h = fg$.

Question 3 (6 marks)

Let $f:(-8,\ \infty) \to R$, where $f(x) = -\sqrt{x+8}+2$.

a. Sketch the graph of f on the set of axes below. Clearly label any axial-intercepts and endpoints with their coordinates.



2 marks

b. Define f^{-1}.

2 marks

c. Find the coordinates of the point of intersection of f and f^{-1}.

2 marks

Question 4 (3 marks)
Find the equation of the normal to the curve with equation $y = \log_2(x+2)$ at the y-intercept.

Question 5 (3 marks)

Solve $\frac{2}{e^x} + 2 = 3e^x$ for x.

Question 6 (3 marks)

A transformation $T : R^2 \to R^2$ that maps the graph of $g : R \setminus \left\{\frac{3}{2}\right\} \to R,\ g(x) = \frac{1}{(2x-3)^2} + 1$ to the graph of $h : R \setminus \{0\} \to R,\ h(x) = \frac{1}{x^2}$ has rule $T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} a & 0 \\ 0 & 1 \end{bmatrix}\left(\begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} b \\ c \end{bmatrix}\right)$, where a, b and c are non-zero real numbers. Find a set of values for a, b and c.

Question 7 (4 marks)

For what values of k and n does the following set of simultaneous equations have an **infinite** number of solutions.

$$kx + 4y = 2n$$

$$2x + (k + 2)y = -1$$

Question 8 (6 marks)

Consider $f : [-1, 4] \to R,\ f(x) = 4 + 3x - x^2$ and $g : (-4,\ 4) \to R,\ g(x) = |x|$.

a. **i**. Find the coordinates of the point where $f'(x) = 0$.

1 mark

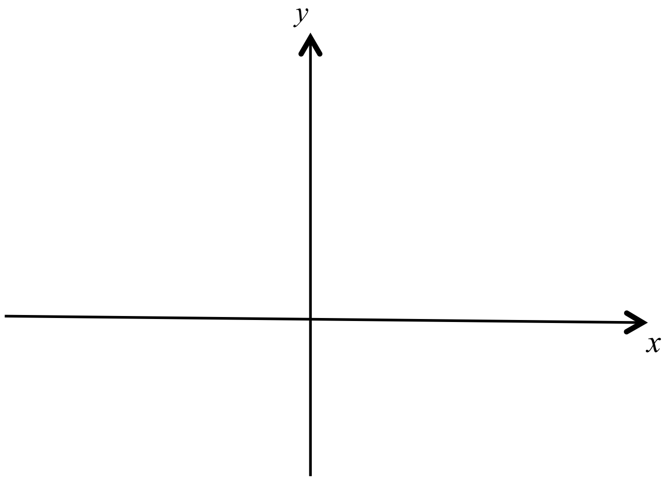
ii. Hence, state the range of f.

1 mark

b. **i.** Write the rule and the domain for $f(g(x))$.

2 marks

ii. Sketch the graph of $f(g(x))$, clearly labelling any axial intercepts and turning points with their coordinates.



2 marks

Question 9 (4 marks)

Consider $y = ae^x \cos\left(2x - \frac{\pi}{3}\right)$, where a is a real constant.

a. Find $\frac{dy}{dx}$.

2 marks

b. If $\frac{dy}{dx} = 1$, when $x = 0$, find the value of a. Write the answer in the form $m\sqrt{3} - n$, where m and n are positive real constants.

2 marks

Question 10 (4 marks)

For events A and B, $\Pr(A) = \frac{3}{4}$ and $\Pr(B) = \frac{1}{5}$, evaluate,

a. $\Pr(A' \cap B')$ when A and B are mutually exclusive, where A' and B' are the complements of A and B respectively;

2 marks

b. $\Pr(A | B)$ when $\Pr(A \cup B) = \frac{33}{40}$.

2 marks

Question 11 (3 marks)

The probability density function for the random variable, X, is given by

$$f(x) = \begin{cases} |a\cos(x)| & 0 \le x \le \pi \\ 0 & \text{elsewhere} \end{cases}.$$

a. Show that $a = \frac{1}{2}$.

2 marks

b. Find $\Pr\left(X > \frac{\pi}{6}\right)$.

1 mark

END OF QUESTION AND ANSWER BOOKLET