Year 2012

VCE

Mathematical Methods CAS

Solutions

Trial Examination 2

KILBAHA MULTIMEDIA PUBLISHING	TEL: (03) 9817 5374
PO BOX 2227	FAX: (03) 9817 4334
KEW VIC 3101	kilbaha@gmail.com
AUSTRALIA	http://kilbaha.com.au

SECTION 1

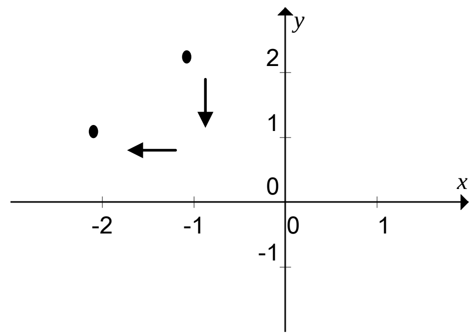
ANSWERS

1	A	**B**	C	D	E
2	**A**	B	C	D	E
3	A	B	C	**D**	E
4	A	B	C	D	**E**
5	A	B	**C**	D	E
6	A	**B**	C	D	E
7	A	B	**C**	D	E
8	A	B	C	D	**E**
9	A	B	**C**	D	E
10	A	B	C	**D**	E
11	A	B	C	**D**	E
12	A	B	C	D	**E**
13	**A**	B	C	D	E
14	A	B	**C**	D	E
15	A	**B**	C	D	E
16	A	B	C	D	**E**
17	**A**	B	C	D	E
18	A	B	C	**D**	E
19	A	**B**	C	D	E
20	A	B	**C**	D	E
21	**A**	B	C	D	E
22	A	B	C	D	**E**

SECTION 1

Question 1 **Answer B**

The point is translated one unit down parallel to the *y*-axis and one unit to the left parallel to the *x*-axis.



$g(x) = f(x+1) - 1$

$f(-1) = 2$ and $g(-2) = 1$

$g(-2) = f(-1) - 1 = 2 - 1 = 1$

Question 2 **Answer A**

$\Delta = \begin{vmatrix} -2 & p-1 \\ p & -6 \end{vmatrix} = 12 - p(p-1) = 12 - p^2 + p$

$\Delta = -(p^2 - p - 12) = -(p-4)(p+3)$

There is no unique solution when $p = 4$ or $p = -3$.

When $p = -3$ the equations become $-2x - 4y = 4$ and $-3x - 6y = 6$, and when $p = 4$ the equations become $-2x + 3y = 4$ and $4x - 6y = -8$, in both cases, the equations are just multiples of each other, and hence there is an infinite number of solutions when $p = 4$ or $p = -3$.

Question 3 **Answer D**

$y_1 = x^2 + 5x$, $y_2 = kx - 2$ $y_1 = y_2$

$x^2 + 5x = kx - 2$

$x^2 + (5-k)x + 2 = 0$

$\Delta = (5-k)^2 - 8 = \left(5 - k + 2\sqrt{2}\right)\left(5 - k - 2\sqrt{2}\right)$

For two distinct solutions, $\Delta > 0 \Rightarrow$ $k > 5 + 2\sqrt{2}$ or $k < 5 - 2\sqrt{2}$

Question 4 **Answer E**

A. B. C. and **D.** are all true , $f(x) = x^3$ $f(a+b) = (a+b)^3 \neq f(a) + f(b) = a^3 + b^3$

Question 5 **Answer C**

A. B. D. and **E.** are all true. When $n = \frac{1}{2}$, the graph of $y = 2\sqrt{x-5} + 4$, has an endpoint at $x = 5$, it is not a vertical asymptote.

Question 6 **Answer B**

$h(x) = f(g(x)) \Rightarrow h(2) = f(g(2)) = f(3) = 4$

$h'(x) = g'(x)\, f'(g(x))$

$h'(2) = g'(2)\, f'(g(2)) = g'(2)\, f'(3) = 5 \times 7 = 35$

Question 7 **Answer C**

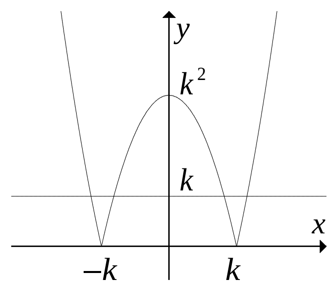
$f(x) = \dfrac{g(x)}{\sqrt{x}}$ using the quotient rule

$$f'(x) = \frac{g'(x)\sqrt{x} - \dfrac{g(x)}{2\sqrt{x}}}{x}$$

$$f'(4) = \frac{g'(4)\sqrt{4} - \dfrac{g(4)}{2\sqrt{4}}}{4} = \frac{6 \times 2 - \dfrac{8}{4}}{4} = \frac{10}{4} = \frac{5}{2}$$

Question 8 **Answer E**

The graphs of $y = \left|x^2 - k^2\right|$ and $y = k$ have 4 points of intersection, when $k > 1$.



Question 9 **Answer C**

Although all the graphs, join up and are continuous at $x = 2$, however **C**. has a vertical asymptote at $x = \dfrac{3}{2}$ and is therefore, not continuous, over its maximal domain.

Question 10 **Answer D**

$a^{2x} = \dfrac{1}{b} \quad \Leftrightarrow \quad 2x = \log_a\left(\dfrac{1}{b}\right)$

$x = \dfrac{1}{2}\log_a\left(\dfrac{1}{b}\right) = -\dfrac{1}{2}\log_a(b) = -\dfrac{1}{2\log_b(a)}$ by change of base rule, **A.** and **B.** are true.

$x = \dfrac{\log_{10}\left(\dfrac{1}{b}\right)}{2\log_{10}(a)} = \dfrac{-\log_{10}(b)}{\log_{10}(a^2)}$ **C.** is true.

$x = \dfrac{1}{2}\log_a\left(\dfrac{1}{b}\right) = \log_a\left(\dfrac{1}{\sqrt{b}}\right)$ **E.** is true.

D. is false.

Question 11 **Answer D**

$f: \quad y=\frac{1}{x^2}+a$ swap x and y

$f^{-1} \quad x=\frac{1}{y^2}+a \quad \Rightarrow \frac{1}{y^2}=x-a \quad \Rightarrow y^2=\frac{1}{x-a} \quad \Rightarrow \quad y=\frac{\pm 1}{\sqrt{x-a}}$

and ran $f=(a,\infty)=\text{dom } f^{-1}$ and ran $f^{-1}=(-\infty,0)=\text{dom } f$, we must take the minus.

$f^{-1}:(a,\infty)\to R$, $f^{-1}(x)=\frac{-1}{\sqrt{x-a}}$

Question 12 **Answer E**

$y=a-\frac{ax}{x-a}=a-\frac{a(x-a)-ax}{x-a}=-\frac{a^2}{x-a}$

this graph has asymptotes at $x=a$ and $y=0$

Question 13 **Answer A**

Let the volume be V cm^3, surface area S cm^2, side length L cm, at time t minutes.

$V=L^3 \quad \frac{dV}{dL}=3L^2 \quad \frac{dV}{dt}=-24=\frac{dV}{dL}\frac{dL}{dt}=3L^2\frac{dL}{dt}$ when $L=2 \Rightarrow \frac{dL}{dt}=-2$

$S=6L^2 \quad \frac{dS}{dL}=12L \quad \frac{dS}{dt}=\frac{dS}{dL}\frac{dL}{dt}=12L\times -2$ when $L=2 \quad \frac{dS}{dt}=-48$

Question 14 **Answer C**

$f(x)=e^{-2x}\cos\left(\frac{\pi x}{4}\right)$, now $f(2)=e^{-4}\cos\left(\frac{\pi}{2}\right)=0 \qquad f(0)=e^{0}\cos(0)=1$

average rate of change over $0\le x\le 2$

$\frac{f(2)-f(0)}{2-0}=\frac{0-1}{2}=-\frac{1}{2}$

Question 15 **Answer B**

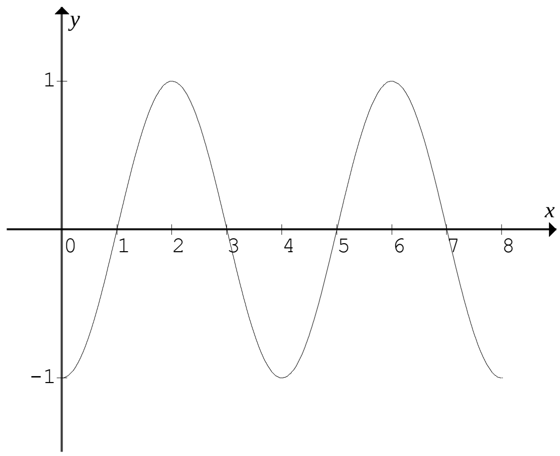
$y=e^{-\frac{x}{2}}\sin\left(\frac{x}{2}\right)$ since $e^{-\frac{x}{2}}\neq 0$ it crosses the x-axis at $\sin\left(\frac{x}{2}\right)=0 \Rightarrow x=2n\pi$

for turning points $\frac{dy}{dx}=\frac{1}{2}e^{-\frac{x}{2}}\left(\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)\right)=0$

$\Rightarrow \tan\left(\frac{x}{2}\right)=1 \Rightarrow x=(4n-3)\frac{\pi}{2}$

Question 16 **Answer E**

$$f:(0,8)\to R\ ,\ \ f(x)=2\left(\sin\left(\frac{\pi(x-4)}{4}\right)\right)^2-1$$



All of **A. B. C.** and **D.** are correct, **E.** is false, the range is $[-1,1]$

Question 17 **Answer A**

$y=\tan\left(\frac{x}{2}\right)$ at $x=\frac{\pi}{3}$ $\tan\left(\frac{\pi}{6}\right)=\frac{\sqrt{3}}{3}$

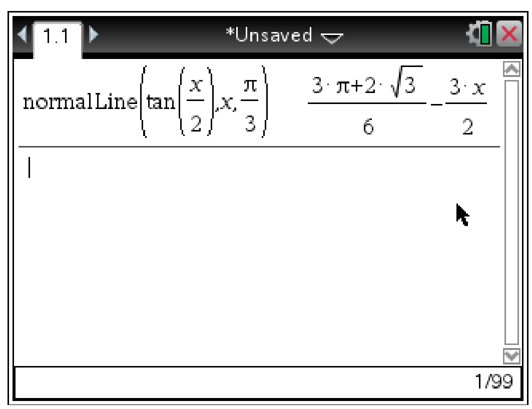
$P\left(\frac{\pi}{3},\frac{\sqrt{3}}{3}\right)$ $\frac{dy}{dx}=\frac{1}{2\cos^2\left(\frac{x}{2}\right)}$ at $x=\frac{\pi}{3}$

$\frac{dy}{dx}=m_T=\frac{1}{2\cos^2\left(\frac{\pi}{6}\right)}=\frac{2}{3}$ $m_N=-\frac{3}{2}$

the equation of the normal

$$y-\frac{\sqrt{3}}{3}=-\frac{3}{2}\left(x-\frac{\pi}{3}\right)$$

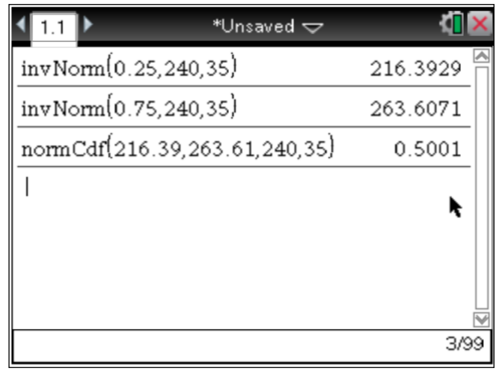
$$y=-\frac{3x}{2}+\frac{\pi}{2}+\frac{\sqrt{3}}{3}$$



Question 18 **Answer D**

$X\overset{d}{=}Bi(n=20,p=0.45)$, $E(X)=np=20\times0.45=9$

$\Pr(X>9)=\Pr(X\geq10)=0.409$



Question 19 **Answer B**

$X \overset{d}{=} N\left(\mu = 240, \sigma^2 = 35^2\right)$

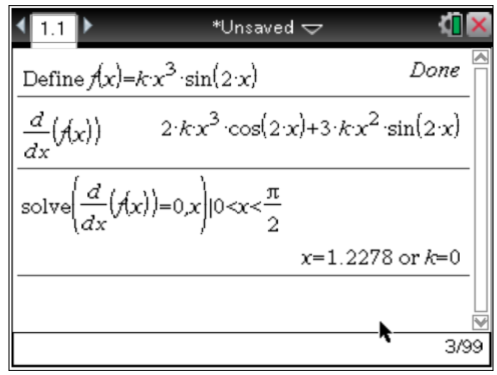
$\Pr(a < X < b) = 0.50$

$a = ? \quad b = ?$

$\Pr(X < a) = 0.25$

$\Pr(X < b) = 0.75$

$a = 216.39 \quad b = 263.61$



Question 20 **Answer C**

The mode is the x value of the highest point on the graph, and occurs when $f'(x) = 0$

$f'(x) = kx^2\left(3\sin(2x) + 2x\cos(2x)\right) = 0$

solving for x, with $0 < x < \dfrac{\pi}{2}$ gives, $x = 1.23$

Question 21 **Answer A**

$\sum \Pr(X = x) = c + 4c + 3c + 3c + 4c + 5c = 20c = 1 \;\Rightarrow\; c = \dfrac{1}{20}$

X	49	50	51	52	53	54
$\Pr(X = x)$	0.05	0.20	0.15	0.15	0.20	0.25

The mean is $E(X) = \sum x \Pr(X = x)$

$E(X) = 49 \times 0.05 + 50 \times 0.20 + 51 \times 0.15 + 52 \times 0.15 + 53 \times 0.20 + 54 \times 0.25 = 52$.

Since $\sum_{49}^{52} \Pr(X = x) = 0.55 > 0.50$ the median is 52, the highest probability, 0.25 occurs when $x = 54$, this is the mode.

Question 22 **Answer E**

$\mu_2 > \mu_1$ and $\sigma_1 > \sigma_2$, since the mean of X_2 is to the right of X_1, but the graph of X_1 is wider (more spread) than the graph of X_2.

END OF SECTION 1 SUGGESTED ANSWERS

SECTION 2

Question 1

a.i. $A(0,2)\quad f(0)=2 \Rightarrow \sqrt{p}=2 \quad \Rightarrow \; p=4$

$B(2,0)\quad f(2)=0 \Rightarrow \sqrt{4+2q}=0 \quad \Rightarrow \; q=-2$ M1

ii. $A=\int_0^2 \sqrt{4-2x}\,dx=\frac{8}{3}=2\frac{2}{3}$ metres^2 A1

iii. $s(x)=\sqrt{x^2+y^2}$ with $y=f(x)=\sqrt{4-2x}$

$s(x)=\sqrt{x^2+\left(\sqrt{4-2x}\right)^2}$ A1

$s(x)=\sqrt{x^2+4-2x}$

iv. $\frac{ds}{dx}=\frac{2x-2}{2\sqrt{x^2+4-2x}}=0$ for a maximum or minimum A1

$\Rightarrow x=1$ A1

and $s_{\min}=s(1)=\sqrt{3}$ metres A1

b.i. $B(2,0)\quad f(2)=0 \Rightarrow 4-e^{2k}-e^{-2k}=0$ let $u=e^{2k}$

$4-u-\frac{1}{u}=0 \quad \Rightarrow \quad u+\frac{1}{u}-4=0$, multiply by u

$u^2-4u+1=0$ M1

$u^2-4u+4=3$

$(u-2)^2=3$ M1

$u-2=\pm\sqrt{3}$

$u=e^{2k}=2\pm\sqrt{3}$ but $k>0$ take the positive M1

$u=e^{2k}=2+\sqrt{3}$

$k=\frac{1}{2}\log_e\left(2+\sqrt{3}\right)$

ii. $A=4-\int_0^2\left(4-e^{kx}-e^{-kx}\right)dx$ with $k=\frac{1}{2}\log_e\left(2+\sqrt{3}\right)$

$A=\frac{4\sqrt{3}}{\log_e\left(2+\sqrt{3}\right)}-4=\frac{2\sqrt{3}}{k}-4$ metres^2 A1

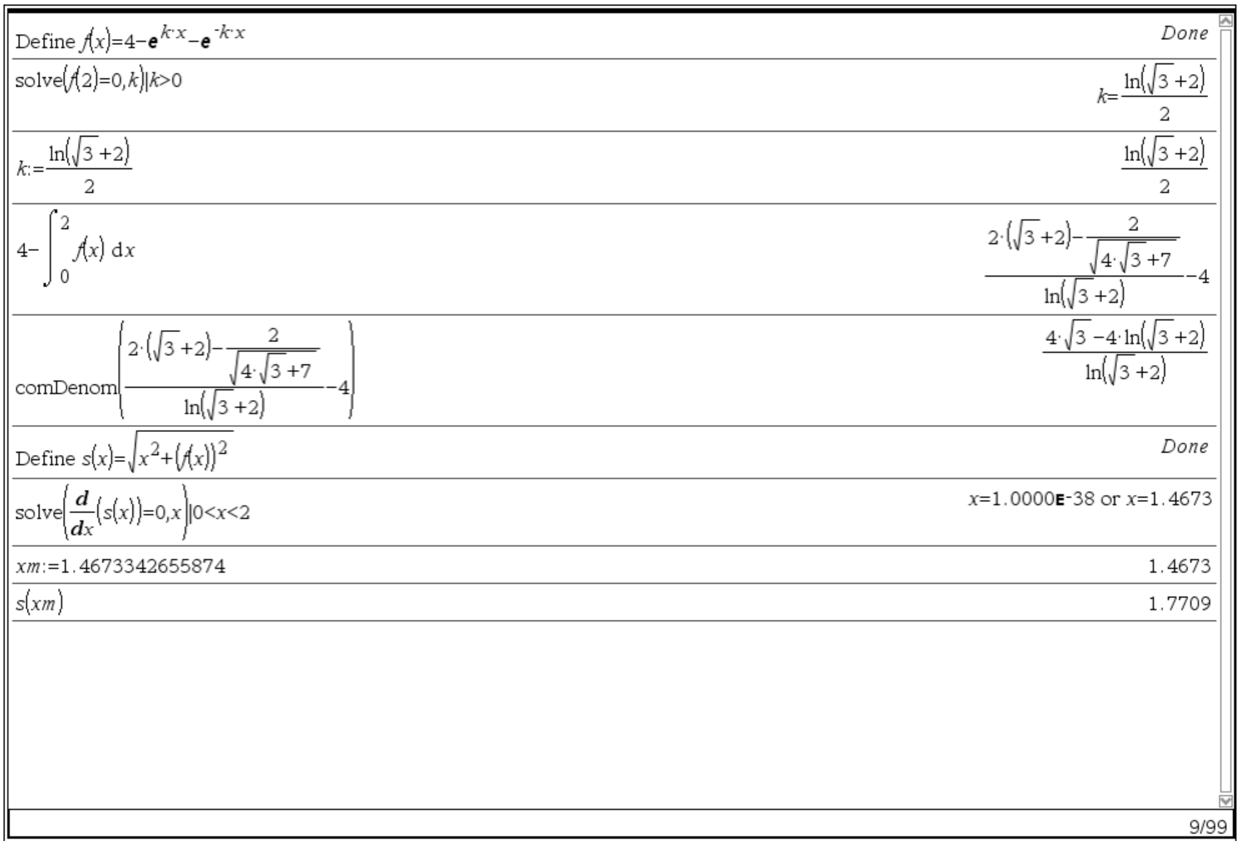
$a=2\quad b=3$ and $c=-4$

iii. $s(x)=\sqrt{x^2+\left(f(x)\right)^2}$

with $f(x)=4-e^{kx}-e^{-kx}$ and $k=\frac{1}{2}\log_e\left(2+\sqrt{3}\right)$ A1

$\frac{ds}{dx}=0$ for a maximum or minimum, $\Rightarrow x=1.467$ A1

$s_{\min}=s(1.467)=1.771$ metres A1



Question 2

a.i. H uses a hand razor, E uses an electric razor.

$H \to E = 0.15$, $E \to H = 0.25$, $H \to H = 0.85$, $E \to E = 0.75$

$\Pr(E \text{ 3times}) = EHEE + EEHE + EEEH$

$= 0.25 \times 0.15 \times 0.75 + 0.75 \times 0.25 \times 0.15 + 0.75^2 \times 0.25$

$= 0.197$ A1

ii. $\Pr(E \text{ onThursday}) = EEEE + EEHE + EHEE + EHHE$

$= 0.75^3 + 0.75 \times 0.25 \times 0.15 + 0.25 \times 0.15 \times 0.75 + 0.25 \times 0.85 \times 0.15$

$= 0.510$ A1

or alternatively $\begin{matrix} & H & E \\ H & 0.85 & 0.25 \\ E & 0.15 & 0.75 \end{matrix}$

$\begin{bmatrix} 0.85 & 0.25 \\ 0.15 & 0.75 \end{bmatrix}^3 \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0.49 \\ 0.51 \end{bmatrix}$

iii. $\dfrac{0.15}{0.15 + 0.25} = 0.375$

or alternatively $\begin{bmatrix} 0.85 & 0.25 \\ 0.15 & 0.75 \end{bmatrix}^n \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0.625 \\ 0.375 \end{bmatrix}$ for $n \geq 19$

in the long run, the probability of using an electric razor is 0.375 A1

b.i. $H \stackrel{d}{=} Bi(n = 20, p = 0.4)$

$\Pr(8 \leq H \leq 12) = 0.5631$ A1

ii. $H \stackrel{d}{=} Bi(n = 20, p = ?)$

$\Pr(9 \leq H \leq 11) = 0.25$

$\binom{20}{9} p^9 (1-p)^{11} + \binom{20}{10} p^{10} (1-p)^{10} + \binom{20}{11} p^{11} (1-p)^9 = 0.25$ by CAS

$-16796 p^9 (p-1)^9 (9p^2 - 9p + 10) = 0.25$ M1

solving with $0 < p < 1$ gives $p = 0.3625$ or 0.6375

but since $E(X) < 10 \Rightarrow p < 0.5$ so $p = 0.3625$ A1

c. X is the time in seconds spent cleaning, $X \overset{d}{=} N\left(\mu = ?, \sigma^2 = ?\right)$

$(1) \quad \Pr(X < 130) = 0.16$

$(2) \quad \Pr(X > 264) = 0.18$

$(1) \Rightarrow \dfrac{130-\mu}{\sigma} = -0.9945$ M1

$(2) \Rightarrow \dfrac{264-\mu}{\sigma} = 0.9154$

$(1) \quad 130 - \mu = -0.9945\sigma$ M1

$(2) \quad 264 - \mu = 0.915\sigma$

now subtract equations $(2)-(1)$ solving gives

$\sigma = 70$ substituting gives $\mu = 200$ seconds A1

d.i. Since the total area under the curve is equal to one.

$\int_0^{\infty} k\,t\,e^{-\frac{t^2}{8}}dt = 4k = 1 \quad \Rightarrow \quad k = \frac{1}{4}$ A1

ii. $\Pr(0 < T < 3) = \frac{1}{4}\int_0^3 t\,e^{-\frac{t^2}{8}}dt = 1 - e^{-\frac{9}{8}}$ M1

$\Pr(0 < T < 3) = 0.6753$ A1

iii. $E(T) = \frac{1}{4}\int_0^{\infty} t^2\,e^{-\frac{t^2}{8}}dt = 2.5066$

mean time $E(T) = 2.51$ minutes A1

$E(T^2) = \frac{1}{4}\int_0^{\infty} t^3\,e^{-\frac{t^2}{8}}dt = 8$

$sd(T) = \sqrt{8 - 2.5066^2} = 1.31$ minutes A1

iv. median m, satisfies $\frac{1}{4}\int_0^{m} t\,e^{-\frac{t^2}{8}}dt = \frac{1}{2}$ solving, with $m > 0$, gives

$m = 2.355$ minutes A1

$\begin{bmatrix}0.85 & 0.25\\0.15 & 0.75\end{bmatrix}^3\cdot\begin{bmatrix}0\\1\end{bmatrix}$	$\begin{bmatrix}0.4900\\0.5100\end{bmatrix}$
$\dfrac{0.15}{0.15+0.25}$	0.3750
$\begin{bmatrix}0.85 & 0.25\\0.15 & 0.75\end{bmatrix}^{19}\cdot\begin{bmatrix}0\\1\end{bmatrix}$	$\begin{bmatrix}0.6250\\0.3750\end{bmatrix}$
binomCdf(20,0.4,8,12)	0.5631
$\text{nCr}(20,9)\cdot p^9\cdot(1-p)^{11}+\text{nCr}(20,10)\cdot p^{10}\cdot(1-p)^{10}+\text{nCr}(20,11)\cdot p^{11}\cdot(1-p)^9$	$-16796\cdot p^9\cdot(p-1)^9\cdot(9\cdot p^2-9\cdot p+10)$
$\text{solve}(-16796\cdot p^9\cdot(p-1)^9\cdot(9\cdot p^2-9\cdot p+10)=0.25,p)\mid 0<p<1$	$p=0.3625$ or $p=0.6375$
$\dfrac{130-m}{s}=\text{invNorm}(0.16,0,1)$	$\dfrac{130-m}{s}=-0.9945$
$\dfrac{264-m}{s}=\text{invNorm}(0.82,0,1)$	$\dfrac{264-m}{s}=0.9154$
$\text{solve}\left(\dfrac{130-m}{s}=-0.99445789074249 \text{ and } \dfrac{264-m}{s}=0.91536508202965,\{m,s\}\right)$	$s=70.1636$ and $m=199.7747$

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Define $f(t)=k\cdot t\cdot e^{\frac{-t^2}{8}}$	*Done*
$\text{solve}\left(\int_0^\infty f(t)\,dt=1,k\right)$	$k=\dfrac{1}{4}$
Define $f(t)=k\cdot t\cdot e^{\frac{-t^2}{8}}\mid k=\dfrac{1}{4}$	*Done*
$\int_0^3 f(t)\,dt$	$\left(e^{\frac{9}{8}}-1\right)\cdot e^{\frac{-9}{8}}$
$\left(e^{\frac{9}{8}}-1\right)\cdot e^{\frac{-9}{8}}$	0.6753
$\int_0^\infty (t\cdot f(t))\,dt$	2.5066
$\int_0^\infty (t^2\cdot f(t))\,dt$	8
$\sqrt{8-(2.5066282746378)^2}$	1.3103
$\text{solve}\left(\int_0^m f(t)\,dt=0.5,m\right)\mid m>0$	$m=2.3548$

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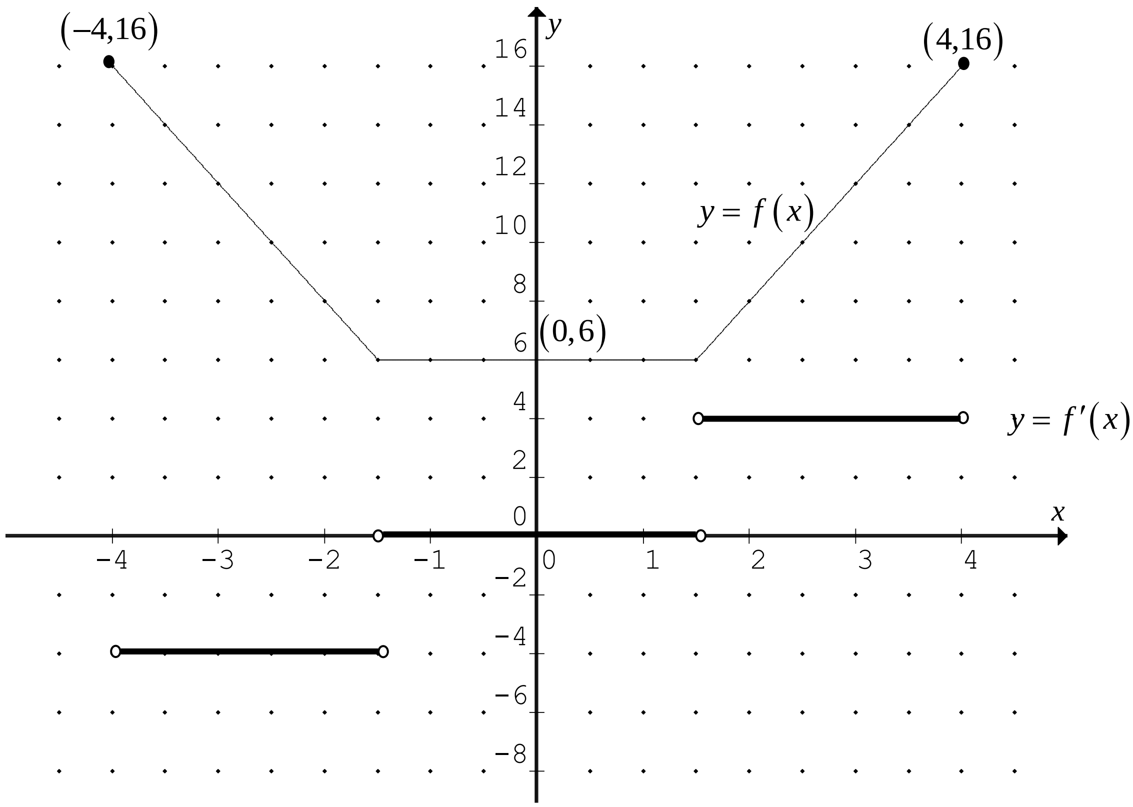
Question 3

a.i. $f(x)=|2x+3|+|2x-3|$

$f(-4)=|5|+|-11|=16$, $f(4)=|11|+|-5|=16$, $f(0)=6$



Correct graph of $y=f(x)$, shape and continuous at $x=\pm\frac{3}{2}$, A1

y-intercept, $(0,6)$, endpoints $(-4,16),(4,16)$. A1

ii. Correct shape of $y=f'(x)$, the graph of $y=f'(x)$ is not defined at A1

$x=\pm4$ and $x=\pm\frac{3}{2}$ and must have open circles at $x=\pm4$ and $x=\pm\frac{3}{2}$ A1

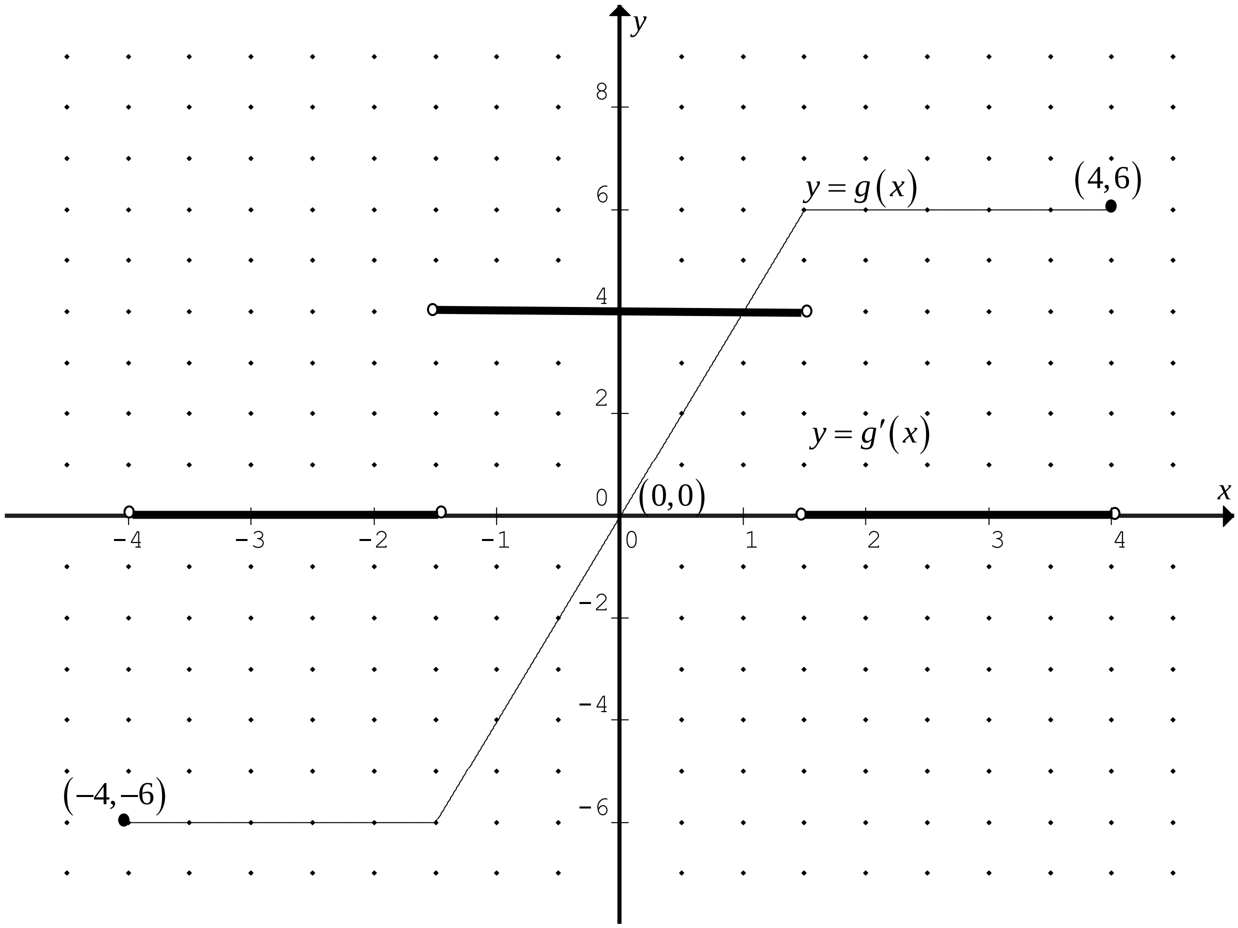
b. $f(x)=|ax+b|+|ax-b|$

$$f'(x)=\begin{cases}2a & \text{for } x\in\left(\frac{b}{a},\infty\right)\\ 0 & \text{for } x\in\left(-\frac{b}{a},\frac{b}{a}\right)\\ -2a & \text{for } x\in\left(-\infty,-\frac{b}{a}\right)\end{cases}$$

must have open brackets A2

c.i. $g(x)=|2x+3|-|2x-3|$

$g(-4)=|-5|-|-11|=-6\ ,\ g(4)=|11|-|5|=6\ ,\ g(0)=0$



Correct graph of $y=g(x)$, shape and continuous at $x=\pm\frac{3}{2}$, A1

passes through the origin $(0,0)$ and endpoints $(-4,-6)$, $(4,6)$. A1

ii. Correct shape of $y=g'(x)$, the graph of $y=g'(x)$ is not defined at A1

$x=\pm4$ and $x=\pm\frac{3}{2}$ and must have open circles at $x=\pm4$ and $x=\pm\frac{3}{2}$ A1

d. $g(x)=|ax+b|-|ax-b|$

$$g'(x)=\begin{cases} 0 & \text{for } x\in\left(\frac{b}{a},\infty\right) \\ 2a & \text{for } x\in\left(-\frac{b}{a},\frac{b}{a}\right) \\ 0 & \text{for } x\in\left(-\infty,-\frac{b}{a}\right) \end{cases}$$ must have open brackets A2

Question 4

a. $A_L = y_A + y_B + y_C + y_D = 1 + \frac{3}{2} + 2 + \frac{3}{2}$

$A_L = 6 \text{ m}^2$ A1

b.i. period $T = \frac{2\pi}{n} = 4 \quad \Rightarrow \quad n = \frac{\pi}{2}$ A1

amplitude of inverted cosine wave $q = -\frac{1}{2}$ A1

translated upwards $p = \frac{3}{2}$ A1

ii. $\overline{g} = \frac{1}{4}\int_0^4 \left(\frac{3}{2} - \frac{1}{2}\cos\left(\frac{\pi x}{2}\right)\right)dx$, accept $\overline{g} = \frac{1}{4}\int_0^4 \left(p + q\cos(nx)\right)dx$ A1

iii. $\overline{g} = \frac{3}{2}$ metres A1

c.i. $f2(x) = a_2x^2 + b_2x + c_2$

$f2(1) = \frac{3}{2} \Rightarrow a_2 + b_2 + c_2 = \frac{3}{2} \quad (1)$

$f2(2) = 2 \Rightarrow 4a_2 + 2b_2 + c_2 = 2 \quad (2)$ A1

$f2(3) = \frac{3}{2} \Rightarrow 9a_2 + 3b_2 + c_2 = \frac{3}{2} \quad (3)$

$f2'(x) = 2a_2x + b_2$

$f2'(2) = 0 \Rightarrow 4a_2 + b_2 = 0 \quad (4)$ A1

ii. $(2)-(1) \Rightarrow 3a_2 + b_2 = \frac{1}{2} \quad (5)$

$(4)-(5) \Rightarrow a_2 = -\frac{1}{2}$ substituting gives $b_2 = 2$ and $c_2 = 0$ A1

checks in all equations, so $f2(x) = -\frac{1}{2}x^2 + 2x$

iii. $f1(x) = a_1x^2 + b_1x + c_1$

$f1(0) = 1 \Rightarrow c_1 = 1 \quad (1)$

$f1(1) = \frac{3}{2} \Rightarrow a_1 + b_1 + c_1 = \frac{3}{2} \Rightarrow a_1 + b_1 = \frac{1}{2} \quad (2)$ A1

$f2'(x) = -x + 2 \quad f2'(1) = -1 + 2 = 1 = f1'(1)$ since the join is smooth

$f1'(x) = 2a_1x + b_1 \quad f1'(1) = 2a_1 + b_1 = 1 \quad (3)$ A1

iv. $(3) - (2) \Rightarrow a_1 = \frac{1}{2}$ substituting gives $b_1 = 0$ since $c_1 = 1$ A1

$f1(x) = \frac{1}{2}x^2 + 1$

v.

- reflect in the *y*-axis A1
- translate 4 units to the right parallel to the *x*-axis. A1

vi. $f3(x) = f1(4 - x) \quad a = -1 \ \ b = 4$ A1

$f3(x) = \frac{1}{2}(4 - x)^2 + 1 = \frac{1}{2}\left(16 - 8x + x^2\right) + 1$

$f3(x) = \frac{1}{2}x^2 - 4x + 9 \quad a_3 = \frac{1}{2}\ ,\ b_3 = -4\ ,\ c_3 = 9$ A1

vii. $\bar{f} = \frac{1}{4}\left[\int_0^1 \left(\frac{1}{2}x^2 + 1\right)dx + \int_1^3 \left(-\frac{1}{2}x^2 + 2x\right)dx + \int_3^4 \left(\frac{1}{2}x^2 - 4x + 9\right)dx\right]$ A1

viii. $\bar{f} = \frac{3}{2}$ metres A1

END OF SECTION 2 SUGGESTED ANSWERS