INSIGHT

YEAR 12 *Trial Exam Paper*

2013

MATHEMATICAL METHODS (CAS)

Written examination 1

STUDENT NAME:

Reading time: 15 minutes
Writing time: 1 hour

QUESTION AND ANSWER BOOK

Structure of book

Number of questions	*Number of questions to be answered*	*Number of marks*
11	11	40

- Students are permitted to bring the following items into the examination: pens, pencils, highlighters, erasers, sharpeners and rulers.
- Students are NOT permitted to bring notes of any kind, sheets of paper, white out liquid/tape or a calculator into the examination.

Materials provided

- The question and answer book of 11 pages, with a separate sheet of miscellaneous formulas.
- Working space is provided throughout the question book.

Instructions

- Write your **name** in the box provided.
- Remove the formula sheet during reading time.
- You must answer the questions in English.

Students are NOT permitted to bring mobile phones or any other electronic devices into the examination.

Instructions

Answer **all** questions in the spaces provided.

A decimal approximation will not be accepted if an **exact** answer is required to a question.

In questions where more than one mark is available, appropriate working must be shown.

Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Question 1 (5 marks)

a. If $y = \sqrt{1+e^{2x}}$, find $\frac{dy}{dx}$.

1 mark

b. If $f(x) = \frac{x}{\cos(x)}$, find $f'(\pi)$.

2 marks

c. Find $\int_1^3 f(3x+1)\,dx$ if $\int_4^{10} f(x)\,dx = 3$.

2 marks

Question 2 (2 marks)

Consider these simultaneous linear equations.

$$ax - 2y = a$$
$$5x + y = 7$$

Find the value(s) of a for which the equations have no solution.

_____

_____

_____

_____

Question 3 (4 marks)

Consider the function $f : [-2, 2] \to R,\ f(x) = 3 - 3\sin(\pi x)$.

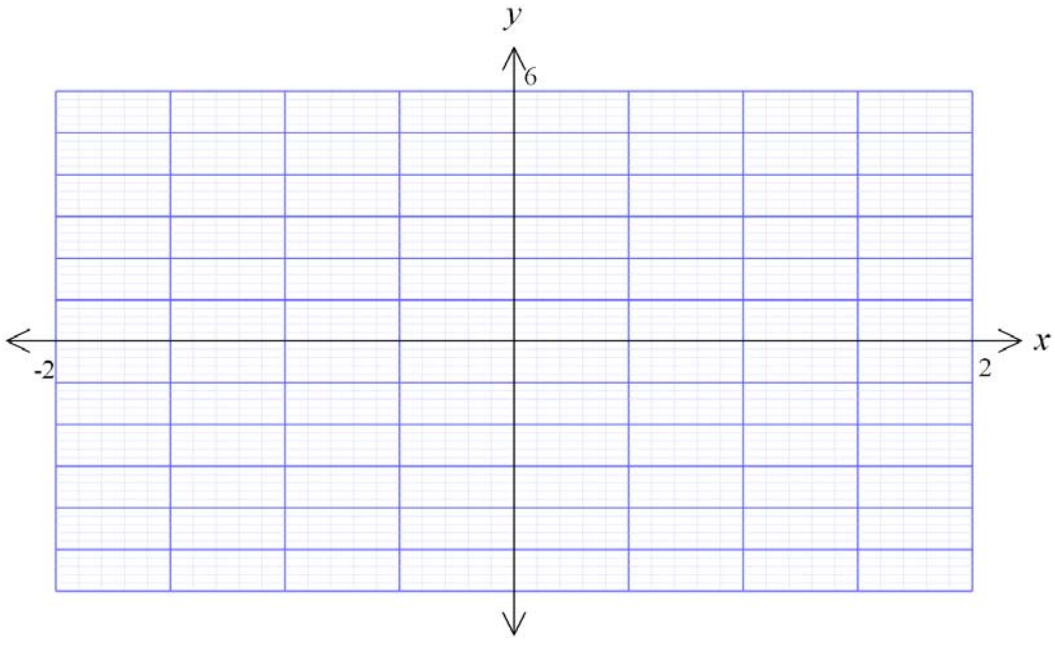
a. State the range and period of the function.

2 marks

_____

b. Sketch the graph of f on the axes below. Label axes intercepts and endpoints with co-ordinates.

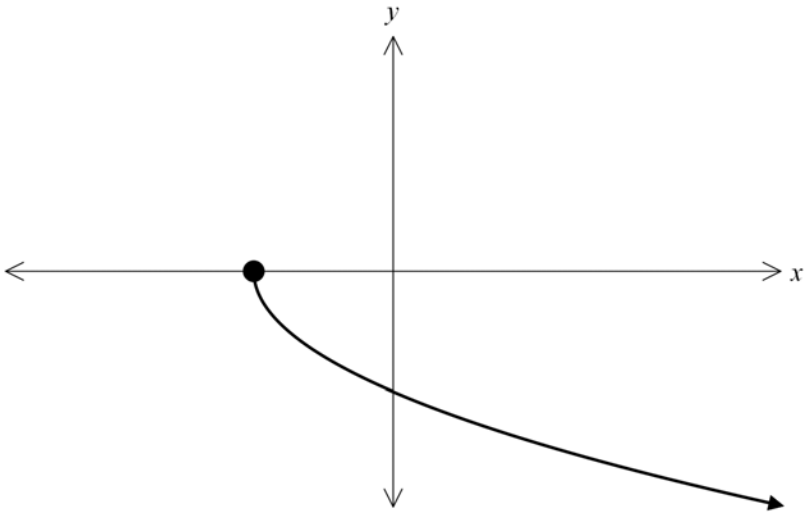
2 marks



TURN OVER

Question 4 (5 marks)

The function $f:[-4, \infty) \to R,\ f(x) = -3\sqrt{x+4}$ is sketched on the axes below.



a. On the same set of axes, sketch the graph of the inverse function f^{-1}.
Label axes intercepts with co-ordinates.

2 marks

b. Find the rule for the inverse function f^{-1}.

2 marks

_____

_____

_____

_____

c. State the domain of the inverse f^{-1}.

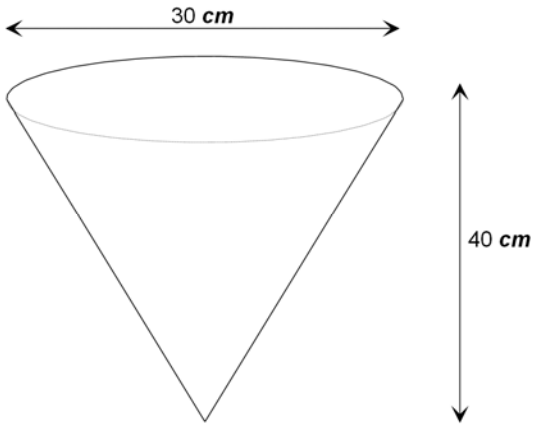
1 mark

_____

_____

Question 5 (3 marks)

An inverted right circular cone has a diameter of 30 cm and a height of 40 cm, as shown in the diagram below.



Water is poured into the cone at a rate of 6 cm^3 min^{-1}.
Find the rate at which the height is increasing when the height is 10 cm.

Question 6 (3 marks)

Let $f(x) = x^2 - 6|x| + 5$.

a. State the values of x for which $f(x)$ is differentiable.

1 mark

_____

_____

b. State a rule for the derivative function.

2 marks

_____

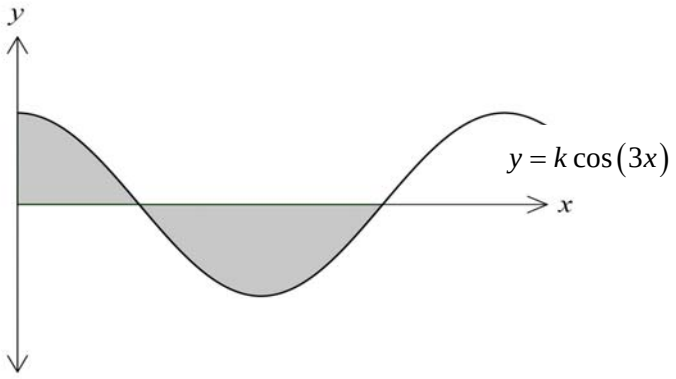
_____

_____

_____

Question 7 (3 marks)

The graph of the function $y = k\cos(3x)$, for $k \in R^+$ is shown below.



If the shaded area is 5π square units, find the value of k.

Question 8 (3 marks)

Max has either a cola or lemonade at morning tea break. If he has cola one morning, the probability he has cola the next is 0.3. If he has lemonade one morning, the probability he has lemonade the next is 0.4. Suppose he has lemonade on a Monday morning. What is the probability that he has cola on the following Wednesday morning?

Question 9 (2 marks)

Let X be a normally distributed random variable with a mean of 72 and a variance of 64.
Let Z be the standard normal random variable.

a. Find the value of c if $\Pr(Z > 1) = \Pr(X < c)$.

1 mark

b. Find the value of d if $\Pr(Z < d) = \Pr(X > 56)$.

1 mark

Question 10 (5 marks)

The probability density function of a continuous random variable X is given by

$$f(x) = \begin{cases} \frac{kx}{12} & 1 \le x \le 5 \\ 0 & \text{otherwise} \end{cases}$$

a. Show that $k = 1$.

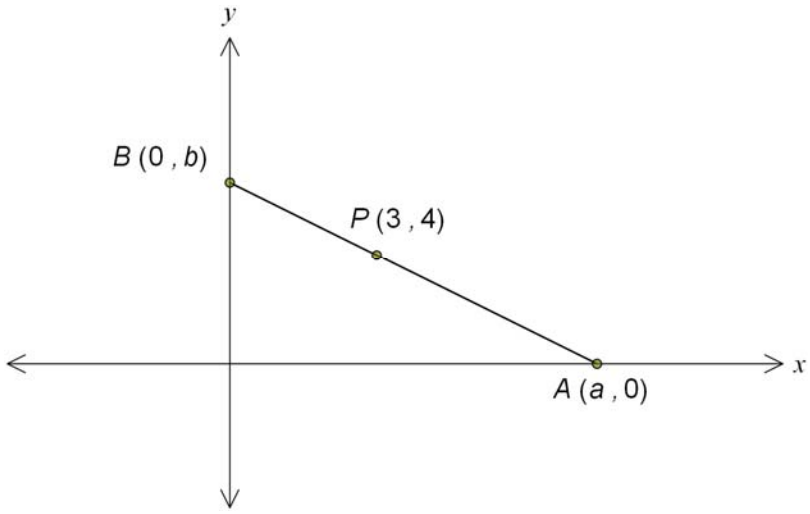
2 marks

b. Find $\Pr(X \le 2 \mid X < 3)$.

3 marks

Question 11 (5 marks)

AB is a line passing through the point (3, 4) with a *y*-intercept of (0, *b*) and an *x*-intercept of (*a*, 0), as shown in the diagram below.



a. Find the gradient of the line segments *PB* and *AP*. Hence, show that $b = \frac{4a}{a-3}$.

2 marks

b. Find the area of the triangle *OAB* in terms of *a*.

1 mark

c. Find the value of a such that the area of triangle OAB is a minimum.
(There is no need to verify that this is a minimum.)

2 marks

_____

_____

_____

_____

_____

_____

END OF QUESTION AND ANSWER BOOK