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Mathematical Methods

2012

Trial Examination 1

Instructions

Answer **all** questions. Do **not** use calculators.
A decimal approximation will not be accepted if an **exact** answer is required to a question.
In questions where more than one mark is available, appropriate working must be shown.
Unless otherwise indicated, the diagrams in this exam are **not** drawn to scale.

Question 1

Let $f(x) = x^2 + x + 1$ and $g(x) = x^2$

a. Solve $f(x) = g(x)$ for x. 1 mark

b. Describe the asymptotic behaviour of $y = \dfrac{f(x)}{g(x)}$. 2 marks

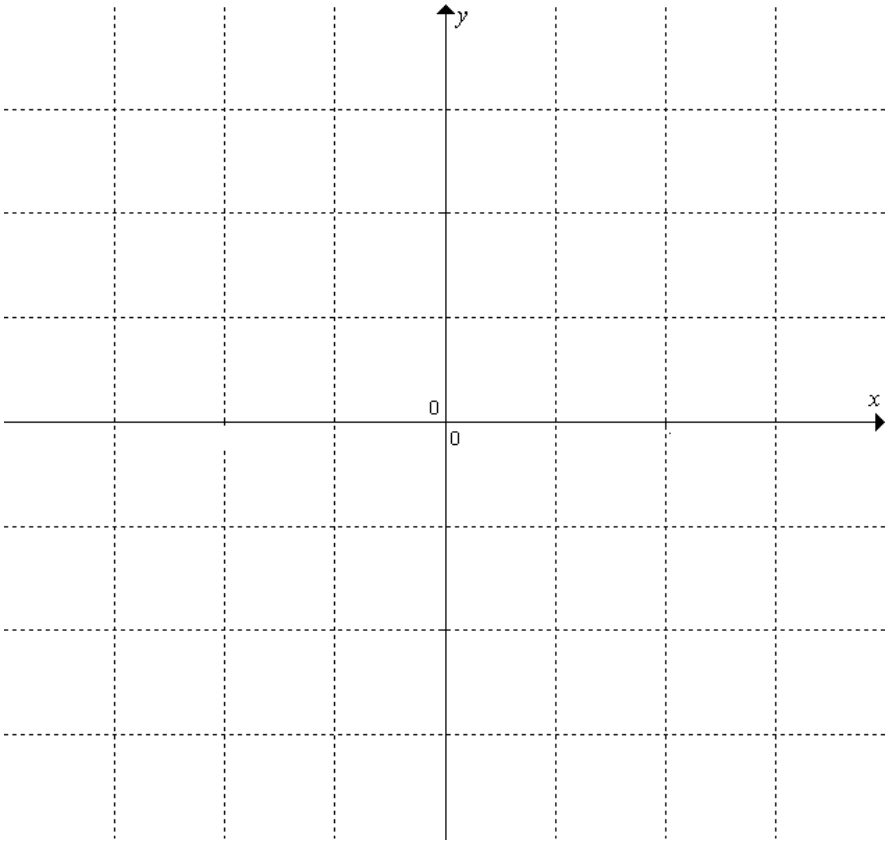
c. Express $f(g(x))$ in terms of $f(x)$ and x in 'simplest' form. 3 marks

Question 2

An ***odd*** polynomial equation of the ***least*** degree, $P(x)=0$, has a ***double*** root at $x=-2$.

a. Find $P(x)$. 2 marks

b. Sketch $y=P(x)$. Show the coordinates of axis intercepts. Do not find the stationary points. 2 marks



Question 3

Let $f(x)=\dfrac{2}{e^x-e^{-x}}$, $x\in R$.

a. Show that $f(x)+f(-x)=0$. 1 mark

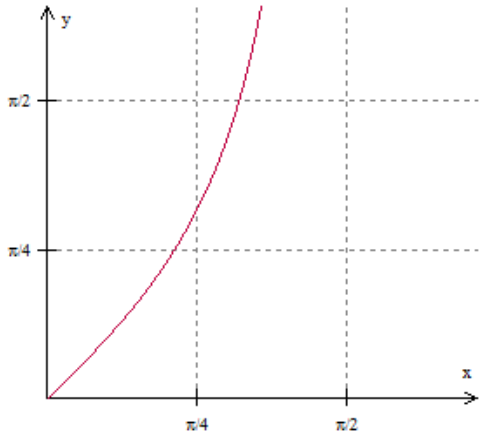
b. Find $f^{-1}(x)$. 3 marks

Question 4

Given $x^2+y^2+z^2+10x+20y-30z+350=0$, find the exact value of $(x-y-z)^2$. 3 marks

Question 5

The graph of $f(x) = \tan x$ for $x \in \left[0, \frac{\pi}{2}\right)$ is shown below.

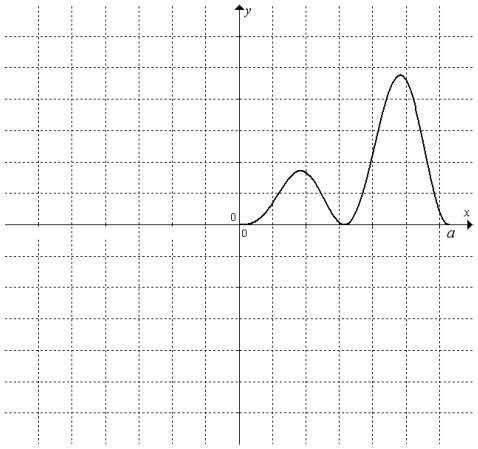


a. Sketch on the grid above the graph of $y = f^{-1}(x)$. 1 mark

b. Find the equation of the tangent to the curve $y = f^{-1}(x)$ at $x = 1$. 3 marks

Question 6

The graph of $y = g(x)$ for $x \in [0, a]$ is shown below.



a. Sketch on the above grid the graph of $y = g(a - x)$ to show that the areas under the two graphs are the same. 2 marks

b. If $f(x)=\dfrac{\cos x}{\cos x+\sin x}$ for $x\in\left[0,\dfrac{\pi}{2}\right]$, show that $f\left(\dfrac{\pi}{2}-x\right)=\dfrac{\sin x}{\cos x+\sin x}$. 2 marks

c. Hence find the exact value of $\displaystyle\int_0^{\frac{\pi}{2}}\frac{\cos x}{\cos x+\sin x}dx$. 2 marks

Question 7

The equation $ke^{2x}-2e^{x}+k=0$ has two solutions for x. Find the possible values of k. 3 marks

Question 8

Consider $f(x) = \dfrac{\log_e x}{x}$.

a. Find $f'(x)$. 2 marks

b. Hence find the exact value of $\displaystyle\int_1^e \frac{\log_e x}{x^2}\,dx$. 2 marks

Question 9

A worker drinks either tea or coffee only. If he had tea before, the probability that he has coffee in his next cup is 0.4. If he had coffee before, the probability that he has tea next is 0.7. Suppose that it is equally likely to have coffee or tea to start the day.

Find the probability that he has only one cup of tea in his *next* two cups. 2 marks

Question 10

The probability distribution of random variable X has a density function given by $f(x) = \begin{cases} 0.1 & 0 \leq x \leq a \\ 0.3 & a < x \leq 5 \\ 0 & elsewhere \end{cases}$

a. Find the exact value of a. 1 mark

b. Find the exact median of X. 1 mark

c. Find the exact value of $\overline{X}$. 2 marks

End of exam 1

