

Year 2012
VCE
Mathematical Methods
CAS
Solutions
Trial Examination 1



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Question 1

a. Let $y = \sqrt{16 - x^3} = (16 - x^3)^{\frac{1}{2}} = u^{\frac{1}{2}}$ where $u = 16 - x^3$

$$\frac{dy}{du} = \frac{1}{2}u^{-\frac{1}{2}} \quad \frac{du}{dx} = -3x^2 \quad \text{using the Chain rule} \quad \text{M1}$$

$$\frac{dy}{dx} = \frac{dy}{du} \frac{du}{dx} = -3x^2 \times \frac{1}{2}u^{-\frac{1}{2}} = -\frac{3x^2}{2\sqrt{u}}$$

$$\frac{dy}{dx} = \frac{-3x^2}{2\sqrt{16 - x^3}} \quad \text{A1}$$

b. $\int \frac{x^2}{\sqrt{16 - x^3}} dx = -\frac{2}{3}\sqrt{16 - x^3}$ A1

Question 2

$$4\cos\left(\frac{\pi x}{3}\right) - 2 = 0 \Rightarrow \cos\left(\frac{\pi x}{3}\right) = \frac{1}{2}$$

$$\frac{\pi x}{3} = 2n\pi \pm \cos^{-1}\left(\frac{1}{2}\right) = 2n\pi \pm \frac{\pi}{3} \quad \text{M1}$$

$$x = 6n \pm 1 \text{ where } n \in \mathbb{Z} \quad \text{A1}$$

Question 3

$$f(x) = \log_e(x+3) \quad g(x) = 5 + 2x - x^2$$

$$f(g(x)) = f(5 + 2x - x^2)$$

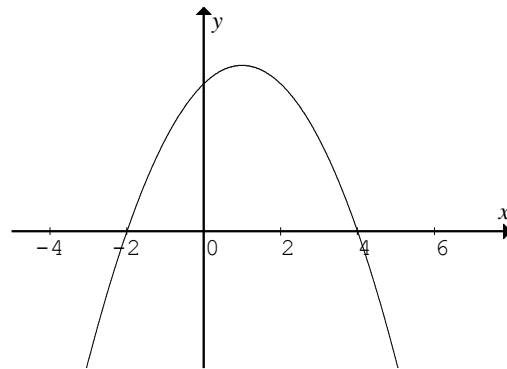
$$f(g(x)) = \log_e(5 + 2x - x^2 + 3)$$

$$f(g(x)) = \log_e(8 + 2x - x^2) \quad \text{A1}$$

$$f(g(x)) = \log_e(-(x^2 - 2x - 8)) = \log_e(-(x-4)(x+2))$$

we require $y = -(x-4)(x+2) > 0$ from the graph

$$-2 < x < 4 = (-2, 4) \quad \text{A1}$$



Question 4

$x' = 2x + 3y - 4$ and $y' = -y + 2$ A1

$\Rightarrow y = 2 - y'$ M1

$2x = x' - 3y + 4 = x' - 3(2 - y') + 4 = x' + 3y' - 2$

so $4x + y = 3$ becomes

$2(x' + 3y' - 2) + 2 - y' = 3$ drop the dashes

$2x + 5y = 5$ so $a = 2$ $b = 5$ $k = 5$ A1

Question 5

i. $q = \frac{10p}{p-10}$

$\frac{dq}{dp} = \frac{10(p-10) - 1(10p)}{(p-10)^2}$ using the quotient rule M1

$\frac{dq}{dp} = \frac{-100}{(p-10)^2}$

ii. $p = 15$ $\Delta p = 0.1$ $\Delta q = ?$

$\frac{dq}{dp} \approx \frac{\Delta q}{\Delta p}$ M1

$\Rightarrow \Delta q \approx \left. \frac{dq}{dp} \right|_{p=15} \Delta p$

$\Delta q = \frac{-100}{5^2} \times 0.1 = -0.4$ cm or 0.4 cm decrease A1

iii. $\frac{dp}{dt} = 0.25$ cm/sec and $\frac{dq}{dt} = \frac{dq}{dp} \cdot \frac{dp}{dt}$ M1

$\frac{dq}{dt} = \frac{-100}{(p-10)^2} \times 0.25$ when $p = 15$

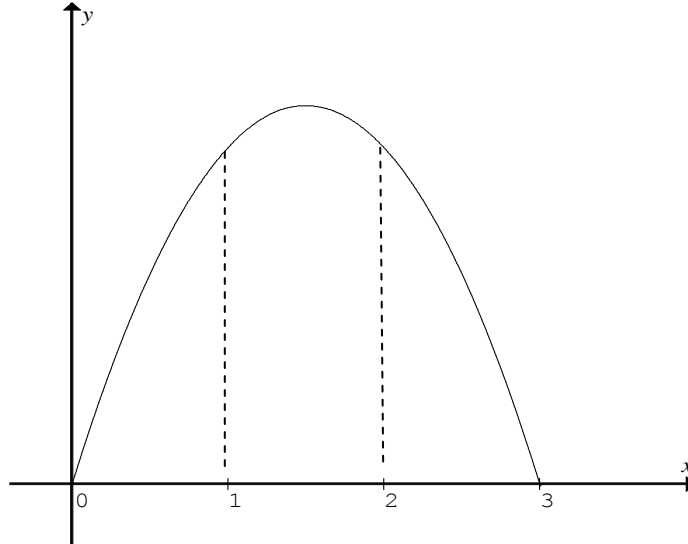
$\left. \frac{dq}{dt} \right|_{p=15} = \frac{-100}{5^2} \times 0.25 = -1$ cm/sec or 1.0 cm/sec decrease A1

Question 6

i. $k \int_0^3 x(3-x) dx = k \int_0^3 (3x - x^2) dx = 1$

$$k \left[\frac{3x^2}{2} - \frac{x^3}{3} \right]_0^3 = k \left(\frac{27}{2} - 9 - 0 \right) = \frac{9k}{2} = 1 \quad \text{M1}$$

$$k = \frac{2}{9}$$



ii. Now $\Pr(2 < X < 3) = \Pr(0 < X < 1)$ by symmetry

$$\begin{aligned} \Pr(0 < X < 1) &= \frac{2}{9} \int_0^1 (3x - x^2) dx \\ &= \frac{2}{9} \left[\frac{3x^2}{2} - \frac{x^3}{3} \right]_0^1 = \frac{2}{9} \left(\frac{3}{2} - \frac{1}{3} - 0 \right) \quad \text{M1} \end{aligned}$$

$$= \frac{7}{27}$$

$$\Pr(X > 2 | X > 1) = \frac{\Pr(X > 2)}{\Pr(X > 1)} = \frac{\Pr(2 < X < 3)}{1 - \Pr(X < 1)} = \frac{\Pr(0 < X < 1)}{1 - \Pr(X < 1)}$$

$$\Pr(X > 2 | X > 1) = \frac{\frac{7}{27}}{1 - \frac{7}{27}} = \frac{7}{20} \quad \text{M1}$$

$$\Pr(X > 2 | X > 1) = \frac{7}{20} \quad \text{A1}$$

Question 7

i. $\sum \Pr(X = x) = \log_8(k+1) + \log_8(2k-4) = 1$

$$\log_8((k+1)(2k-4)) = 1$$

$$(k+1)(2k-4) = 8$$

M1

$$2k^2 - 2k - 4 = 8$$

$$2k^2 - 2k - 12 = 0$$

$$2(k^2 - k - 6) = 0$$

M1

$$2(k-3)(k+2) = 0$$

$$k = 3 \text{ or } k = -2 \text{ but } k > 2$$

$$k = 3 \text{ only}$$

A1

ii. $E(X) = \log_8(k+1) + 2\log_8(2k-4)$ with $k = 3$

$$E(X) = \log_8(4) + 2\log_8(2) = \log_8(4) + \log_8(4) = \log_8(16) = m$$

M1

$$8^m = 16 \quad (2^3)^m = 2^4 \quad \Rightarrow 3m = 4$$

$$m = E(X) = \frac{4}{3}$$

A1

Question 8

$$y = f(x) = a(x-h)^3 + k$$

since there is a stationary point at $x = 2 \Rightarrow h = 2$

crosses the x -axis at $x = 4 \quad f(4) = 0 \Rightarrow 8a + k = 0$ so $k = -8a$

A1

$$y = f(x) = a(x-2)^3 - 8a = a((x-2)^3 - 8)$$

$$A = a \int_0^4 ((x-2)^3 - 8) dx = 64$$

M1

$$A = a \left[\frac{1}{4}(x-2)^4 - 8x \right]_0^4 = a \left(\left(\frac{1}{4}(2^4) - 32 \right) - \left(\frac{1}{4}(-2)^4 - 0 \right) \right) = -32a = 64$$

M1

$$a = -2 \quad k = 16 \quad h = 2$$

A1

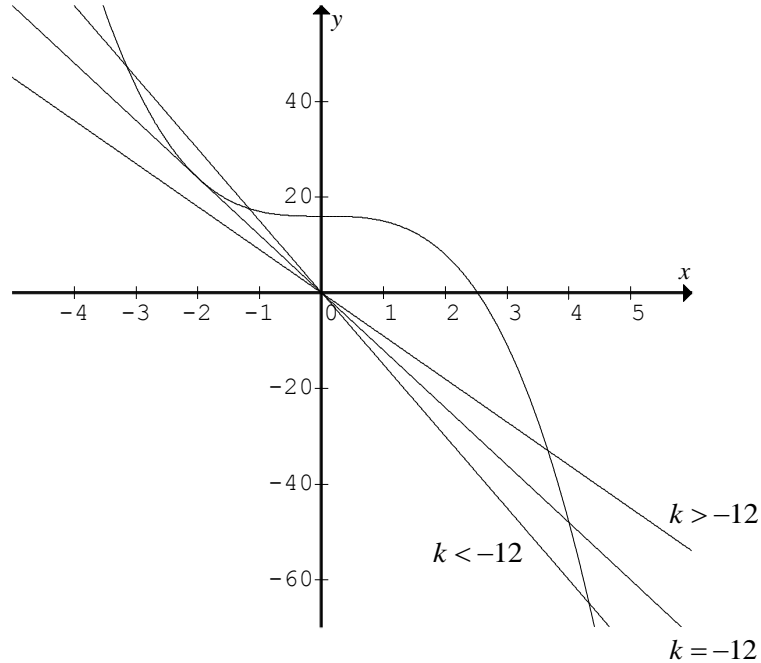
Question 9

$y = 16 - x^3$ at the point $(a, f(a)) = (a, 16 - a^3)$ $\frac{dy}{dx} = -3x^2$ $m_T = -3a^2$

$-3a^2 = \frac{16 - a^3 - 0}{a - 0} \Rightarrow 16 - a^3 = -3a^3$ M1

at the point of contact $-2a^3 = 16$ $a^3 = -8$ so $a = -2$ A1

The tangent is $y = -12x$ A1



i. one solution $k > -12$

ii. two solutions $k = -12$

iii. three solutions $k < -12$

A2

Question 10

$v(t) = \frac{24}{\sqrt{4t+9}}$

$s = \int_0^4 \frac{24}{\sqrt{4t+9}} dt$

$s = \frac{24}{4} \times 2 \left[\sqrt{4t+9} \right]_0^4$ A1

$s = 12(\sqrt{25} - \sqrt{9})$

$s = 24$ metres. A1

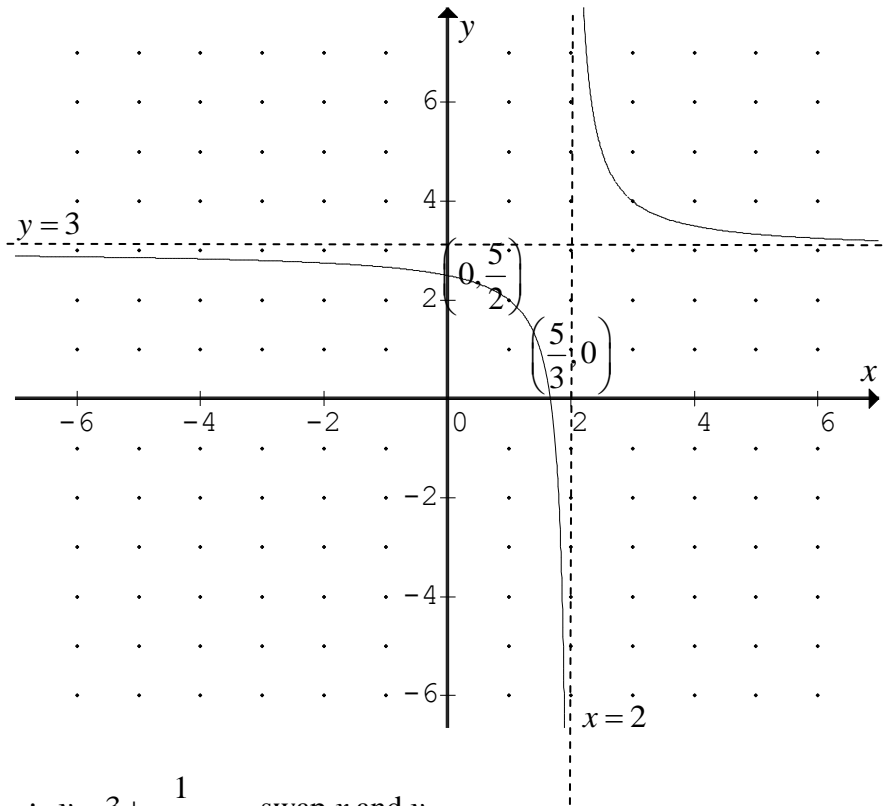
Question 11

i. $f(x) = \frac{3x-5}{x-2} = \frac{3(x-2)+1}{x-2} = 3 + \frac{1}{x-2}$ A1

ii. crosses the x -axis $y=0 \Rightarrow 3x-5=0 \Rightarrow x = \frac{5}{3} \left(\frac{5}{3}, 0\right)$

crosses the y -axis $x=0 \Rightarrow y = \frac{5}{2} \left(0, \frac{5}{2}\right)$ A1

the line $x=2$ is a vertical asymptote and
the line $y=3$ is a horizontal asymptote. A1



iii. $f : y = 3 + \frac{1}{x-2}$ swap x and y

$f^{-1} : x = 3 + \frac{1}{y-2} \Rightarrow \frac{1}{y-2} = x-3$ A1

$y-2 = \frac{1}{x-3} \Rightarrow y = f^{-1}(x) = 2 + \frac{1}{x-3}$ A1

must state the domain of the function, $\text{dom } f^{-1} = \text{ran } f = R \setminus \{3\}$

$f^{-1} : R \setminus \{3\} \rightarrow R \quad f^{-1}(x) = 2 + \frac{1}{x-3}$ A1

END OF SUGGESTED SOLUTIONS