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Mathematical Methods (CAS)

2013

Trial Examination 1

Instructions

Answer **all** questions. Do **not** use CAS/calculators.
A decimal approximation will not be accepted if an **exact** answer is required to a question.
In questions where more than one mark is available, appropriate working must be shown.
Unless otherwise indicated, the diagrams in this exam are **not** drawn to scale.

Question 1 Consider quadratic function $f(x) = ax^2 + bx + c$ and linear function $g(x) = 2x + 3$.

a. Find $f'(x)$ in terms of a and b. 1 mark

b. Express $f(g(x))$ in the form $Ax^2 + Bx + C$. 2 marks

c. Find a value for each of a, b and c in $f(x) = ax^2 + bx + c$ such that $f(f'(x)) = f'(f(x))$. 3 marks

Question 2 A worker usually has 5 cups of drink with or without sugar each day. If he has sugar in one drink, the probability of having sugar in the next drink is 0.30. If he has no sugar in one drink, the probability of not having sugar in the next drink is 0.40.

a. Find the probability that he has sugar in his fifth drink if he does not have sugar in his third drink.
2 marks

b. Find out whether he has sugar or not in his first drink if the probability that he has sugar in his third drink is 0.42.
2 marks

Question 3 Let $f(x) = x^3 - 8x^2 - 4x + 32$.

a. Show that $f(x) = (x-2)(x+2)(x-8)$. 2 marks

b. Hence solve $2^{3y} - 2^{2y+3} - 2^{y+2} + 2^5 = 0$ for y. 3 marks

Question 4 Consider $y = \log_2 x$.

a. Find the gradient of the tangent to the graph of $y = \log_2 x$ at $x = 2$. 2 marks

b. Find the exact coordinates of the y-intercept of the tangent to the graph of $y = \log_2 x$ at $x = 2$. 2 marks

Question 5 Given $f(x) = e^{x-a} + 1$ and $a \in R^+$, find the value of a such that the graphs of $y = f^{-1}(x)$ and $y = f(x)$ touch each other (i.e. intersect at one point only). 3 marks

Question 6

a. Find the exact value of $\cos\left(\frac{\pi}{12} - \frac{3\pi}{4}\right)$. 1 mark

b. Hence solve $\sin\left(\frac{\pi}{2} - 3x\right) + \frac{1}{2} = 0$ for $\frac{\pi}{6} < x < \frac{\pi}{2}$. 2 marks

Question 7 The probability density function of random variable X is given by

$$f(x)=\begin{cases} 0, & x<0 \\ \dfrac{x+1}{4}, & 0\le x\le a \\ 0, & x>a \end{cases}$$

a. Show that $a=2$. 1 mark

b. Find the average value of $f(x)$ in the interval $0\le x\le 2$. 1 mark

c. Find the average value of X. 2 marks

Question 8 A small cube has one red face, two blue faces and three green faces. It is rolled three times.

a. Find the number of distinct probability values of the possible outcomes. 1 mark

b. Find the probability that each colour appeared once in three rolls of the cube. 2 marks

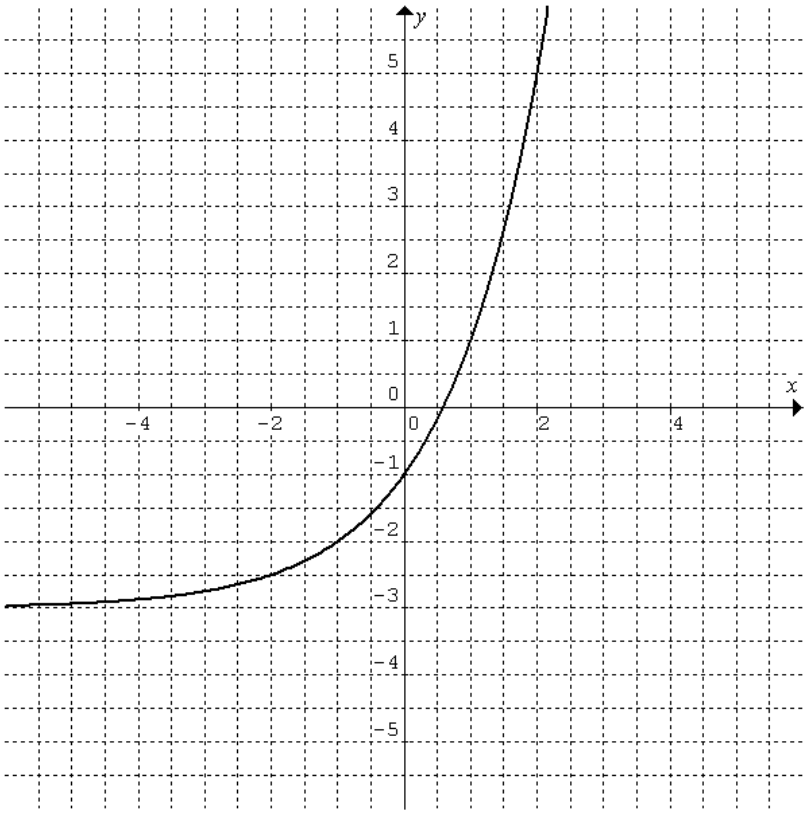
c. Find the probability that exactly one blue face appeared in three rolls of the cube. 2 marks

Question 9 Consider $f(x)=\frac{1}{5}\sin(5x-1)$.

a. Find the value of $f'(0.2)$. 1 mark

b. Use $f(a+h)\approx f(a)+h\times f'(a)$ to find the linear approximation of $f(0.21)$. 1 mark

Question 10 The graph of $y = 2 - f(1 - x)$ is shown below.



a. State a sequence of transformations on the graph of $y = 2 - f(1 - x)$ to produce the image given by the equation $x = 1 - f(2 - y)$. 3 marks

b. Sketch the graph of $x = 1 - f(2 - y)$ on the grid shown above. 1 mark

End of exam 1