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Engage
Education
Foundation

Units 3 and 4 Maths Methods (CAS): Exam 1

Technology Free Practice Exam Question and Answer Booklet

Duration: 15 minutes reading time, 60 minutes writing time

Structure of book:

Number of questions	Number of questions to be answered	Number of marks
10	10	40

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers and rulers.
- Students are not permitted to bring into the examination room: blank sheets of paper and/or white out liquid/tape.
- No calculator is allowed in this examination.

Materials supplied:
- This question and answer booklet of 7 pages, including a formula sheet on the last page.

Instructions:
- You must complete all questions of the examination.
- Write all your answers in the spaces provided in this booklet.

Instructions

Answer all questions in the spaces provided.
In all questions where a numerical answer is required an exact value must be given unless otherwise specified.
In questions where more than one mark is available, appropriate working must be shown.
Unless otherwise indicated, the diagrams in this book are not drawn to scale.

Questions

Question 1

a. Differentiate $f(x) = xlog_e(x)$.

2 marks

b. Hence, find $\int_1^2 \log_e(x)dx$.

2 marks

Total: 4 marks

Question 2

A random variable X follows a binomial distribution with mean 5 and variance 4. Find n and p.

3 marks

Question 3

Find and classify the stationary points of $f(x) = |x^4 - x^2|$.

4 marks

Question 4

a. Solve $log_e(2) = log_2(x)$ for x.

2 marks

b. Solve $25^x - 5^{x+1} = -6$ for x.

3 marks

Total: 5 marks

Question 5

Consider the simultaneous equations containing the real constant k:

$$(k-1)x + y = 3$$
$$6x + ky = 3k$$

Find the values of k for which there are infinitely many solutions.

4 marks

Question 6

a. Solve $2\sin\left(x+\frac{\pi}{2}\right)+1=0$ for x over $x \in [0,2\pi]$.

3 marks

b. Hence or otherwise, state the solutions to the equation $4\cos\left(x+\frac{\pi}{3}\right)+5=3$ over $x \in [0,2\pi]$.

1 mark

Total: 4 marks

Question 7

Find the value of a such that the area bounded by $y=x^2$ and $y=ax$ is $^9/_2$.

4 marks

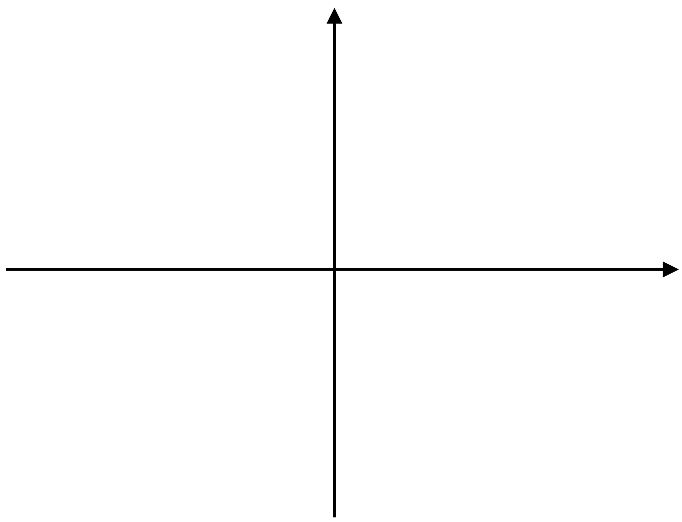
Question 8

A spherical balloon is being inflated at the rate $10\text{cm}^3/\text{s}$. At the point where the radius of the balloon is 5cm, find the rate of change of the radius with respect to time.

4 marks

Question 9

Sketch the functions $f(x) = e^x$, $g(x) = -e^{-x}$, and $(f + g)(x)$ on the axes below. State whether or not $(f + g)(x)$ has any intercepts or stationary points, and give their coordinates if it does.



4 marks

Question 10

a. Using left rectangles of width $\frac{1}{2}$, approximate the area under $y = \frac{1}{x^2}$ from $x = 2$ to $x = 3$. Give an exact decimal answer.

2 marks

b. Evaluate $\int_2^3 \frac{1}{x^2}\,dx$.

1 mark

c. Was your approximation from part a smaller or larger than the actual area? Why?

1 mark

Total: 4 marks

Formula sheet

Mensuration

area of a trapezium	$\frac{1}{2}(a+b)h$	volume of a pyramid	$\frac{1}{3}Ah$
curved surface area of a cylinder	$2\pi rh$	volume of a sphere	$\frac{4}{3}\pi r^3$
volume of a cylinder	$\pi r^2 h$	area of a triangle	$\frac{1}{2}bc\sin A$
volume of a cone	$\frac{1}{3}\pi r^2 h$		

Calculus

$$\frac{d}{dx}(x^n) = nx^{n-1}$$

$$\int x^n dx = \frac{1}{n+1}x^{n+1} + c, n \neq -1$$

$$\frac{d}{dx}(e^{ax}) = ae^{ax}$$

$$\int e^{ax} dx = \frac{1}{a}e^{ax} + c$$

$$\frac{d}{dx}(\log_e x) = \frac{1}{x}$$

$$\int \frac{1}{x} dx = \log_e|x| + c$$

$$\frac{d}{dx}(\sin(ax)) = a\cos(ax)$$

$$\int \sin(ax)\, dx = -\frac{1}{a}\cos(ax) + c$$

$$\frac{d}{dx}(\cos(ax)) = -a\sin(ax)$$

$$\int \cos(ax)\, dx = \frac{1}{a}\sin(ax) + c$$

$$\frac{d}{dx}(\tan(ax)) = \frac{a}{\cos^2(ax)} = a\sec^2(ax)$$

product rule $\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$

quotient rule $\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{\left(v\frac{du}{dx} - u\frac{dv}{dx}\right)}{v^2}$

chain rule $\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx}$

approximation $f(x+h) = f(x) + hf'(x)$

Probability

$\Pr(A) = 1 - \Pr(A')$

$\Pr(A \cup B) = \Pr(A) + \Pr(B) - \Pr(A \cap B)$

$\Pr(A|B) = \frac{\Pr(A \cap B)}{\Pr(B)}$

transition matrices $S_n = T^n \times S_0$

mean $\mu = E(X)$

variance $var(X) = \sigma^2 = E((X-\mu)^2) = E(X^2) - \mu^2$

probability distribution		mean	variance
discrete	$\Pr(X = x) = p(x)$	$\mu = \Sigma xp(x)$	$\sigma^2 = \Sigma(x-\mu)^2 p(x)$
continuous	$\Pr(a < X < b) = \int_a^b f(x)dx$	$\mu = \int_{-\infty}^{\infty} xf(x)dx$	$\sigma^2 = \int_{-\infty}^{\infty} (x-\mu)^2 f(x)dx$