FOR PUBLIC RELEASE

FINAL

REVISED
10:25 am, Jul 05, 2013

Year 2013
VCE
Mathematical Methods
Trial Examination 2

KILBAHA MULTIMEDIA PUBLISHING PO BOX 2227 KEW VIC 3101 AUSTRALIA	TEL: (03) 9018 5376 FAX: (03) 9817 4334 kilbaha@gmail.com http://kilbaha.com.au

Victorian Certificate of Education
2013

STUDENT NUMBER

									Letter
Figures									
Words									

MATHEMATICAL METHODS CAS
Trial Written Examination 2

Reading time: 15 minutes
Total writing time: 2 hours

QUESTION AND ANSWER BOOK

Structure of book

Section	*Number of questions*	*Number of questions to be answered*	*Number of marks*
1	22	22	22
2	4	4	58
			Total 80

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers, a protractor, set-squares, aids for curve sketching, one bound reference, one approved CAS calculator (memory DOES NOT need to be cleared) and, if desired, one scientific calculator. For approved computer-based CAS, their full functionality may be used.
- Students are NOT permitted to bring into the examination room: blank sheets of paper and/or white out liquid/tape.

Materials supplied

- Question and answer booklet of 30 pages with a detachable sheet of miscellaneous formulas at the end of this booklet.
- Answer sheet for multiple-choice questions.

Instructions

- Detach the formula sheet from the end of this book during reading time.
- Write your **student number** in the space provided above on this page.
- Write your **name** and **student number** on your answer sheet for multiple-choice questions, and sign your name in the space provided.
- All written responses must be in English.

At the end of the examination

- Place the answer sheet for multiple-choice questions inside the front cover of this book.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

SECTION 1

Instructions for Section I

Answer **all** questions in pencil on the answer sheet provided for multiple-choice questions.
Choose the response that is **correct** for the question.
A correct answer scores 1 mark, an incorrect answer scores 0.
Marks will **not** be deducted for incorrect answers.
No mark will be given if more than one answer is completed for any question.

Question 1

For the function with the rule $f(x)=\sqrt{b^2-x^2}$, where $b \in R^+$, the average rate of change of $f(x)$ with respect to x on the interval $\left[0,\frac{b}{2}\right]$ is

A. $b\left(\sqrt{3}-2\right)$

B. $\sqrt{3}-2$

C. $-\frac{\sqrt{3}}{3}$

D. $-\frac{b\sqrt{3}}{3}$

E. $\frac{b}{2}\left(\sqrt{3}-2\right)$

Question 2

Given the function $f:[-a,3a)\to R$, $f(x)=2a-x$ where $a \in R\backslash\{0\}$, then the range of the function of f is

A. $[-a,3a)$

B. $(-a,3a]$

C. $(-a,3a)$

D. $[-a,3a]$

E. $(3a,-a]$

Question 3

The function with the rule $f(x) = a\tan\left(\dfrac{b\pi x}{3}\right)$ where $a, b \in R^+$, has period equal to

A. $\dfrac{6}{b}$

B. $\dfrac{3}{b}$

C. $\dfrac{3\pi}{b}$

D. $\dfrac{2\pi b}{3}$

E. $\dfrac{\pi ab}{3}$

Question 4

Given that $f(x) = g(x)\log_e(h(x))$ and that

$g(2) = 3$, $g'(2) = 4$, $h(2) = e^2$ and $h'(2) = 2$, Then $f'(2)$ is equal to

A. $8 + \dfrac{6}{e^2}$

B. $4\log_e(2)$

C. $8 + \dfrac{3}{e^2}$

D. 8

E. 2

Question 5

The system of linear equations

$$x - ky + z = 14$$
$$2x - y + kz = 10$$
$$x + y + z = -2k$$

will have infinitely many solutions if

A. $k = 2$

B. $k = -1$

C. $k \in R \setminus \{-1\}$

D. $k \in R \setminus \{2\}$

E. $k \in R \setminus \{-1, 2\}$

Question 6

Using the linear approximation $f(x+h) \approx f(x) + hf'(x)$, with $f(x) = \tan(x)$, the value of $\tan(61^0)$ is closest to

A. 1.7472

B. 1.7623

C. 1.8019

D. 1.8040

E. 1.8717

Question 7

The function $f : (-\infty, a] \to R$, $f(x) = -x^3 + 4x^2 + 3x - 2$ will have an inverse provided

A. $a \le -\frac{1}{3}$

B. $a \ge -\frac{1}{3}$

C. $a \le 3$

D. $a \in \left[-\frac{1}{3}, 3\right]$

E. $a \le 0$

Question 8

The transformation $T: R^2 \to R^2$, which maps the curve with equation $y = \log_e(x)$ to the curve with equation $y = 2 - 2\log_e(2x+2)$, has the rule

A. $T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} 2 & 0 \\ 0 & -2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} 1 \\ 2 \end{bmatrix}$

B. $T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} -\frac{1}{2} & 0 \\ 0 & -2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} 1 \\ 2 \end{bmatrix}$

C. $T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} 2 & 0 \\ 0 & -2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} 2 \\ 2 \end{bmatrix}$

D. $T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} \frac{1}{2} & 0 \\ 0 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} -2 \\ -2 \end{bmatrix}$

E. $T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} \frac{1}{2} & 0 \\ 0 & -2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} -1 \\ 2 \end{bmatrix}$

Question 9

The normal to the graph of $y = \cos(\pi b x)$ at the point $x = \frac{1}{6b}$ where $b \in R^+$ has a gradient of 4. The value of b is equal to

A. $-\frac{8}{\pi}$

B. $-\frac{1}{2\pi}$

C. $\frac{1}{2\pi}$

D. $\frac{8}{\pi}$

E. $-\frac{16\sqrt{3}}{3\pi}$

Question 10

Consider the function $f : R \to R$, $f(x) = ax^2 - 2bx$, where $a, b \in R \setminus \{0\}$. Then

A. if $a > 0$ and $b > 0$ the function is strictly decreasing for $0 < x < 2b$.

B. if $a > 0$ and $b < 0$ the function is strictly decreasing for $x > 0$.

C. if $a > 0$ and $b > 0$ the function is strictly decreasing for $x > \frac{b}{a}$.

D. if $a < 0$ and $b > 0$ the function is strictly decreasing for $x < \frac{b}{a}$.

E. if $a < 0$ and $b > 0$ the function is strictly decreasing for $x > \frac{b}{a}$.

Question 11

Ray either drives to work, or catches the train. If on one day he catches the train, the probability that he catches the train the following day is 0.7. If he drives to work one day, the probability that he catches the train the following day is 0.2. The long-term probability that he drives to work is

A. $\frac{1}{5}$

B. $\frac{2}{5}$

C. $\frac{3}{5}$

D. $\frac{4}{5}$

E. $\frac{7}{10}$

Question 12

A hexagon with a side length of x cm, has an area of $\frac{3\sqrt{3}}{2}x^2$. If the area of a hexagon is increasing at a rate of 36 cm^2/sec, then the rate at which the side length is increasing at in cm/sec when the side length is 2 cm, is equal to

A. $\frac{\sqrt{3}}{6}$

B. $\sqrt{3}$

C. $2\sqrt{3}$

D. $4\sqrt{3}$

E. $108\sqrt{3}$

Question 13

The average value of the function $f: R \to R$, $f(x) = e^{2x}$ over the interval $\left[0, \frac{k}{2}\right]$ where $k \in R^+$ is equal to 3. The value of k, correct to three decimal places is equal to

A. 0.619

B. 0.763

C. 1.386

D. 1.904

E. 2.918

Question 14

Given the functions with the rules and maximal domains $f(x)=|x|$ and $g(x)=x^2$,

some students stated the following.

Peter stated that $f(x)=g^{-1}(x)g(x)$

Quentin stated that $f(x)=g^{-1}(g(x))$

Sara stated that $f(x)=g(g^{-1}(x))$

Tanya stated that $g(x)=f(g(x))$.

Then

A. All of Peter, Quentin, Sara and Tanya are all correct.

B. Only Quentin is correct.

C. Both Quentin and Sara are correct.

D. Only Tanya is correct

E. Both Quentin and Tanya are correct.

Question 15

Given the continuous probability distribution defined by $f(x)=\begin{cases} k\sin(3x) & \text{for } 0 \le x \le \frac{\pi}{3} \\ 0 & \text{elsewhere} \end{cases}$

Which of the following is **false?**

A. $k=\frac{3}{2}$

B. The median is equal to $\frac{\pi}{6}$

C. $E(X)=\frac{\pi}{6}$

D. $E(X^2)=\frac{\pi^2}{36}$

E. $\Pr\left(0<X<\frac{\pi}{4}\right)=\frac{2+\sqrt{2}}{4}$

Question 16

A discrete random variable X has the probability function $\Pr(X = k) = \dfrac{e^{-\lambda}\lambda^k}{k!}$, where k is a non-negative integer and $\lambda \in R^+$. The probability that X is more than 2 is equal to

A. $1 - e^{-\lambda}(\lambda + 1)$

B. $e^{-\lambda}(\lambda + 1)$

C. $1 - \dfrac{\lambda^2 e^{-\lambda}}{2}$

D. $\dfrac{e^{-\lambda}}{2}(\lambda^2 + 2\lambda + 2)$

E. $1 - \dfrac{e^{-\lambda}}{2}(\lambda^2 + 2\lambda + 2)$

Question 17

The velocity $v(t)$ of a moving particle at a time t seconds is given by $v(t) = \dfrac{27}{(3t+4)^2}$ m/s for $t \ge 0$. Initially the particle is at rest. Which of the following is **false?**

A. The initial velocity is $\dfrac{27}{16}$ m/s

B. The distance travelled in the first two seconds $\dfrac{27}{20}$ metres.

C. The acceleration of the particle is given by $\dfrac{-162}{(3t+4)^3}$ m/s^2

D. The distance travelled at a time t is equal to $\dfrac{-9}{3t+4}$ metres.

E. The position $x(t) = \dfrac{27t}{4(3t+4)}$ metres.

Question 18

The cubic function $f: R \to R$, $f(x) = ax^3 + bx^2 + cx + d$, where $a, b, c, d \in R \setminus \{0\}$ has no turning points when

A. $b^2 > 3ac$

B. $b^2 \leq 3ac$

C. $b^2 < 4ac$

D. $b^2 > 4ac$

E. $b^2 = 4ac$

Question 19

The function $f(x)$ is continuous and is such that $f(x) > 0$ for $x \in [a-b, 2b]$ where $a, b \in R$ and $b > a > 0$.

Let $I_1 = \int_a^b f(x)\,dx$ $\quad I_2 = \int_{a+b}^{2b} f(x \quad b)\,dx$ and $I_3 = \int_{a-b}^{0} f(x+b)\,dx$. Then

A. $I_1 = I_2 = I_3$

B. $I_1 = I_2$ and $I_1 \neq I_3$

C. $I_1 = I_3$ and $I_1 \neq I_2$

D. $I_2 = I_3$ and $I_1 \neq I_2$

E. $I_1 \neq I_2$, $I_2 \neq I_3$ and $I_1 \neq I_3$

Question 20

The continuous random variable X has a normal distribution with mean 15 and variance 16. The continuous random variable Z has the standard normal distribution. Which of the following is **not equal** to the probability that Z is between -1 and 2

A. $1-\left[\Pr(Z>2)+\Pr(Z<-1)\right]$

B. $\Pr(0<Z<1)+\Pr(0<Z<2)$

C. $\Pr(11<X<23)$

D. $1-\Pr(7<X<19)$

E. $1-\left[\Pr(X<7)+\Pr(X>19)\right]$

Question 21

The random variable X has the following probability distribution.

X	1	2	3
$\Pr(X=x)$	a	$\frac{a}{2}$	$\frac{a}{3}$

Which of the following statements is **false?**

A. $a=\frac{6}{11}$

B. $E(X)=\frac{18}{11}$

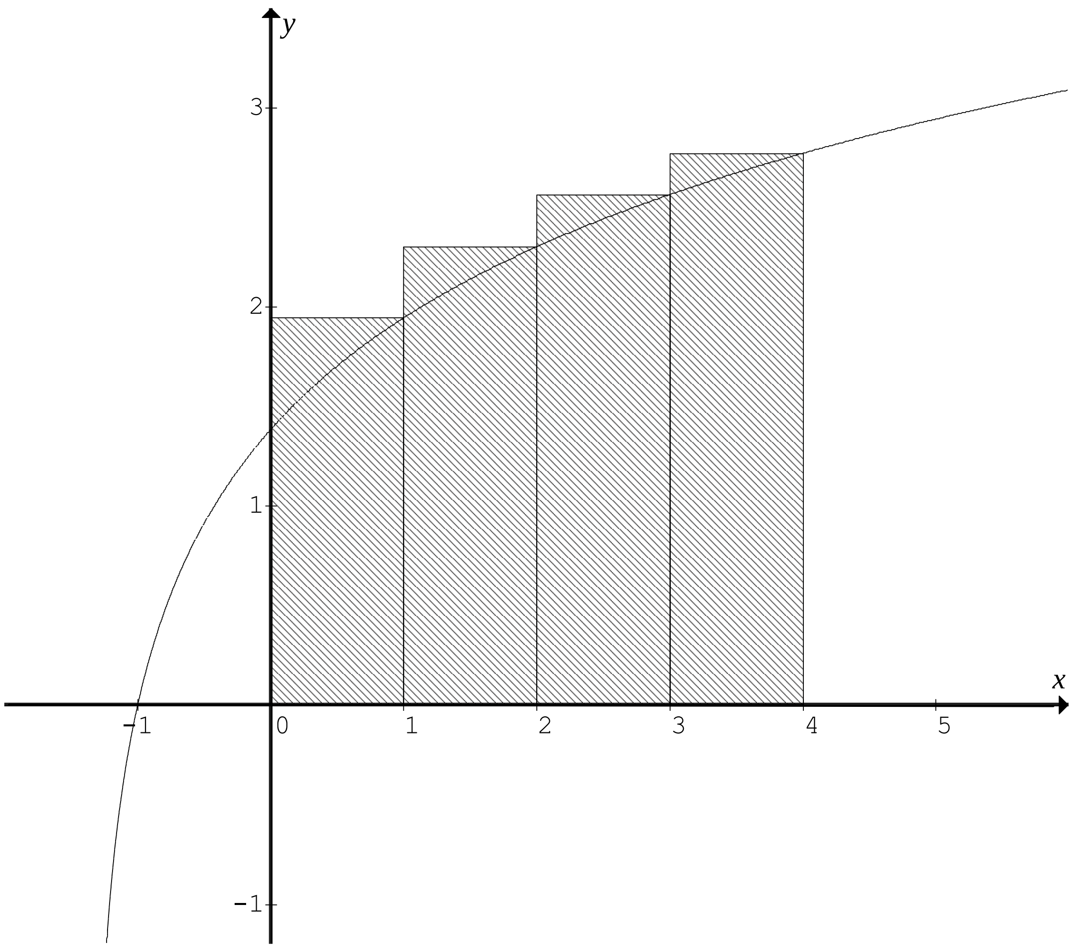
C. $E(X^2)=\frac{36}{11}$

D. $\text{var}(X)=\frac{72}{121}$

E. $E\left(\frac{1}{X}\right)=\frac{11}{18}$

Question 22

Part of the function $f(x) = \log_e(3x+4)$ is shown below. In order to find an approximation to the area bounded by the graph of *f*, the co-ordinate axes and the line $x = 4$, four rectangles are drawn. The area of the rectangles is equal to



A. $\frac{56}{3}\log_e(2) - 4$

B. $\log_e(14560)$

C. $\log_e(7280)$

D. $\log_e(3640)$

E. 8.938

END OF SECTION 1

SECTION 2

Instructions for Section 2

Answer **all** questions in the spaces provided.
In all questions where a numerical answer is required an exact value must be given unless otherwise specified.
In questions where more than one mark is available, appropriate working **must** be shown.
Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Question 1 18 marks

a. When Lilly goes shopping, she either pays by cash or pays by credit card. If she purchases an item on cash, the probability that her next purchase using cash is 0.35, while if she bought an item using a credit card, the probability that her next purchase using a credit card is 0.45. On certain day, Lilly bought four items, for her first purchase she used a credit card. Find the each of the following probabilities, giving all answers correct to three decimal places, that

i. she used a credit card exactly three times.

2 marks

ii. on her fourth purchase, she used a credit card.

1 mark

b. Let p be the probability that people are paying monthly interest on their credit cards.

Out of a group of 16 people who use their credit cards,

i. if the probability that 8 or 9 pay monthly interest is equal to 0.25, and we expect more than half are paying interest, find the value of p, correct to four decimal places.

2 marks

ii. If in fact $p = 0.65$, find the probability that more than half are paying monthly interest on their credit cards, give your answer correct to four decimal places.

1 mark

c. The amount of money spent per month on a credit card is found to be normally distributed. In one month, it is found that 14% of families spend more than \$3,700 on a credit card, while 26% of families spend less than \$2,990 on a credit card. Determine the mean and standard deviation of the amounts spent on a monthly credit card, giving your answers to the nearest dollar.

3 marks

d. The time *t* hours, where $t \geq 0$, that Lilly spends shopping, on a certain day is found to satisfy a continuous probability density function *T* with the rule

$$T(t) = \begin{cases} a e^{\frac{t}{3}} & \text{for } 0 \leq t \leq 3 \\ b \sin\left(\frac{\pi t}{6}\right) & \text{for } 3 \leq t \leq 6 \\ 0 & \text{elsewhere} \end{cases}$$

where *a* and *b* are positive constants.

i. Explain why and show that $b = ae$ where $a = \dfrac{\pi}{3\left(e\left(\pi+2\right)-\pi\right)}$. 2 marks

ii. Sketch the graph of $y = T\left(t\right)$ on the axes provided, clearly labelling the scale.

2 marks

iii. Find the exact probability that Lilly spends more than four hours shopping.

1 mark

iv. Find the exact probability that Lilly spends less than two hours shopping, if it is known that she spent less than three hours shopping.

2 marks

v. Find the expected time in hours, correct to two decimal places that Lilly spends shopping.

2 marks

Question 2 14 marks

Given the cubic function $f: R \to R$, $f(x) = x^3 + bx^2 + cx + 6$ where *b* and *c* are real constants.

i. Find $f'(x)$ in terms of *b* and *c*. 1 mark

ii. Find in terms of *b* and *c*, the equation of the tangent $tp(x)$ to the cubic at the point *P* where $x = 2$. 2 marks

iii. This tangent $tp(x)$ intersects the cubic again at the point $Q(-1, 6)$.

Write down simultaneous equations involving *b* and *c*, and hence show that $b = -3$ and $c = -4$. 2 marks

iv. Write down a definite integral A_1 not involving b and c, which gives the area between the tangent $tp(x)$ and the cubic.

2 marks

v. Find the equation of the tangent $tq(x)$ to the cubic at the point $Q(-1,6)$. 2 marks

vi. This tangent $tq(x)$ intersects the cubic again at the point R, find the coordinates of the point R.

1 mark

vii. Write down a definite integral A_2 not involving b and c, which gives the area between the tangent $tq(x)$ and the cubic.

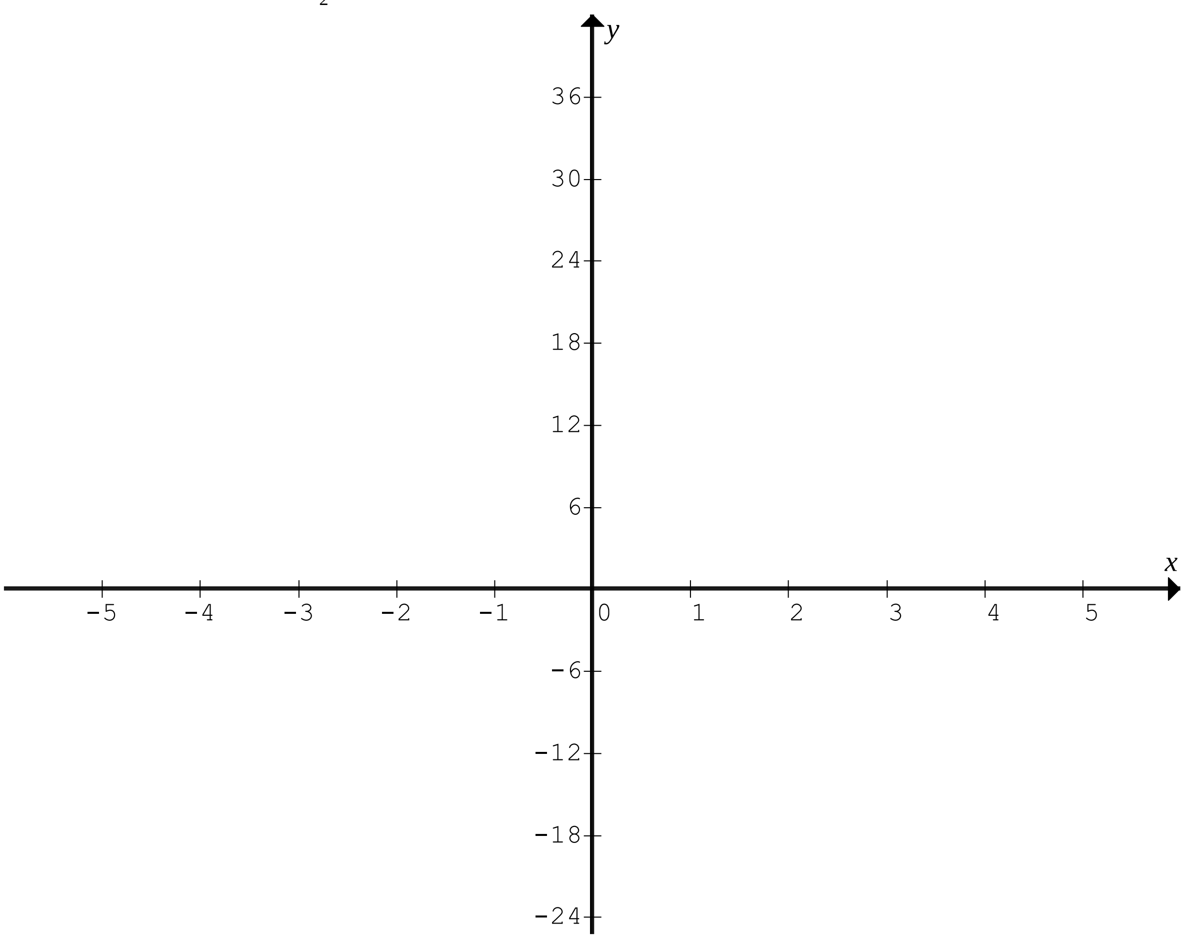
1 mark

viii. Find the value of $\frac{A_2}{A_1}$

1 mark

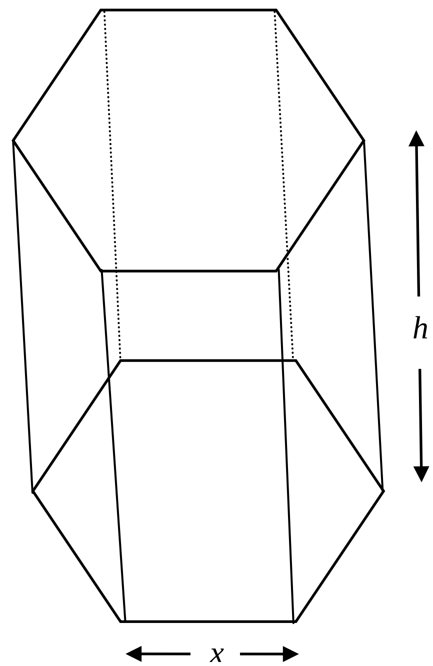
ix. On the axes below, sketch the graphs of $y = f(x)$ and the tangents at P and Q and shade the area A_2.

2 marks



Question 3 14 marks

A vase which is open at the top, has a base which is in the shape of a regular hexagon of side length x cm. The height of the vase is h cm, as shown in the diagram.



a. Show that the total surface area S of the vase in sq cm, is given by $S=\frac{3\sqrt{3}}{2}x^2+6xh$.

2 marks

b. Find an expression for the total volume V of the vase in cm^3, in terms of x and h.

1 mark

c.i. If the total surface area S is a constant, show that the volume can be expressed as

$V(x)=\dfrac{\sqrt{3}}{8}\left(2S\,x-3\sqrt{3}x^{3}\right)$ and find the domain for the possible values for x.

3 marks

ii. Find $\dfrac{dV}{dx}$ and if the volume of the vase is a maximum when the surface area is constant, express x in terms of S.

2 marks

d.i. If the volume V is a constant, express the surface area in terms of x and V.

2 marks

ii. If the surface area is a minimum, and the volume constant, show that $x = \sqrt[3]{\frac{4V}{9}}$.

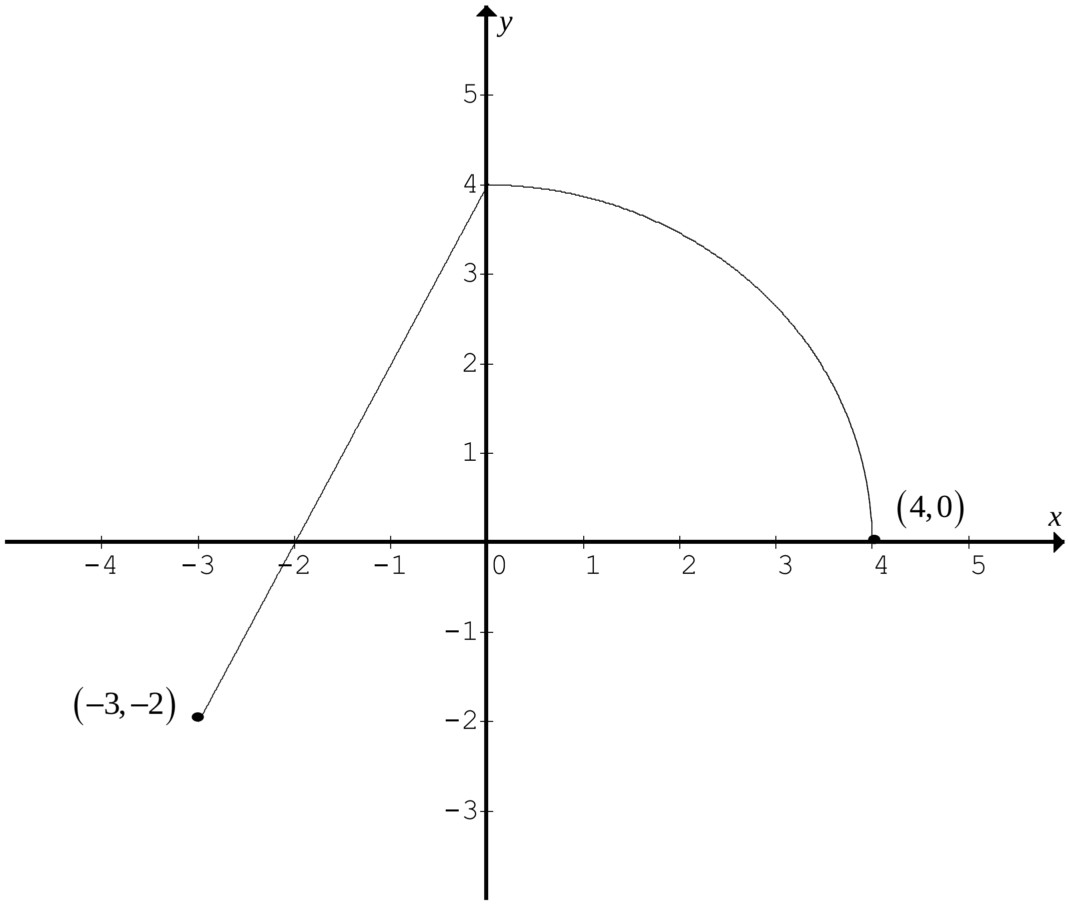
2 marks

e. If the vase is to be made with a minimum surface area and maximum volume, with $V = 9S$, find the values of x and h.

2 marks

Question 4 12 marks

The continuous function f is defined on the interval $[-3,4]$. The graph of the function f consists of one straight line segment and one quarter circle as shown below.



Let $g:[-3,4]\to R$, $g(x)=\int_0^x f(t)\,dt-2x$

a. Find the values of

i. $g(-2)$ 1 mark

ii. $g(4)$ 1 mark

iii. $g(-3)$ 1 mark

b. Find $g'(x)$, expressing your final answer in terms of *x*, not involving the function $f(x)$, clearly stating its domain.

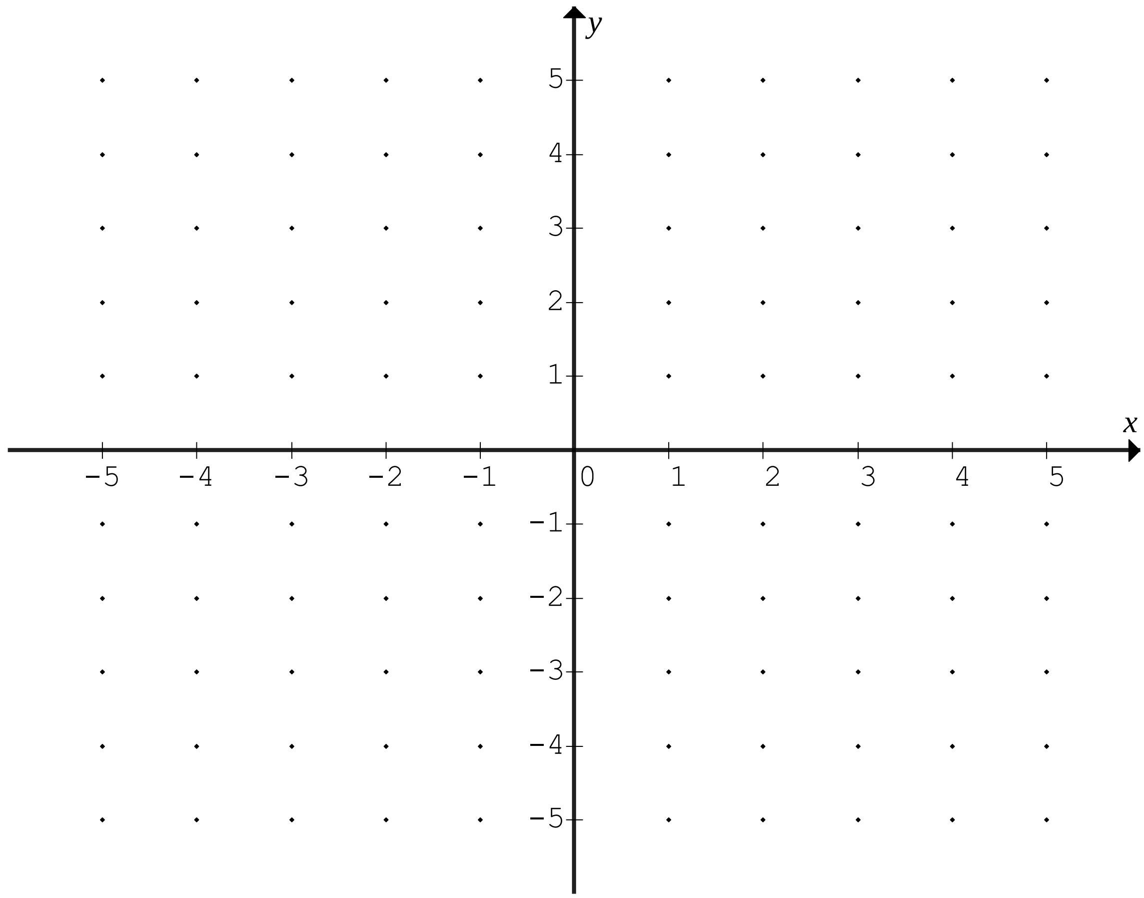
2 marks

c. Find the coordinates of the stationary points of the function $g(x)$.

4 marks

d. Find the coordinates of the point of inflexion of the function $g(x)$. 1 mark

e. Sketch the graph of the function $g(x)$ on the axes below. 2 marks



END OF EXAMINATION

EXTRA WORKING SPACE

MATHEMATICAL METHODS CAS

Written examination 2

FORMULA SHEET

Directions to students

Detach this formula sheet during reading time. This formula sheet is provided for your reference.

Mathematical Methods and CAS Formulas

Mensuration

area of a trapezium:	$\frac{1}{2}(a+b)h$	Volume of a pyramid:	$\frac{1}{3}Ah$
curved surface area of a cylinder:	$2\pi rh$	volume of a sphere:	$\frac{4}{3}\pi r^3$
volume of a cylinder:	$\pi r^2 h$	area of triangle:	$\frac{1}{2}bc\sin(A)$
volume of a cone:	$\frac{1}{3}\pi r^2 h$		

Calculus

$$\frac{d}{dx}\left(x^n\right) = nx^{n-1}$$

$$\frac{d}{dx}\left(e^{ax}\right) = ae^{ax}$$

$$\frac{d}{dx}\left(\log_e(x)\right) = \frac{1}{x}$$

$$\frac{d}{dx}\left(\sin(ax)\right) = a\cos(ax)$$

$$\frac{d}{dx}\left(\cos(ax)\right) = -a\sin(ax)$$

$$\frac{d}{dx}\left(\tan(ax)\right) = \frac{a}{\cos^2(ax)} = a\sec^2(ax)$$

$$\int x^n dx = \frac{1}{n+1}x^{n+1} + c\ ,\ n \neq -1$$

$$\int e^{ax} dx = \frac{1}{a}e^{ax} + c$$

$$\int \frac{1}{x} dx = \log_e|x| + c$$

$$\int \sin(ax)\, dx = -\frac{1}{a}\cos(ax) + c$$

$$\int \cos(ax)\, dx = \frac{1}{a}\sin(ax) + c$$

product rule: $\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$

quotient rule: $\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$

Chain rule: $\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx}$

approximation: $f(x+h) \approx f(x) + h f'(x)$

Probability

$$\Pr(A) = 1 - \Pr(A')$$

$$\Pr(A \cup B) = \Pr(A) + \Pr(B) - \Pr(A \cap B)$$

$$\Pr(A / B) = \frac{\Pr(A \cap B)}{\Pr(B)}$$

Transition Matrices $S_n = T^n \times S_0$

mean: $\mu = E(X)$

variance: $\operatorname{var}(X) = \sigma^2 = E\left((X-\mu)^2\right) = E(X^2) - \mu^2$

probability distribution		mean	variance
discrete	$\Pr(X = x) = p(x)$	$\mu = \sum x\, p(x)$	$\sigma^2 = \sum (x-\mu)^2\, p(x)$
continuous	$\Pr(a < X < b) = \int_a^b f(x)\,dx$	$\mu = \int_{-\infty}^{\infty} x\, f(x)\,dx$	$\sigma^2 = \int_{-\infty}^{\infty} (x-\mu)^2 f(x)\,dx$

ANSWER SHEET

STUDENT NUMBER

									Letter
Figures									
Words									

SIGNATURE ______________________________

SECTION 1

1	**A**	**B**	**C**	**D**	**E**
2	**A**	**B**	**C**	**D**	**E**
3	**A**	**B**	**C**	**D**	**E**
4	**A**	**B**	**C**	**D**	**E**
5	**A**	**B**	**C**	**D**	**E**
6	**A**	**B**	**C**	**D**	**E**
7	**A**	**B**	**C**	**D**	**E**
8	**A**	**B**	**C**	**D**	**E**
9	**A**	**B**	**C**	**D**	**E**
10	**A**	**B**	**C**	**D**	**E**
11	**A**	**B**	**C**	**D**	**E**
12	**A**	**B**	**C**	**D**	**E**
13	**A**	**B**	**C**	**D**	**E**
14	**A**	**B**	**C**	**D**	**E**
15	**A**	**B**	**C**	**D**	**E**
16	**A**	**B**	**C**	**D**	**E**
17	**A**	**B**	**C**	**D**	**E**
18	**A**	**B**	**C**	**D**	**E**
19	**A**	**B**	**C**	**D**	**E**
20	**A**	**B**	**C**	**D**	**E**
21	**A**	**B**	**C**	**D**	**E**
22	**A**	**B**	**C**	**D**	**E**