The Mathematical Association of Victoria
MATHEMATICAL METHODS (CAS)
SOLUTIONS: Trial Exam 2013

Written Examination 1

Question 1

$\int\left(\frac{1}{(1-2x)^3}\right)dx$

$=\int\left((1-2x)^{-3}\right)dx$

$=\frac{1}{-2\times-2}(1-2x)^{-2}+c$ **1M** must have + c

$=\frac{1}{4(1-2x)^2}+c$

An antiderivative is $\frac{1}{4(1-2x)^2}$. **1A**

Question 2

$f:(2,\ 3]\to R,\ f(x)=(x-1)^{\frac{4}{5}}$ and $g:\left(-\frac{1}{4},\ 5\right]\to R,\ g(x)=\sqrt[5]{(x-1)}$

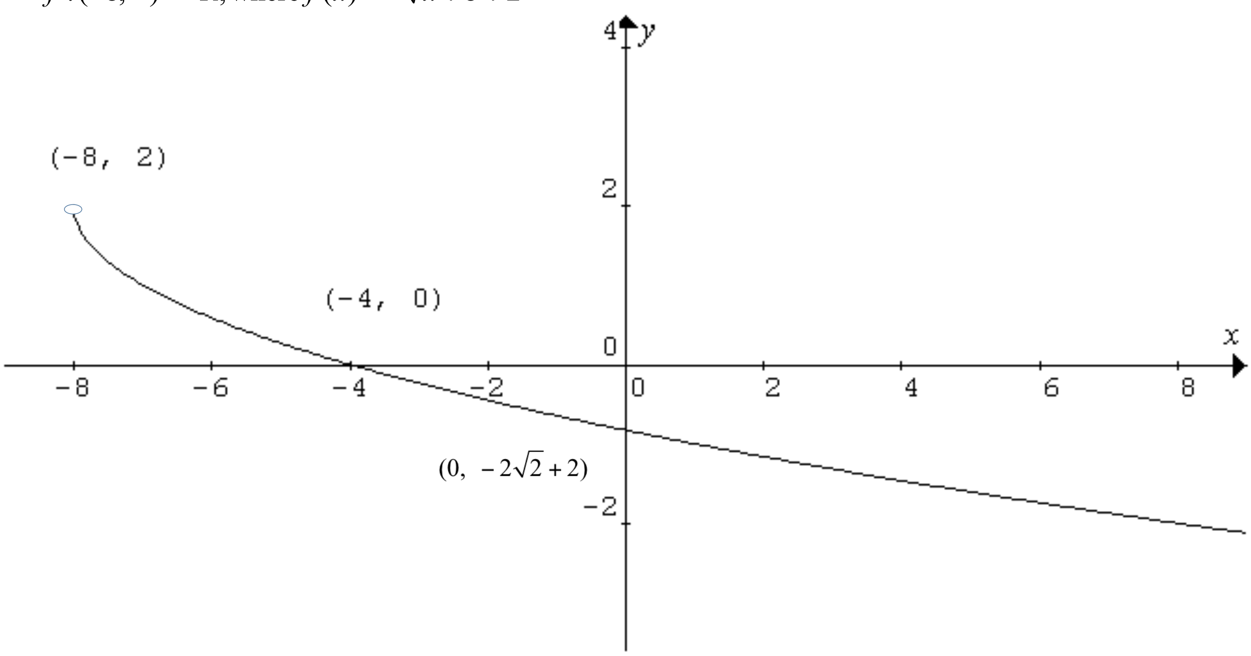
$h(x)=f(x)\times g(x)=(x-1)^{\frac{4}{5}}(x-1)^{\frac{1}{5}}=x-1$ **1A** rule

domain h $(2,3]\cap\left(-\frac{1}{4},5\right]=(2,3]$

$h:(2,\ 3]\to R,$ where $h(x)=x-1$ **1A** domain

Question 3

a. $f:(-8,\infty)\to R,$ where $f(x)=-\sqrt{x+8}+2$



Shape **1A**
Correct coordinates **1A**

b. Let $y=-\sqrt{x+8}+2$

Inverse swap x and y.

$x=-\sqrt{y+8}+2$ **1M**

$2-x=\sqrt{y+8}$

$y=(2-x)^2-8$

$f^{-1}:(-\infty,\ 2)\to R,\ f^{-1}(x)=(2-x)^2-8$ **1A** must have domain and f^{-1}

c. As there is only one point of intersection, it occurs along the line $y = x$.

Solve $x=-\sqrt{x+8}+2$ for x. **1M**

$2-x=\sqrt{x+8}$

$(2-x)^2=x+8$

$x^2-4x+4=x+8$

$x^2-5x-4=0$

Quadratic formula

$x=\dfrac{5\pm\sqrt{41}}{2},\ x\in(-\infty,2)$

$\left(\dfrac{5-\sqrt{41}}{2},\ \dfrac{5-\sqrt{41}}{2}\right)$ **1A**

Or

Completing the square

$\left(x-\dfrac{5}{2}\right)^2-\dfrac{41}{4}=0$

$\left(x-\dfrac{5}{2}-\dfrac{\sqrt{41}}{2}\right)\left(x-\dfrac{5}{2}+\dfrac{\sqrt{41}}{2}\right)=0$

$x=\dfrac{5\pm\sqrt{41}}{2},\ x\in(-\infty,\ 2)$

$\left(\dfrac{5-\sqrt{41}}{2},\dfrac{5-\sqrt{41}}{2}\right)$ **1A**

Question 4

$y=\log_2(x+2)=\dfrac{\log_e(x+2)}{\log_e(2)}$

y-intercept (0, 1) **1A**

$\dfrac{dy}{dx}=\dfrac{1}{\log_e(2)(x+2)}$

$m_T=\dfrac{1}{2\log_e(2)}$

$m_N=-2\log_e(2)$ **1A**

The equation of the normal is

$y=-2\log_e(2)x+1$ **1A**

Question 5

$\dfrac{2}{e^x}+2=3e^x$

$2+2e^x=3e^{2x}$ **1M**

$3e^{2x}-2e^x-2=0$

Let $a = e^x$

$3a^2 - 2a - 2 = 0$

$a = \frac{2 \pm \sqrt{28}}{6} = \frac{1 \pm \sqrt{7}}{3}$ **1A**

$e^x = \frac{1-\sqrt{7}}{3}$ no solution

$e^x = \frac{1+\sqrt{7}}{3}$

$x = \log_e\left(\frac{1+\sqrt{7}}{3}\right)$ **1A**

Question 6

$g : R \backslash \left\{\frac{3}{2}\right\} \to R,$ where $g(x) = \frac{1}{(2x-3)^2} + 1$

$g(x) - 1 = \frac{1}{(2x-3)^2}$

$T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} a & 0 \\ 0 & 1 \end{bmatrix}\left(\begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} b \\ c \end{bmatrix}\right)$

$x' = ax + ab = 2x - 3$

$y' = y + c = y - 1$

Equate coefficients

$a = 2,\ ab = -3,\ b = -\frac{3}{2},\ c = -1$ **3A**

Or

$x' = ax + ab,\ x = \frac{x' - ab}{a}$

$y' = y + c,\ y = y' - c$

$y' - c = \frac{1}{\left(2\left(\frac{x' - ab}{a}\right) - 3\right)^2} + 1$

$y' = \frac{1}{\left(\frac{2}{a}(x' - ab) - 3\right)^2} + 1 + c$

Equating coefficients:

$1 + c = 0,\ \ c = -1$

$\frac{2}{a} = 1 \qquad a = 2$

$\frac{-2ab}{a} - 3 = 0$

$-2b - 3 = 0,\ b = -\frac{3}{2}$

$a = 2,\ b = -\frac{3}{2},\ c = -1$ **3A**

Question 7

$kx + 4y = 2n$

$2x + (k+2)y = -1$

$\begin{vmatrix} k & 4 \\ 2 & k+2 \end{vmatrix} = 0$ **1M**

$k(k+2) - 8 = 0$

$k^2 + 2k - 8 = 0$

$(k+4)(k-2) = 0$

$k = -4$ or $k = 2$ **1A**

For infinite number of solutions

$\frac{k}{2} = \frac{2n}{-1}$

When $k = -4$, $n = 1$ **1A**

When $k = 2$, $n = -\frac{1}{2}$ **1A**

Or

$y = \frac{2n - kx}{4}$ (1)

$y = \frac{-1 - 2x}{(k+2)}$ (2) **1M**

For there to be infinite solutions, the gradients must be the same and the *y*-intercepts must be the same.

Gradients:

$-\frac{k}{4} = -\frac{2}{k+2}$

$k^2 + 2k = 8$

$k^2 + 2k - 8 = 0$

$(k+4)(k-2) = 0$

$k = -4, 2$ **1A**

y-intercepts:

$\frac{2n}{4} = -\frac{1}{k+2}$

$2nk + 4n = -4$

When $k = -4,\ n = 1$ **1A**

When $k = 2,\ n = -\frac{1}{2}$ **1A**

Question 8

a. i. $f'(x) = 3 - 2x = 0$

$x = \frac{3}{2}$

$f\left(\frac{3}{2}\right) = 4 + 3 \times \frac{3}{2} - \left(\frac{3}{2}\right)^2$

$= \frac{25}{4}$

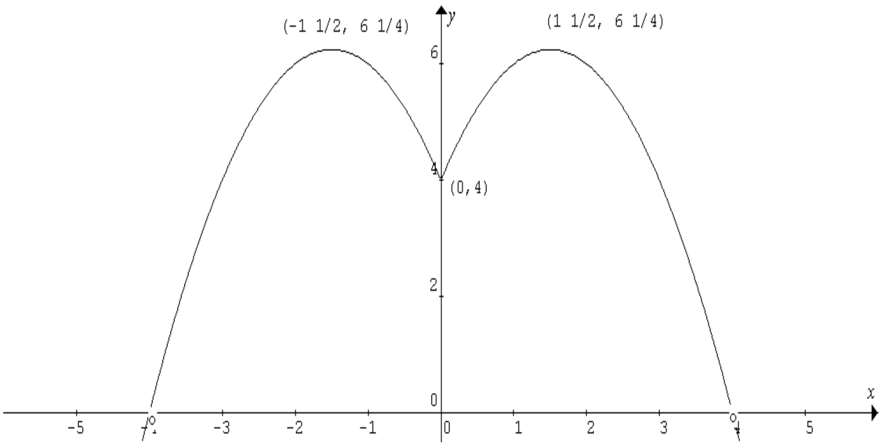
coordinates: $\left(\frac{3}{2}, \frac{25}{4}\right)$ or $\left(1\frac{1}{2}, 6\frac{1}{4}\right)$ **1A**

ii. range: $\left[0, 6\frac{1}{4}\right]$ or $0 \le y \le 6\frac{1}{4}$ **1A**

b. i. $f(g(x)) = 4 + 3|x| - x^2$ **1A**

domain: $(-4,\ 4)$ **1A**

ii. $y = f(g(x)) = 4 + 3|x| - x^2$



Shape **1 A**

Intercepts and Turning Points **1 A**

Question 9

a. $\frac{dy}{dx} = ae^x \cos\left(2x - \frac{\pi}{3}\right) - 2ae^x \sin\left(2x - \frac{\pi}{3}\right)$ **1M (product rule)** **1A**

b. $ae^0\left(\cos\left(-\frac{\pi}{3}\right) - 2\sin\left(-\frac{\pi}{3}\right)\right) = 1$

$a\left(\frac{1}{2} + \sqrt{3}\right) = 1$ **1A**

$a\left(\frac{1 + 2\sqrt{3}}{2}\right) = 1$

$a = \frac{2}{1 + 2\sqrt{3}}$

$a = \frac{2}{1 + 2\sqrt{3}} \times \frac{1 - 2\sqrt{3}}{1 - 2\sqrt{3}}$

$= \frac{2\left(1 - 2\sqrt{3}\right)}{1 - 12}$

$$= \frac{2\left(1-2\sqrt{3}\right)}{-11}$$

$$a = \frac{4}{11}\sqrt{3} - \frac{2}{11}$$ **1A**

Question 10

a. $\Pr(A \cup B) = \Pr(A) + \Pr(B)$

$$= \frac{3}{4} + \frac{1}{5}$$

$$= \frac{19}{20}$$ **1A**

$$\Pr(A' \cap B') = 1 - \frac{19}{20} \quad = \frac{1}{20}$$ **1A**

b. $\Pr(A \cap B) = \Pr(A) + \Pr(B) - \Pr(A \cup B)$

$$= \frac{19}{20} - \frac{33}{40}$$

$$= \frac{1}{8}$$ **1A**

$$\Pr(A \mid B) = \frac{\Pr(A \cap B)}{\Pr(B)}$$

$$= \frac{\frac{1}{8}}{\frac{1}{5}} = \frac{5}{8}$$ **1A**

Question 11

a. $a\int_0^{\frac{\pi}{2}} \left|\cos(x)\right| dx - a\int_{\frac{\pi}{2}}^{\pi} \left|\cos(x)\right| dx = 1$ **1M**

$$= a\left(\left[\left|-\sin(x)\right|\right]_0^{\frac{\pi}{2}} - \left[\left|-\sin(x)\right|\right]_{\frac{\pi}{2}}^{\pi}\right) = 1$$

$$= a\left(\left[\left|-\sin\left(\frac{\pi}{2}\right)\right| - \left|-\sin(0)\right|\right] - \left[\left|-\sin(\pi)\right| - \left|-\sin\left(\frac{\pi}{2}\right)\right|\right]\right) = 1$$

$$a(1+1) = 1$$

$$a = \frac{1}{2}$$ **1M** *show that*

or

$$2a\int_0^{\frac{\pi}{2}} \left|\cos(x)\right| dx = 1$$ **1M**

$$2a\left[\left|-\sin(x)\right|\right]_0^{\frac{\pi}{2}} = 1$$

$$a = \frac{1}{2}$$ **1M** *show that*

b. $\Pr\left(X > \frac{\pi}{6}\right) = 1 - \frac{1}{2}\left(\int_0^{\frac{\pi}{6}} \left|\cos(x)\right| dx\right)$

$$= 1 - \frac{1}{2}\left(\left[\left|-\sin(x)\right|\right]_0^{\frac{\pi}{6}}\right)$$

$$= 1 - \frac{1}{2}\left(\frac{1}{2} - 0\right)$$

$$= \frac{3}{4}$$ **1A**