Year 2012

VCE

Mathematical Methods CAS

Trial Examination 2

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Victorian Certificate of Education
2012

STUDENT NUMBER

									Letter
Figures									
Words									

MATHEMATICAL METHODS CAS
Trial Written Examination 2

Reading time: 15 minutes
Total writing time: 2 hours

QUESTION AND ANSWER BOOK

Structure of book

Section	*Number of questions*	*Number of questions to be answered*	*Number of marks*
1	22	22	22
2	4	4	58
			Total 80

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers, a protractor, set-squares, aids for curve sketching, one bound reference, one approved CAS calculator (memory DOES NOT need to be cleared) and, if desired, one scientific calculator. For approved computer-based CAS, their full functionality may be used.
- Students are NOT permitted to bring into the examination room: blank sheets of paper and/or white out liquid/tape.

Materials supplied

- Question and answer booklet of 30 pages with a detachable sheet of miscellaneous formulas at the end of this booklet.
- Answer sheet for multiple-choice questions.

Instructions

- Detach the formula sheet from the end of this book during reading time.
- Write your **student number** in the space provided above on this page.
- Write your **name** and **student number** on your answer sheet for multiple-choice questions, and sign your name in the space provided.
- All written responses must be in English.

At the end of the examination

- Place the answer sheet for multiple-choice questions inside the front cover of this book.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

SECTION 1

Instructions for Section I

Answer **all** questions in pencil on the answer sheet provided for multiple-choice questions.
Choose the response that is **correct** for the question.
A correct answer scores 1 mark, an incorrect answer scores 0.
Marks will **not** be deducted for incorrect answers.
No mark will be given if more than one answer is completed for any question.

Question 1

The graph of the function $y = f(x)$ passes through the point $(-1,2)$. The graph of the function $y = g(x)$ passes through the point $(-2,1)$. Then

A. $g(x) = f(x-1)-1$

B. $g(x) = f(x+1)-1$

C. $g(x) = 1-f(x-1)$

D. $g(x) = 1-f(1-x)$

E. $g(x) = f^{-1}(x)$

Question 2

Given the system of linear simultaneous equations
$-2x+(p-1)y=4$ and $px-6y=-2p$ where p is a real constant, then

A. There is an infinite number of solutions when $p=-3$ or when $p=4$.

B. There is a unique solution when $p=-3$.

C. There is a unique solution when $p=4$.

D. There is no solution when $p=-3$.

E. There is no solution when $p=4$.

Question 3

The graph of $y = kx - 2$ intersects the graph of $y = x^2 + 5x$ at two distinct points for

A. $k = 3$

B. $k = 8$

C. $5 - 2\sqrt{2} < k < 5 + 2\sqrt{2}$

D. $k > 5 + 2\sqrt{2}$ or $k < 5 - 2\sqrt{2}$

E. $3 < k < 8$

Question 4

Let $f(x) = x^3$. If *a* and *b* are non-zero real numbers, then which of the following is **false?**

A. $f(ab) = f(a)f(b)$

B. $\dfrac{f(a)}{f(b)} = f\left(\dfrac{a}{b}\right)$

C. $f(b) + f(-b) = 0$

D. $f(2a) = 8f(a)$

E. $f(a+b) = f(a) + f(b)$

Question 5

Consider the function $f: R \to R,\ f(x) = 2(x-5)^n + 4$. Which of the following is **false?**

A. If $n = 4$, the point $(5, 4)$ is a minimum turning point.

B. If $n = 3$, the point $(5, 4)$ is a stationary point of inflexion.

C. If $n = \dfrac{1}{2}$, the line $x = 5$ is a vertical asymptote.

D. If $n = -1$, the line $x = 5$ is a vertical asymptote.

E. If $n = -2$, the range of the function is $(4, \infty)$.

Question 6

If f and g are two continuous and differentiable functions with the following properties

$f(2)=1 \quad f(3)=4 \quad f'(2)=6 \quad f'(3)=7$

$g(2)=3 \quad g(3)=8 \quad g'(2)=5 \quad g'(3)=2$. If $h(x)=f(g(x))$ then

A. $h(2)=4$ and $h'(2)=12$

B. $h(2)=4$ and $h'(2)=35$

C. $h(2)=4$ and $h'(2)=105$

D. $h(2)=3$ and $h'(2)=30$

E. $h(2)=3$ and $h'(2)=90$

Question 7

If $f(x)=\dfrac{g(x)}{\sqrt{x}}$ and $g(4)=8$ and $g'(4)=6$, then $f'(4)$ is equal to

A. 5

B. 3

C. $\dfrac{5}{2}$

D. $\dfrac{3}{2}$

E. $-\dfrac{3}{8}$

Question 8

The equation $\left|x^2-k^2\right|=k$ where $k>1$ has

A. no solutions.

B. one solution.

C. two solutions.

D. three solutions.

E. four solutions.

Question 9

Which of the following is **not** a continuous function?

A. $f(x)=\begin{cases} x^2-3 & \text{for } x<2 \\ 1 & \text{for } x\geq 2 \end{cases}$

B. $f(x)=\begin{cases} \sqrt{2x-3} & \text{for } x<2 \\ 1 & \text{for } x\geq 2 \end{cases}$

C. $f(x)=\begin{cases} \dfrac{1}{(2x-3)^2} & \text{for } x<2 \\ 1 & \text{for } x\geq 2 \end{cases}$

D. $f(x)=\begin{cases} \log_2(x) & \text{for } x<2 \\ 1 & \text{for } x\geq 2 \end{cases}$

E. $f(x)=\begin{cases} \sin\left(\dfrac{\pi x}{4}\right) & \text{for } x<2 \\ 1 & \text{for } x\geq 2 \end{cases}$

Question 10

If $a^{2x}=\dfrac{1}{b}$ where $a>1$ and $b>1$, then which of the following is **not** true?

A. $x=-\dfrac{1}{2}\log_a b$

B. $x=\dfrac{-1}{2\log_b a}$

C. $x=-\dfrac{\log_{10} b}{\log_{10} a^2}$

D. $x=\dfrac{\ln\left(\sqrt{b}\right)}{\ln(a)}$

E. $x=\log_a\left(\dfrac{1}{\sqrt{b}}\right)$

Question 11

The inverse of $f:(-\infty,0)\to R$, $f(x)=\frac{1}{x^2}+a$ where $a>0$ is

A. $f^{-1}:R\backslash\{a\}\to R$, $f^{-1}(x)=x^2+\frac{1}{a}$

B. $f^{-1}:R\backslash\{a\}\to R$, $f^{-1}(x)=\frac{1}{\sqrt{x+a}}$

C. $f^{-1}:(a,\infty)\to R$, $f^{-1}(x)=\frac{1}{\sqrt{x-a}}$

D. $f^{-1}:(a,\infty)\to R$, $f^{-1}(x)=\frac{-1}{\sqrt{x-a}}$

E. $f^{-1}:[a,\infty)\to R$, $f^{-1}(x)=-\sqrt{x-a}$

Question 12

The graph of $y=a-\frac{ax}{x-a}$ where $a\in R\backslash\{0\}$ has asymptotes at

A. $x=a$ only.

B. $y=a$ only.

C. $x=a$ and $y=a$.

D. $x=-a$ and $y=a$.

E. $x=a$ and $y=0$.

Question 13

An ice block in the shape of a cube, is placed in a glass of lemonade. It melts so that its volume is decreasing at a rate of $24\ \text{cm}^3/\text{minute}$. When the side length is 2 cm, the rate in $\text{cm}^2/\text{minute}$ at which its surface area is decreasing at, is equal to

A. 48

B. 24

C. 12

D. 6

E. 4

Question 14

The average rate of change of the function with the rule $f(x) = e^{-2x}\cos\left(\frac{\pi x}{4}\right)$ over $0 \le x \le 2$ is equal to

A. $\frac{\sqrt{2}}{2e^2}$

B. 0

C. $-\frac{1}{2}$

D. –2

E. $-\frac{\pi}{4e^4}$

Question 15

The graph of $y = e^{-\frac{x}{2}}\sin\left(\frac{x}{2}\right)$

A. crosses the *x*-axis at $x = (2n+1)\frac{\pi}{2}$ and has turning points at $x = (4n-3)\frac{\pi}{2}$ where $n \in Z$.

B. crosses the *x*-axis at $x = 2n\pi$ and has turning points at $x = (4n-3)\frac{\pi}{2}$ where $n \in Z$.

C. crosses the *x*-axis at $x = (2n+1)\frac{\pi}{2}$ and has turning points at $x = (2n-1)\frac{\pi}{2}$ where $n \in Z$.

D. crosses the *x*-axis at $x = 2n\pi$ and has turning points at $x = (2n-1)\frac{\pi}{2}$ where $n \in Z$.

E. crosses the *x*-axis at $x = (2n-1)\frac{\pi}{2}$ and has turning points at $x = 2n\pi$ where $n \in Z$.

Question 16

Consider the function $f:(0,8)\to R$, $f(x)=2\left(\sin\left(\frac{\pi(x-4)}{4}\right)\right)^2-1$.

Which of the following is **false?**

A. The function is increasing for $x\in(0,2)\cup(4,6)$.

B. The function is decreasing for $x\in(2,4)\cup(6,8)$.

C. The graph crosses the *x*-axis at $x=1,3,5,7$.

D. The graph has turning points at $x=2,4,6$.

E. The range of the function is $[-3,1]$.

Question 17

The equation of the normal to the curve $y=\tan\left(\frac{x}{2}\right)$ at the point where $x=\frac{\pi}{3}$ is given by

A. $y=-\frac{3x}{2}+\frac{\pi}{2}+\frac{\sqrt{3}}{3}$

B. $y=-\frac{3x}{2}+\frac{\pi}{2}+\sqrt{3}$

C. $y=\frac{3x}{2}-\frac{\pi}{2}+\frac{\sqrt{3}}{3}$

D. $y=\frac{2x}{3}-\frac{2\pi}{9}+\sqrt{3}$

E. $y=\frac{2x}{3}-\frac{2\pi}{9}+\frac{\sqrt{3}}{3}$

Question 18

It is known that 45% of all people aged over 85 years of age have Alzheimer's disease. A nursing home contains 20 elderly residents over 85 years of age. The probability that the nursing home contains more residents with Alzheimer's disease, than expected is closest to

A. 0.159

B. 0.177

C. 0.250

D. 0.409

E. 0.586

Question 19

The time taken to run a fun run, is found to be normally distributed, with a mean of four hours and a standard deviation of 35 minutes. 50% of the competitors complete the run with times in minutes between

A. 222 and 258.

B. 216 and 264.

C. 205 and 275.

D. 170 and 310.

E. 135 and 345.

Question 20

Given the continuous probability distribution defined by

$$f(x) = \begin{cases} k\,x^3 \sin(2x) & \text{for } 0 \le x \le \dfrac{\pi}{2} \\ 0 & \text{elsewhere} \end{cases}$$

Then the mode is closest to

A. 1.11

B. 1.14

C. 1.23

D. 1.29

E. 1.32

Question 21

Given the probability distribution of X, where X is the number of matches in a matchbox.

X	49	50	51	52	53	54
$\Pr(X = x)$	c	$4c$	$3c$	$3c$	$4c$	$5c$

Then

A. the mean is 52, the median is 52 and the mode is 54.

B. the mean is 52, the median is 53 and the mode is 54.

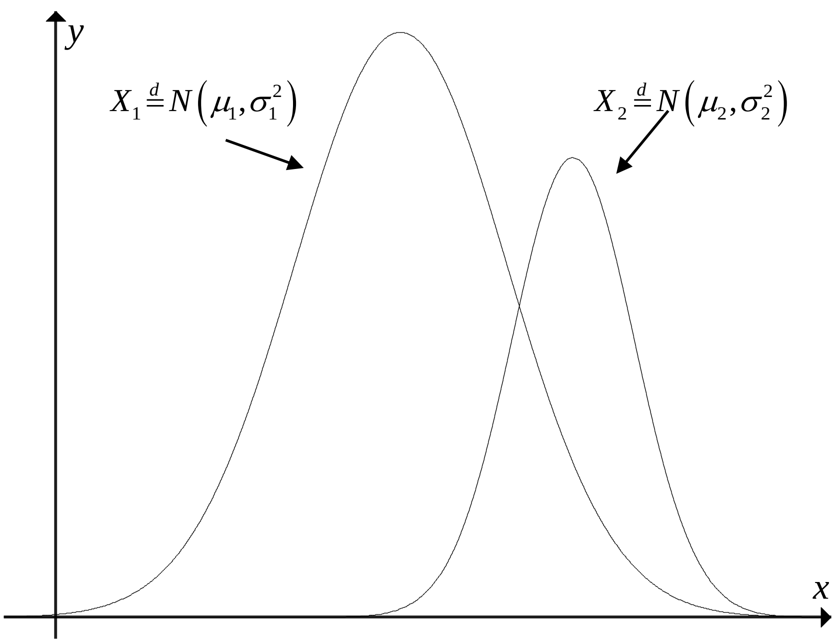
C. the mean is 52, the median is 54 and the mode is 54.

D. the mean is 51, the median is 53 and the mode is 53.

E. the mean is 51, the median is 53 and the mode is 53.

Question 22

Given the graphs of two normal distributions, then



A. $\mu_2 < \mu_1$ and $\sigma_1 = \sigma_2$

B. $\mu_2 < \mu_1$ and $\sigma_1 < \sigma_2$

C. $\mu_2 < \mu_1$ and $\sigma_1 > \sigma_2$

D. $\mu_2 > \mu_1$ and $\sigma_1 < \sigma_2$

E. $\mu_2 > \mu_1$ and $\sigma_1 > \sigma_2$

END OF SECTION 1

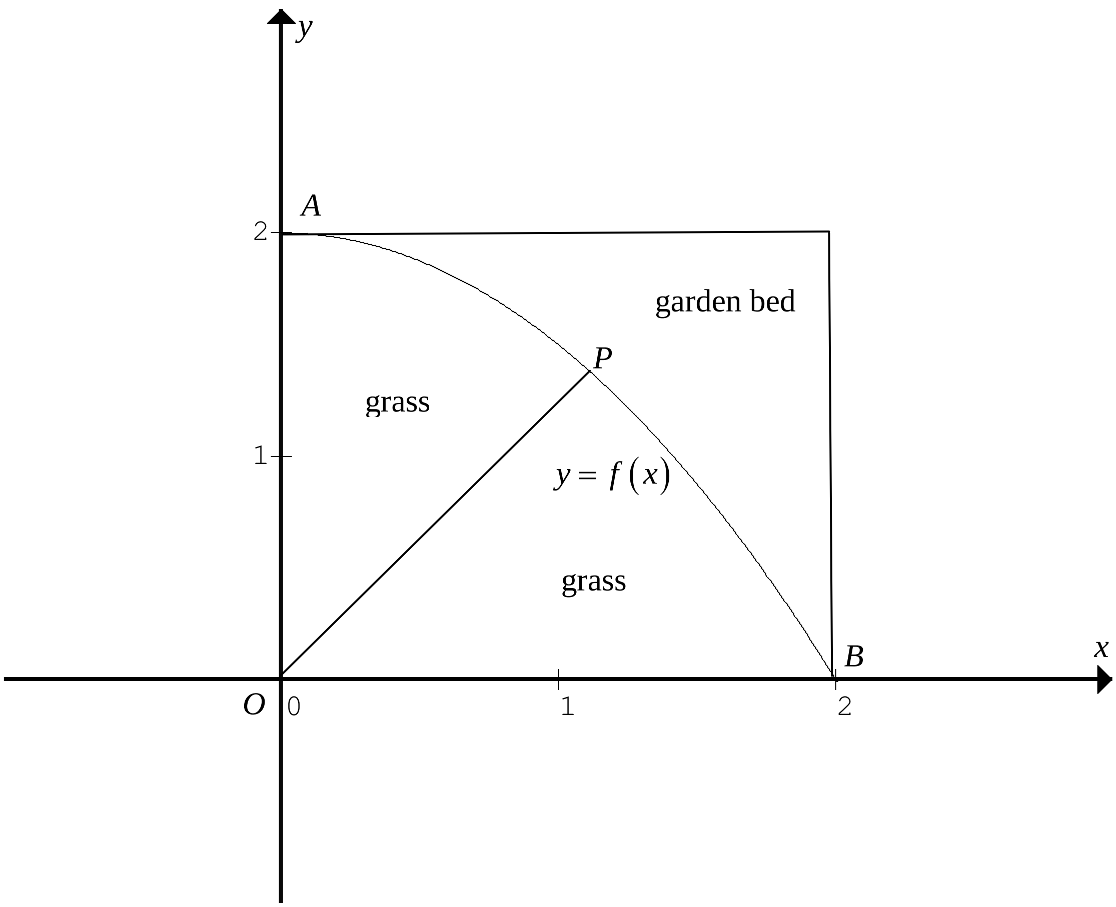
SECTION 2

Instructions for Section 2

Answer **all** questions in the spaces provided.
In all questions where a numerical answer is required an exact value must be given unless otherwise specified.
In questions where more than one mark is available, appropriate working **must** be shown.
Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Question 1

A square garden patch has a length of 2 metres and a width of 2 metres as shown in the diagram below.



Coordinate axes have been placed with the origin O at the bottom left-hand corner, with units in metres. The lower section is to contain grass, while the upper section is to contain a garden bed as shown on the diagram. The patch is divided by a curve having the form $y = f(x)$, which passes through the points $A(0,2)$ and $B(2,0)$. A straight line path OP of distance $s(x)$ is required from the origin to a point P on the curve with co-ordinates $(x, f(x))$, where $0 < x < 2$.

a. One model for the curve is a function given by $f:[0,2] \to R$, $f(x) = \sqrt{p+qx}$, for this model,

i. show that $p = 4$ and $q = -2$.

1 mark

ii. Find the area of the grassed section in square metres.

1 mark

iii. Show that $s(x) = \sqrt{x^2 + 4 - 2x}$.

1 mark

iv. Find $\frac{ds}{dx}$ and hence find the value of x for which the distance $s(x)$ is a minimum.

Determine this minimum distance.

3 marks

b. Another model for the curve is a function given by $f:[0,2] \to R$, $f(x) = 4 - e^{kx} - e^{-kx}$ where k is a positive constant.

i. By solving an appropriate equation, show that $k = \frac{1}{2}\log_e\left(2+\sqrt{3}\right)$.

3 marks

ii. The area of the garden bed in square metres can be expressed in the form $\frac{a\sqrt{b}}{k}+c$.

State the values of the integers *a, b* and *c*.

1 mark

iii. Find the value of *x* for which the distance $s(x)$ is a minimum and determine the minimum distance, giving both answers correct to three decimal places.

3 marks

Total 13 marks

Question 2

A survey was carried out, to find out the shaving habits of men. The survey investigated if men, who shaved once every day, used either a hand razor or an electric razor.

a. For one group of men who shave once every day, it was found that if they used a hand razor one day, the probability that they used an electric razor the next day was 0.15, while if they used an electric razor one day, the probability that they used a hand razor the next day was 0.25. Given that on Monday a man from this group used an electric razor, find the probability, giving all answers correct to three significant figures, that

i. from Monday to Thursday he used an electric razor exactly three times.

1 mark

ii. on Thursday, he used an electric razor.

1 mark

iii. for this group of men find the long term probability of using an electric razor.

1 mark

b. Of the men who shave once every day, let *p* be the probability that they use a hand razor. Out of a group of 20 men,

i. if $p = 0.4$, find the probability, correct to four decimal places that from 8 to 12 men inclusive, use a hand razor.

1 mark

ii. Assuming that less than 10 men use a hand razor, find the value of *p*, correct to four decimal places, if the probability that from 9 to 11 men inclusive who use a hand razor is equal to 0.25.

2 marks

c. The time taken to clean an electric razor, was found to be normally distributed. 16% of men take less than 130 seconds to clean the razor, while 18% of men take longer than 264 seconds to clean the razor. Find the mean and standard deviation of the times taken, to clean an electric razor, giving your answers to the nearest second.

3 marks

d. The time T in minutes, taken to shave using a hand razor, was found to satisfy a probability density function, defined by $T(t)=\begin{cases} k\,t\,e^{-\frac{t^2}{8}} & \text{for } t \ge 0 \\ 0 & \text{elsewhere} \end{cases}$

i. Explain why $k=\frac{1}{4}$.

1 mark

ii. Find the probability that men who use a hand razor take less than three minutes to shave. Give your answer correct to four decimal places.

2 marks

iii. Find the mean and standard deviation of the shaving times, in minutes correct to two decimal places, for men who use a hand razor.

2 marks

iv. Find the median time in minutes correct to three decimal places, for men who use a hand razor.

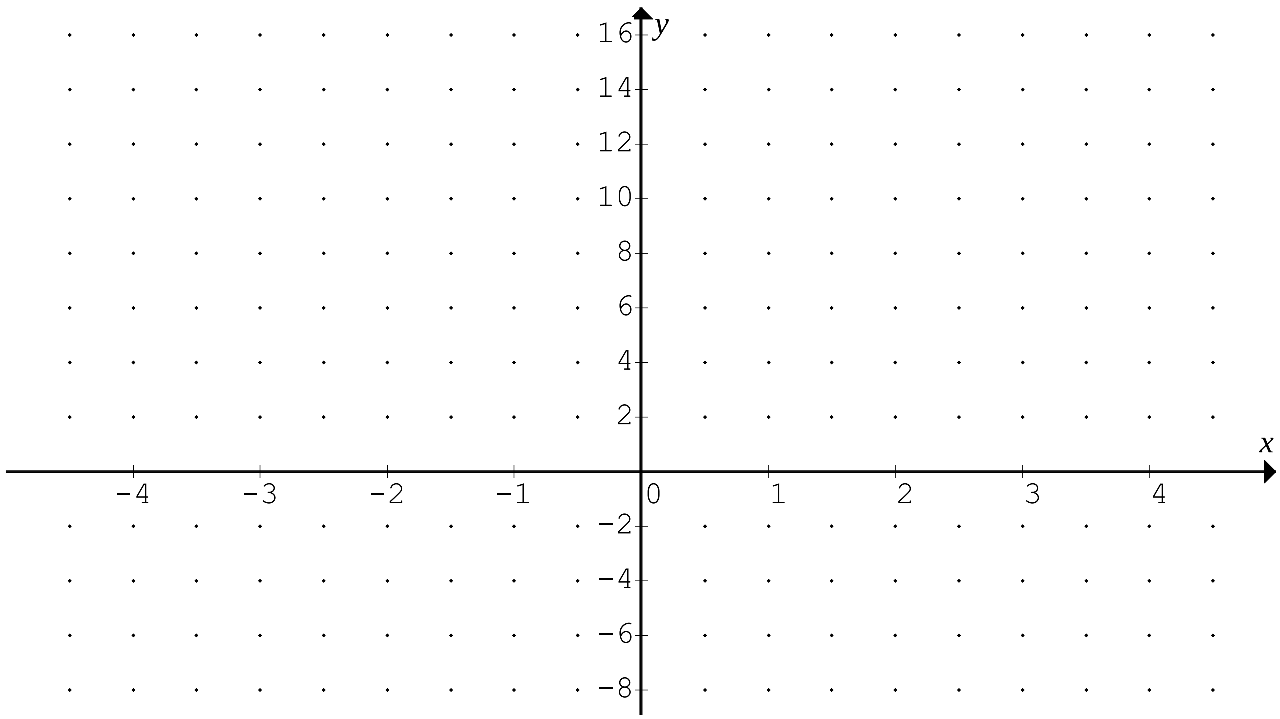
1 mark

Total 15 marks

Question 3

a. Consider the function $f:[-4,4]\rightarrow R$, $f(x)=|2x+3|+|2x-3|$

i. Sketch the graph of $y=f(x)$ on the axes below, clearly indicating the coordinates of the end-points and any axial intercepts.



2 marks

ii. Sketch the graph of $y=f'(x)$ on the axes above.

2 marks

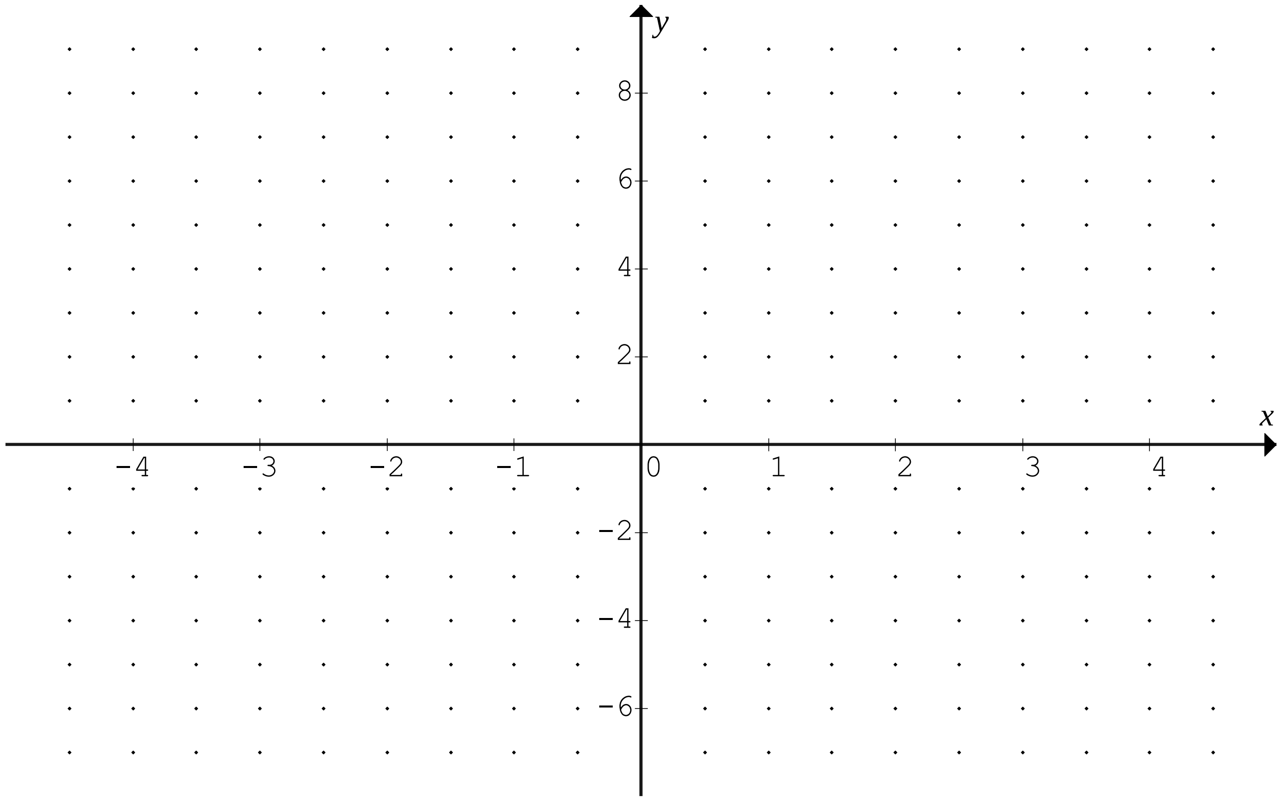
b. Consider now the function $f:R\rightarrow R$, $f(x)=|ax+b|+|ax-b|$, where *a* and *b* are both positive real constants. Complete the following to specify the gradient function $f'(x)$ in terms of *a* and *b* as a hybrid function.

$$f'(x)=\begin{cases} & \text{for } x\in \\ & \text{for } x\in \\ & \text{for } x\in \end{cases}$$

2 marks

c. Consider the function $g:[-4,4]\to R$, $g(x)=|2x+3|-|2x-3|$

i. Sketch the graph of $y=g(x)$ on the axes below, clearly indicating the coordinates of the end-points and any axial intercepts.



2 marks

ii. Sketch the graph of $y=g'(x)$ on the axes above.

2 marks

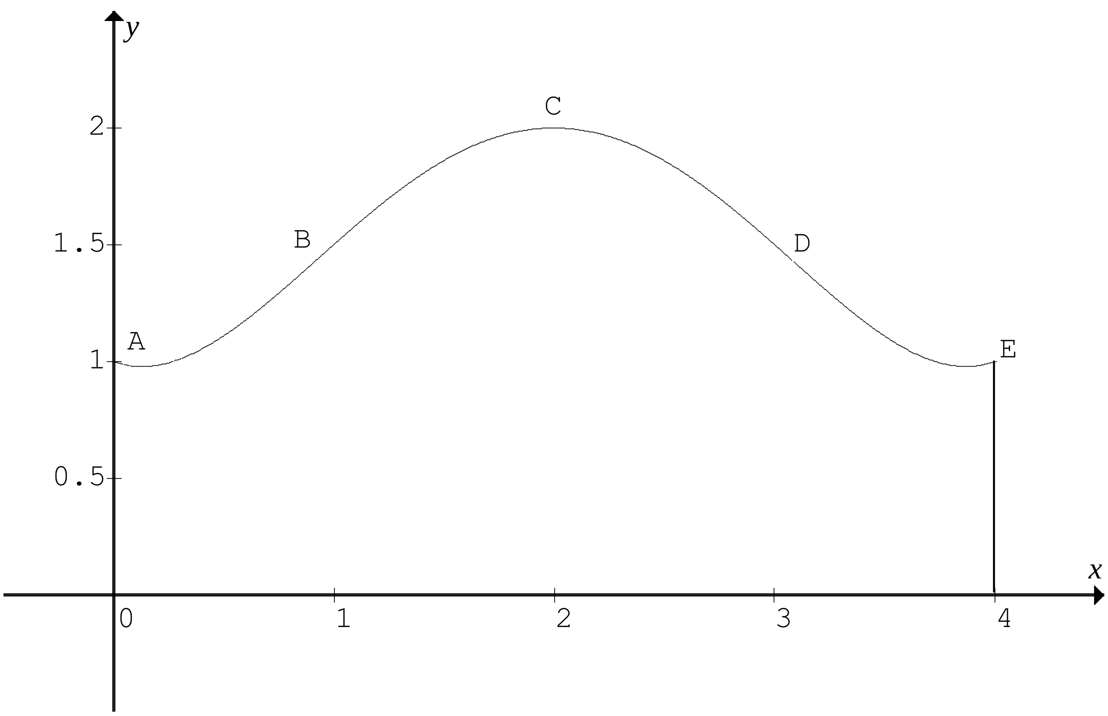
d. Consider now the function $g:R\to R$, $g(x)=|ax+b|-|ax-b|$, where *a* and *b* are both positive real constants. Complete the following to specify the gradient function $g'(x)$ in terms of *a* and *b* as a hybrid function.

$$g'(x)=\begin{cases} \qquad\qquad \text{for} \quad x\in \\ \qquad\qquad \text{for} \quad x\in \\ \qquad\qquad \text{for} \quad x\in \end{cases}$$

2 marks
Total 12 marks

Question 4

The design of a section of fencing ABCDE is shown below, along with the *x* and *y* axes with dimensions in metres. The fence must pass through the points $A(0,1)$, $B\left(1,\frac{3}{2}\right)$, $C(2,2)$ $D\left(3,\frac{3}{2}\right)$ and $E(4,1)$. The point C is the highest point on the fence, while points A and E are the lowest points on the fence.



a. If the area of the fence is approximated by four equally spaced left rectangles, determine in square metres the area of the fence.

1 mark

b. One design for the fence is a function of the form $g(x) = p + q\cos(nx)$ for $0 \le x \le 4$.

i. Find the values of p, q and n.

3 marks

ii. Using this design, write down a definite integral which gives the average height of the fence.

1 mark

iii. Find the average height of the fence in metres.

1 mark

c. Another design for the fence is a hybrid function of the form

$$f(x)=\begin{cases} f1(x) & \text{for } \ 0\le x\le 1 \\ f2(x) & \text{for } \ 1\le x\le 3 \\ f3(x) & \text{for } \ 3\le x\le 4 \end{cases}$$

where $f1(x)=a_1x^2+b_1x+c_1$, $f2(x)=a_2x^2+b_2x+c_2$ and $f3(x)=a_3x^2+b_3x+c_3$ and the fence is smoothly joined at B and D.

i. Write down a set of four linear simultaneous equations involving a_2, b_2 , c_2.

2 marks

ii. Solve these equations to show that $a_2=-\frac{1}{2}$, $b_2=2$ and $c_2=0$.

1 mark

iii. Write down a set of three linear simultaneous equations involving a_1, b_1 and c_1.

2 marks

iv. By solving these equations show that $a_1 = \frac{1}{2}$, $b_1 = 0$ and $c_1 = 1$.

1 mark

v. Write down in words, two transformations, from which the rule for the function $f3(x)$ can be obtained from the rule for the function $f1(x)$.

2 marks

vi. The rule for $f3(x)$ can be expressed as $f3(x) = f1(ax+b)$, write down the values of *a* and *b*, and hence write down the values of a_3, b_3 and c_3.

2 marks

vii. Using this design, write down a definite integral which gives the average height of the fence.

1 mark

viii. Find the average height of the fence in metres.

1 mark
Total 18 marks

END OF EXAMINATION

EXTRA WORKING SPACE

MATHEMATICAL METHODS CAS

Written examination 2

FORMULA SHEET

Directions to students

Detach this formula sheet during reading time. This formula sheet is provided for your reference.

Mathematical Methods and CAS Formulas

Mensuration

area of a trapezium:	$\frac{1}{2}(a+b)h$	Volume of a pyramid:	$\frac{1}{3}Ah$
curved surface area of a cylinder:	$2\pi rh$	volume of a sphere:	$\frac{4}{3}\pi r^3$
volume of a cylinder:	$\pi r^2 h$	area of triangle:	$\frac{1}{2}bc\sin(A)$
volume of a cone:	$\frac{1}{3}\pi r^2 h$		

Calculus

$$\frac{d}{dx}\left(x^n\right)=nx^{n-1}$$

$$\int x^n dx=\frac{1}{n+1}x^{n+1}+c\ ,\ n\neq -1$$

$$\frac{d}{dx}\left(e^{ax}\right)=ae^{ax}$$

$$\int e^{ax}dx=\frac{1}{a}e^{ax}+c$$

$$\frac{d}{dx}\left(\log_e(x)\right)=\frac{1}{x}$$

$$\int \frac{1}{x}dx=\log_e|x|+c$$

$$\frac{d}{dx}\left(\sin(ax)\right)=a\cos(ax)$$

$$\int \sin(ax)\,dx=-\frac{1}{a}\cos(ax)+c$$

$$\frac{d}{dx}\left(\cos(ax)\right)=-a\sin(ax)$$

$$\int \cos(ax)\,dx=\frac{1}{a}\sin(ax)+c$$

$$\frac{d}{dx}\left(\tan(ax)\right)=\frac{a}{\cos^2(ax)}=a\sec^2(ax)$$

product rule: $\frac{d}{dx}(uv)=u\frac{dv}{dx}+v\frac{du}{dx}$

quotient rule: $\frac{d}{dx}\left(\frac{u}{v}\right)=\frac{v\frac{du}{dx}-u\frac{dv}{dx}}{v^2}$

Chain rule: $\frac{dy}{dx}=\frac{dy}{du}\frac{du}{dx}$

approximation: $f(x+h)\approx f(x)+h\,f'(x)$

Probability

$\Pr(A)=1-\Pr(A')$

$\Pr(A\cup B)=\Pr(A)+\Pr(B)-\Pr(A\cap B)$

$\Pr(A/B)=\frac{\Pr(A\cap B)}{\Pr(B)}$

Transition Matrices $S_n=T^n\times S_0$

mean: $\mu=E(X)$

variance: $\mathrm{var}(X)=\sigma^2=E\left((X-\mu)^2\right)=E(X^2)-\mu^2$

probability distribution		mean	variance
discrete	$\Pr(X=x)=p(x)$	$\mu=\sum x\,p(x)$	$\sigma^2=\sum(x-\mu)^2\,p(x)$
continuous	$\Pr(a<X<b)=\int_a^b f(x)dx$	$\mu=\int_{-\infty}^{\infty} x\,f(x)dx$	$\sigma^2=\int_{-\infty}^{\infty}(x-\mu)^2\,f(x)dx$

ANSWER SHEET

STUDENT NUMBER

Figures										Letter
Words										

SIGNATURE ______________________________

SECTION 1

1	**A**	**B**	**C**	**D**	**E**
2	**A**	**B**	**C**	**D**	**E**
3	**A**	**B**	**C**	**D**	**E**
4	**A**	**B**	**C**	**D**	**E**
5	**A**	**B**	**C**	**D**	**E**
6	**A**	**B**	**C**	**D**	**E**
7	**A**	**B**	**C**	**D**	**E**
8	**A**	**B**	**C**	**D**	**E**
9	**A**	**B**	**C**	**D**	**E**
10	**A**	**B**	**C**	**D**	**E**
11	**A**	**B**	**C**	**D**	**E**
12	**A**	**B**	**C**	**D**	**E**
13	**A**	**B**	**C**	**D**	**E**
14	**A**	**B**	**C**	**D**	**E**
15	**A**	**B**	**C**	**D**	**E**
16	**A**	**B**	**C**	**D**	**E**
17	**A**	**B**	**C**	**D**	**E**
18	**A**	**B**	**C**	**D**	**E**
19	**A**	**B**	**C**	**D**	**E**
20	**A**	**B**	**C**	**D**	**E**
21	**A**	**B**	**C**	**D**	**E**
22	**A**	**B**	**C**	**D**	**E**