Neap:

Trial Examination 2012

# VCE Mathematical Methods (CAS) Units 3 & 4

Written Examination 1

## Suggested Solutions

**Question 1**

**a.** Using the product rule, i.e. $u = x$, $\frac{du}{dx} = 1$, $v = \sin^2(x)$ and $\frac{dv}{dx} = 2\sin(x)\cos(x)$. M1

Hence $h'(x) = 2x\sin(x)\cos(x) + \sin^2(x)$. A1

$\sin\left(\frac{\pi}{6}\right) = \frac{1}{2}$ and $\cos\left(\frac{\pi}{6}\right) = \frac{\sqrt{3}}{2}$

$h'\left(\frac{\pi}{6}\right) = 2 \times \frac{\pi}{6} \times \frac{1}{2} \times \frac{\sqrt{3}}{2} + \left(\frac{1}{2}\right)^2$

$h'\left(\frac{\pi}{6}\right) = \frac{\sqrt{3}\pi}{12} + \frac{1}{4}$ A1

**Question 2**

$\int_{\sqrt{2}}^{2} \frac{1}{3-x}dx = [-\log_e|3-x|]_{\sqrt{2}}^{2}$ A1

$= -\log_e(1) + \log_e(3-\sqrt{2})$

$= \log_e(3-\sqrt{2})$

So, $\log_e(3-\sqrt{2}) = \log_e(k)$ and hence $k = 3-\sqrt{2}$. A1

**Question 3**

**a.** $f(g(x)) = \frac{1}{(x+2)^2}$ A1

$\frac{1}{(x+2)^2} = 4$ and so $(x+2)^2 = \frac{1}{4}$.

$x + 2 = \pm\frac{1}{2}$

Hence $x = -\frac{5}{2}, -\frac{3}{2}$. A1

**b.** Interchanging $x$ and $y$ we obtain $x = \frac{1}{y+2}$.

Solving for $y$ we obtain $y = \frac{1}{x} - 2$, i.e. $g^{-1}(x) = \frac{1}{x} - 2$. A1

The domain of $g^{-1}$ is $R\backslash\{0\}$ and the range of $g^{-1}$ is $R\backslash\{-2\}$. A1

**Question 4**

Using $\cos\left(\pm\frac{\pi}{4}\right) = \frac{1}{\sqrt{2}}$. A1

More generally, $\cos\left(\pm\frac{\pi}{4} + 2k\pi\right) = \frac{1}{\sqrt{2}}$. M1

Since $\cos\left(\frac{x}{2} + \frac{\pi}{3}\right) = \frac{1}{\sqrt{2}}$

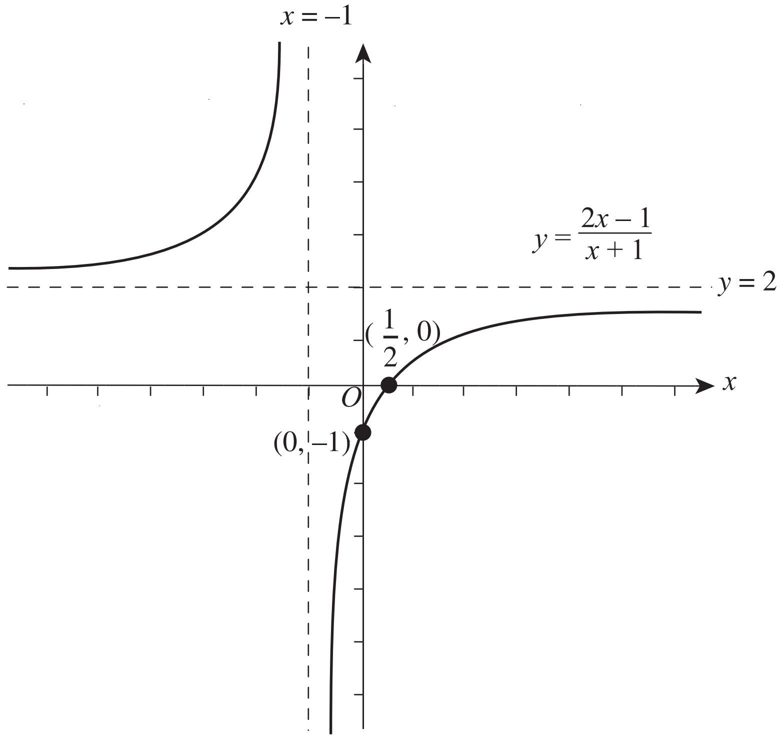
$$\frac{x}{2} + \frac{\pi}{3} = \pm\frac{\pi}{4} + 2k\pi$$

So $x = -\frac{2\pi}{3} \pm \frac{\pi}{2} + 4k\pi$ M1

Substituting $k = 1$ gives $x = \frac{17\pi}{6}$. A1

**Question 5**

**a.**




Attempting to express $y = \frac{2x-1}{x+1}$ as $y = 2 - \frac{3}{x+1}$. M1

Asymptotes $x = -1$, $y = 2$. A1

Axes intercepts $(0, -1)$, $\left(\frac{1}{2}, 0\right)$. A1

Two correct branches. A1

**b.** Referring to the graph, $0 < y < 2$ for $x > \frac{1}{2}$ A1

**Question 6**

From $\int_0^1 (ax+b)dx = 1$ we obtain $\left[\frac{ax^2}{2} + bx\right]_0^1 = 1$.

Hence $\frac{a}{2} + b = 1$. (1) A1

From $\int_0^1 x(ax+b)dx = \frac{7}{12}$ we obtain $\left[\frac{ax^3}{3} + \frac{bx^2}{2}\right]_0^1 = \frac{7}{12}$.

Hence $\frac{a}{3} + \frac{b}{2} = \frac{7}{12}$. (2) A1

Attempting elimination or substitution, for example $2 \times (2) - (1)$ gives $\frac{a}{6} = \frac{1}{6}$. M1

Hence $a = 1$ and $b = \frac{1}{2}$. A1

**Question 7**

Using $\Pr(A \cap B) = \Pr(A|B) \times \Pr(B)$ we obtain $\Pr(A \cap B) = \frac{3}{5} \times \frac{1}{5}$ i.e. $\Pr(A \cap B) = \frac{3}{25}$. M1

Now $\Pr(B|A) = \frac{\Pr(A \cap B)}{\Pr(A)}$ and so $\Pr(B|A) = \frac{\frac{3}{25}}{\frac{2}{5}}$, i.e. $\Pr(B|A) = \frac{3}{10}$. M1 A1

**Question 8**

Given that $\Sigma \Pr(X = x) = 1$ we obtain $5p^2 + 4p - 1 = 0$. M1

Factorising the LHS we obtain $(p+1)(5p-1) = 0$.

We reject $p = -1$ since $0 < p < 1$, and so $p = \frac{1}{5}$. A1

**Question 9**

**a.** Given $y = 4 - e^{2x}$ we obtain $\frac{dy}{dx} = -2e^{2x}$.

At $x = \log_e(2)$, $\frac{dy}{dx} = -2e^{2\log_e(2)}$ i.e. $\frac{dy}{dx} = -8$.

So the gradient of the normal is $\frac{1}{8}$. M1

Using $y - y_1 = m(x - x_1)$ we obtain $y = \frac{1}{8}(x - \log_e(2))$.

Hence the equation of the normal is $y = \frac{1}{8}x - \frac{1}{8}\log_e(2)$. A1

**b.** The area bounded by the curve and the two axes is given by

$$\int_0^{\log_e(2)} (4 - e^{2x})dx = \left[4x - \frac{1}{2}e^{2x}\right]_0^{\log_e(2)}$$

$$= \left[4\log_e(2) - \frac{1}{2}e^{2\log_e(2)}\right] - \left[0 - \frac{1}{2}e^0\right] \quad \text{M1}$$

$$= 4\log_e(2) - 2 + \frac{1}{2}$$

$$= 4\log_e(2) - \frac{3}{2}$$

The area of the triangle bounded by the normal and the two axes can be calculated as follows:

When $x = 0$, $y = -\frac{1}{8}\log_e(2)$.

The area of the triangle is $\frac{1}{2} \times \frac{1}{8}\log_e(2) \times \log_e(2)$, i.e. $\frac{1}{16}(\log_e(2))^2$.

So the total area is $4\log_e(2) - \frac{3}{2} + \frac{1}{16}(\log_e(2))^2$ (square units). A1 A1

**Question 10**

**a.** Given $f(x) = x + \sqrt{x^2 + 1}$,

$$= x + (x^2 + 1)^{\frac{1}{2}}$$

$$f'(x) = 1 + \frac{1}{2}(2x)(x^2 + 1)^{-\frac{1}{2}} \quad \text{M1}$$

$$f'(x) = 1 + \frac{x}{\sqrt{x^2 + 1}} \quad \text{A1}$$

**b.** Given that $g(x) = \log_e(f(x))$, $g'(x) = \dfrac{f'(x)}{f(x)}$.

So, $g'(x) = \dfrac{1 + \dfrac{x}{\sqrt{x^2 + 1}}}{x + \sqrt{x^2 + 1}}$ M1

$= \dfrac{\dfrac{\sqrt{x^2 + 1} + x}{\sqrt{x^2 + 1}}}{x + \sqrt{x^2 + 1}}$ A1

$= \dfrac{\sqrt{x^2 + 1 + x}}{\sqrt{x^2 + 1}} \times \dfrac{1}{x + \sqrt{x^2 + 1}}$

$= \dfrac{1}{\sqrt{x^2 + 1}}$

Hence $g'(x) = \dfrac{1}{\sqrt{x^2 + 1}}$. A1

**c.** As $g(x) = \log_e(f(x))$, $\displaystyle\int g'(x)dx = \log_e(f(x)) + c$.

So, $\displaystyle\int_0^1 \frac{1}{\sqrt{x^2 + 1}}\,dx = [\log_e(x + \sqrt{x^2 + 1})]_0^1$ A1

$= \log_e(1 + \sqrt{2})$ A1

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