Year 2012
VCE
Mathematical Methods
CAS
Solutions
Trial Examination 1

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Question 1

a. Let $y=\sqrt{16-x^3}=\left(16-x^3\right)^{\frac{1}{2}}=u^{\frac{1}{2}}$ where $u=16-x^3$

$\frac{dy}{du}=\frac{1}{2}u^{-\frac{1}{2}}$ $\quad\frac{du}{dx}=-3x^2$ using the Chain rule M1

$\frac{dy}{dx}=\frac{dy}{du}\frac{du}{dx}=-3x^2\times\frac{1}{2}u^{-\frac{1}{2}}=-\frac{3x^2}{2\sqrt{u}}$

$\frac{dy}{dx}=\frac{-3x^2}{2\sqrt{16-x^3}}$ A1

b. $\int\frac{x^2}{\sqrt{16-x^3}}\,dx=-\frac{2}{3}\sqrt{16-x^3}$ A1

Question 2

$4\cos\left(\frac{\pi x}{3}\right)-2=0 \quad\Rightarrow\quad \cos\left(\frac{\pi x}{3}\right)=\frac{1}{2}$

$\frac{\pi x}{3}=2n\pi\pm\cos^{-1}\left(\frac{1}{2}\right)=2n\pi\pm\frac{\pi}{3}$ M1

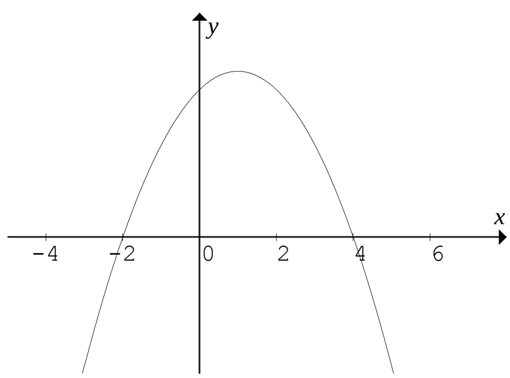
$x=6n\pm1$ where $n\in Z$ A1

Question 3

$f\left(x\right)=\log_e\left(x+3\right)\quad g\left(x\right)=5+2x-x^2$

$f\left(g\left(x\right)\right)=f\left(5+2x-x^2\right)$

$f\left(g\left(x\right)\right)=\log_e\left(5+2x-x^2+3\right)$



$f\left(g\left(x\right)\right)=\log_e\left(8+2x-x^2\right)$ A1

$f\left(g\left(x\right)\right)=\log_e\left(-\left(x^2-2x-8\right)\right)=\log_e\left(-\left(x-4\right)\left(x+2\right)\right)$

we require $y=-\left(x-4\right)\left(x+2\right)>0$ from the graph

$-2<x<4=\left(-2,4\right)$ A1

Question 4

$x' = 2x + 3y - 4$ and $y' = -y + 2$ A1

$\Rightarrow y = 2 - y'$

$2x = x' - 3y + 4 = x' - 3(2 - y') + 4 = x' + 3y' - 2$ M1

so $4x + y = 3$ becomes

$2(x' + 3y' - 2) + 2 - y' = 3$ drop the dashes

$2x + 5y = 5$ so $a = 2$ $b = 5$ $k = 5$ A1

Question 5

i. $q = \dfrac{10p}{p - 10}$

$\dfrac{dq}{dp} = \dfrac{10(p-10) - 1(10p)}{(p-10)^2}$ using the quotient rule M1

$\dfrac{dq}{dp} = \dfrac{-100}{(p-10)^2}$

ii. $p = 15$ $\Delta p = 0.1$ $\Delta q = ?$

$\dfrac{dq}{dp} \approx \dfrac{\Delta q}{\Delta p}$

$\Rightarrow \Delta q \approx \left.\dfrac{dq}{dp}\right|_{p=15} \Delta p$ M1

$\Delta q = \dfrac{-100}{5^2} \times 0.1 = -0.4$ cm or 0.4 cm decrease A1

iii. $\dfrac{dp}{dt} = 0.25$ cm/sec and $\dfrac{dq}{dt} = \dfrac{dq}{dp} \cdot \dfrac{dp}{dt}$ M1

$\dfrac{dq}{dt} = \dfrac{-100}{(p-10)^2} \times 0.25$ when $p = 15$

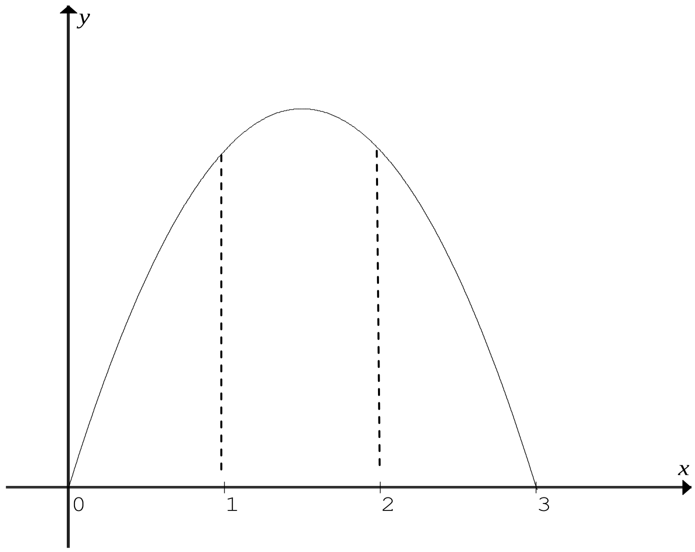
$\left.\dfrac{dq}{dt}\right|_{p=15} = \dfrac{-100}{5^2} \times 0.25 = -1$ cm/sec or 1.0 cm/sec decrease A1

Question 6

i. $k\int_0^3 x(3-x)\,dx = k\int_0^3 \left(3x-x^2\right)dx = 1$

$k\left[\frac{3x^2}{2}-\frac{x^3}{3}\right]_0^3 = k\left(\frac{27}{2}-9-0\right) = \frac{9k}{2} = 1$ M1

$k = \frac{2}{9}$



ii. Now $\Pr(2 < X < 3) = \Pr(0 < X < 1)$ by symmetry

$\Pr(0 < X < 1) = \frac{2}{9}\int_0^1 \left(3x - x^2\right)dx$

$= \frac{2}{9}\left[\frac{3x^2}{2}-\frac{x^3}{3}\right]_0^1 = \frac{2}{9}\left(\frac{3}{2}-\frac{1}{3}-0\right)$ M1

$= \frac{7}{27}$

$\Pr(X > 2 \mid X > 1) = \frac{\Pr(X > 2)}{\Pr(X > 1)} = \frac{\Pr(2 < X < 3)}{1-\Pr(X < 1)} = \frac{\Pr(0 < X < 1)}{1-\Pr(X < 1)}$

$\Pr(X > 2 \mid X > 1) = \frac{\frac{7}{27}}{1-\frac{7}{27}} = \frac{\frac{7}{27}}{\frac{20}{27}}$ M1

$\Pr(X > 2 \mid X > 1) = \frac{7}{20}$ A1

Question 7

i. $\sum \Pr(X = x) = \log_8(k+1) + \log_8(2k-4) = 1$

$\log_8((k+1)(2k-4)) = 1$

$(k+1)(2k-4) = 8$ M1

$2k^2 - 2k - 4 = 8$

$2k^2 - 2k - 12 = 0$

$2(k^2 - k - 6) = 0$ M1

$2(k-3)(k+2) = 0$

$k = 3$ or $k = -2$ but $k > 2$

$k = 3$ only A1

ii. $E(X) = \log_8(k+1) + 2\log_8(2k-4)$ with $k = 3$

$E(X) = \log_8(4) + 2\log_8(2) = \log_8(4) + \log_8(4) = \log_8(16) = m$

$8^m = 16 \quad (2^3)^m = 2^4 \quad \Rightarrow 3m = 4$ M1

$m = E(X) = \frac{4}{3}$ A1

Question 8

$y = f(x) = a(x-h)^3 + k$

since there is a stationary point at $x = 2 \Rightarrow h = 2$

crosses the *x*-axis at $x = 4 \quad f(4) = 0 \quad \Rightarrow 8a + k = 0$ so $k = -8a$ A1

$y = f(x) = a(x-2)^3 - 8a = a((x-2)^3 - 8)$

$A = a\int_0^4 ((x-2)^3 - 8)\,dx = 64$ M1

$A = a\left[\frac{1}{4}(x-2)^4 - 8x\right]_0^4 = a\left(\left(\frac{1}{4}(2^4) - 32\right) - \left(\frac{1}{4}(-2)^4 - 0\right)\right) = -32a = 64$ M1

$a = -2 \quad k = 16 \quad h = 2$ A1

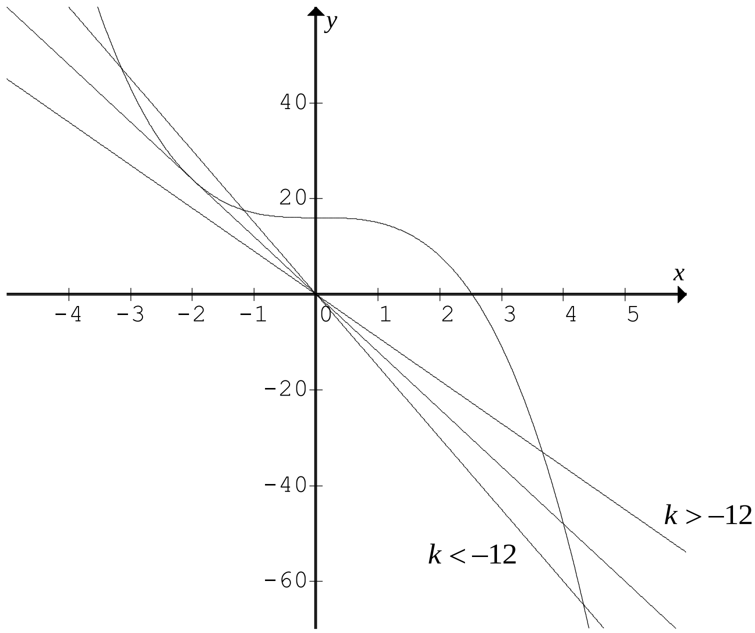
Question 9

$y = 16 - x^3$ at the point $\left(a, f(a)\right) = \left(a, 16 - a^3\right)$ $\dfrac{dy}{dx} = -3x^2$ $m_T = -3a^2$

$-3a^2 = \dfrac{16 - a^3 - 0}{a - 0}$ $\Rightarrow 16 - a^3 = -3a^3$ M1

at the point of contact $-2a^3 = 16$ $a^3 = -8$ so $a = -2$ A1

The tangent is $y = -12x$ A1



i. one solution $k > -12$

ii. two solutions $k = -12$

iii. three solutions $k < -12$ A2

Question 10

$v(t) = \dfrac{24}{\sqrt{4t + 9}}$

$s = \displaystyle\int_0^4 \frac{24}{\sqrt{4t + 9}}\, dt$

$s = \dfrac{24}{4} \times 2\left[\sqrt{4t + 9}\right]_0^4$ A1

$s = 12\left(\sqrt{25} - \sqrt{9}\right)$

$s = 24$ metres. A1

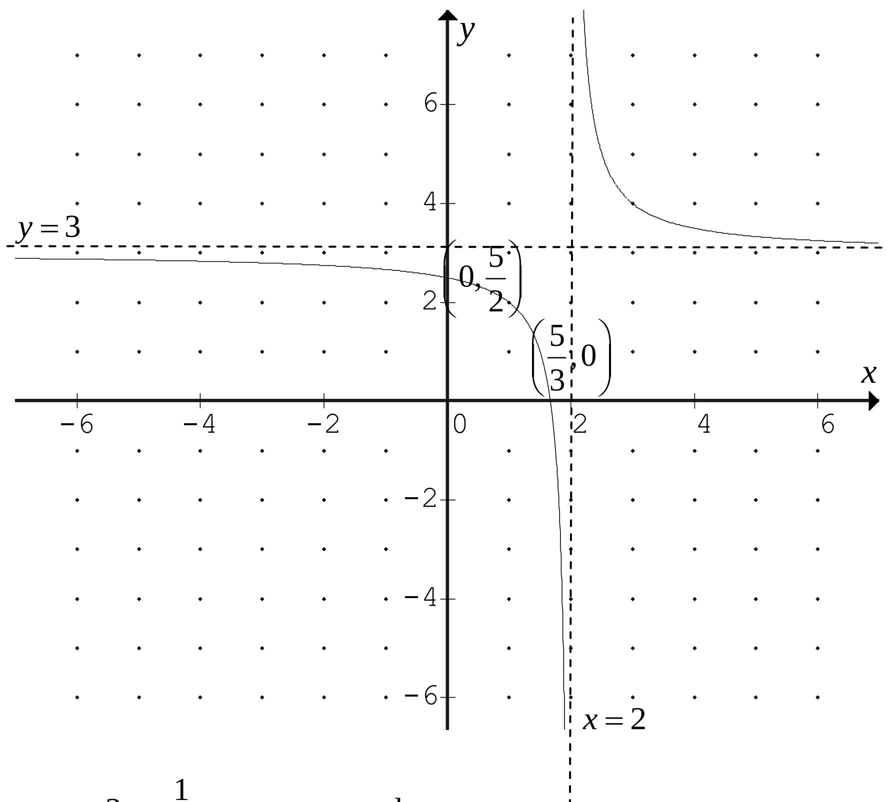
Question 11

i. $f(x)=\frac{3x-5}{x-2}=\frac{3(x-2)+1}{x-2}=3+\frac{1}{x-2}$ A1

ii. crosses the x-axis $y=0 \Rightarrow 3x-5=0 \Rightarrow x=\frac{5}{3}$ $\left(\frac{5}{3},0\right)$

crosses the y-axis $x=0 \Rightarrow y=\frac{5}{2}$ $\left(0,\frac{5}{2}\right)$ A1

the line $x=2$ is a vertical asymptote and
the line $y=3$ is a horizontal asymptote. A1



iii. $f \; : \; y=3+\frac{1}{x-2}$ swap x and y

$f^{-1} \; : \; x=3+\frac{1}{y-2} \quad \Rightarrow \frac{1}{y-2}=x-3$

$y-2=\frac{1}{x-3} \quad y=f^{-1}(x)=2+\frac{1}{x-3}$ A1

must state the domain of the function, dom $f^{-1}=\text{ran } f=R\backslash\{3\}$

$f^{-1}:R\backslash\{3\}\rightarrow R \quad f^{-1}(x)=2+\frac{1}{x-3}$ A1

END OF SUGGESTED SOLUTIONS