INSIGHT

YEAR 12 *Trial Exam Paper*

2012

MATHEMATICAL METHODS (CAS)

Written examination 2

STUDENT NAME:

QUESTION AND ANSWER BOOK

Reading time: 15 minutes
Writing time: 2 hours

Structure of book

Section	*Number of questions*	*Number of questions to be answered*	*Number of marks*
1	22	22	22
2	4	4	58
			Total 80

- Students are permitted to bring the following items into the examination: pens, pencils, highlighters, erasers, sharpeners, rulers, a protractor, set-squares, aids for curve sketching, one bound reference, one approved CAS calculator (memory DOES NOT need to be cleared) and, if desired, one scientific calculator. For approved computer based CAS, their full functionality may be used.
- Students are NOT permitted to bring the following items into the examination: blank sheets of paper and/or white out liquid/tape.

Materials provided

- The question and answer book of 23 pages, with a separate sheet of miscellaneous formulas.
- An answer sheet for multiple-choice questions.

Instructions

- Write your **name** in the box provided and on the answer sheet for multiple-choice questions.
- Remove the formula sheet during reading time.
- You must answer the questions in English.

At the end of the exam

- Place the answer sheet for multiple-choice questions inside the front cover of this question book.

Students are NOT permitted to bring mobile phones or any other electronic devices into the examination.

SECTION 1

Instructions for Section 1

Answer **all** questions in pencil on the multiple-choice answer sheet.
Select the response that is **correct** for the question.
A correct answer scores 1 mark, an incorrect answer scores 0.
Marks will not be deducted for incorrect answers.
If more than one answer is selected no marks will be awarded.

Question 1

The linear function $f : D \to R,\ \ f(x) = 7 - 3x$ has the range $[-4, 12]$.
Hence, the domain D is

A. R

B. $[-4, 12]$

C. R^+

D. $\left[-\frac{5}{3}, \frac{11}{3}\right]$

E. $\left[-\frac{11}{3}, \frac{5}{3}\right]$

Question 2

Let $g(x) = x^2 + 2x$ and $f(x) = e^{3x-5}$.
Then $f(g(x))$ is given by

A. e^{x^2+2x}

B. e^{3x^2+6x}

C. e^{3x^2+6x-5}

D. e^{x^2+2x-5}

E. $e^{3x^2} + 2e^x - 5$

Question 3

When $y = \log_a(6x - 2b) + 3$, x is equal to

A. $\frac{1}{6}(a^{-3} + 2b)$

B. $\frac{1}{6}(1 + 2b)$

C. $\dfrac{y - 3 + \log_a(2b)}{\log_a(6)}$

D. $\dfrac{y - 3}{6\log_a(2b)}$

E. $\frac{1}{6}(a^{y-3} + 2b)$

Question 4

The average rate of change of the function with rule $f(x) = x^2 - \sqrt{2x+1}$ between $x = 0$ and $x = 4$ is

A. $3\frac{1}{2}$

B. $3\frac{1}{3}$

C. 14

D. $1\frac{3}{4}$

E. $3\frac{1}{6}$

Question 5

The simultaneous linear equations

$$mx + 8y = 16$$

$$8x + my = m$$

has no solution when

A. $m = \pm 8$

B. $m = 8$

C. $m = -8$

D. $m \in R \setminus \{8\}$

E. $m \in R \setminus \{\pm 8\}$

Question 6

The range of the function $f : [\frac{\pi}{6}, \frac{7\pi}{6}) \to R,\ f(x) = 2\left|\cos(2x) - 1\right| + 3$ is

A. $[3, 7]$

B. $[1, 5]$

C. R

D. R^+

E. $[3, 4]$

Question 7

Which one of the following is **not** true about the function $f : R \to R,\ f(x) = \left|x^2 - 4\right|$?

A. The graph of *f* is continuous everywhere.

B. The graph of f' is continuous everywhere.

C. $f(x) \geq 0$ for all values of *x*.

D. $f'(x) = 0$ for *x* = 0.

E. $f(x) = 0$ for *x* = 2 and *x* = –2.

Question 8

If $k = \int_2^5 \frac{2}{x}\, dx$, then $e^{\frac{k}{2}}$ is equal to

A. $\frac{25}{4}$

B. $\frac{5}{2}$

C. 5

D. $e^5 - e^2$

E. $e^{\frac{5}{2}} - e^1$

Question 9

The graph of $y = kx - 2$ intersects the graph of $y = x^2 + 6x$ at two distinct points for

A. $k > 0$

B. $k < 6 \pm 2\sqrt{2}$

C. $k < \pm 6 - 2\sqrt{2}$

D. $k < 6 - 2\sqrt{2}$ or $k > 6 + 2\sqrt{2}$

E. $6 - 2\sqrt{2} < k < 6 + 2\sqrt{2}$

Question 10

The solution set of the equation $e^{6x} - 9e^{3x} + 8 = 0$ over R is

A. R

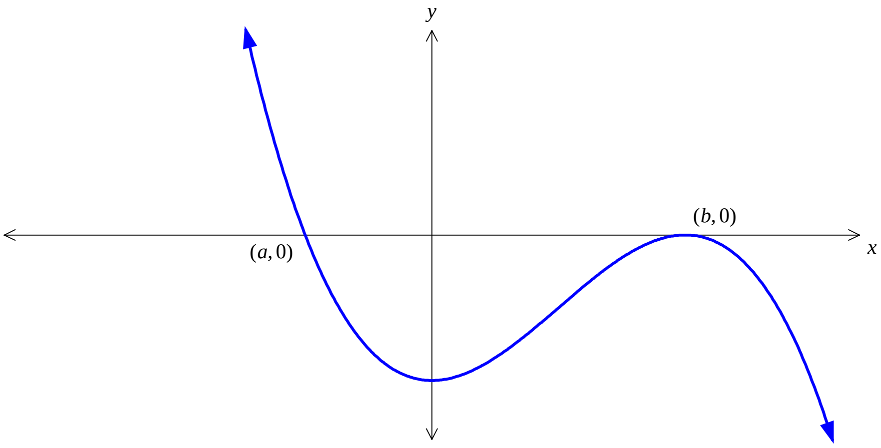
B. R^+

C. $\{0\}$

D. $\{\log_e 2\}$

E. $\{0, \log_e 2\}$

Question 11



For the graph of $y = f(x)$ shown above with $f'(0) = 0$, an interval over which $f(x)$ and $f'(x)$ are simultaneously negative is

A. (a, b)

B. (a, ∞)

C. $(a, 0)$

D. $(-\infty, 0)$

E. $(-\infty, a)$

Question 12

For $y = \log_e(\sqrt{f(x)})$, $\dfrac{dy}{dx}$ is equal to

A. $\dfrac{1}{\sqrt{f(x)}}$

B. $\dfrac{f'(x)}{2f(x)}$

C. $\dfrac{f'(x)}{f(x)}$

D. $\dfrac{f'(x)}{2\sqrt{f(x)}}$

E. $\dfrac{\log_e f(x)}{2}$

Question 13

For $f(x) = |\cos(x)|$ over the interval $[-2\pi, 2\pi]$, the derivative $f'(x)$ is defined as

A. $f'(x) = \begin{cases} -\sin(x), & x \in \left[-2\pi, -\frac{3\pi}{2}\right] \cup \left[-\frac{\pi}{2}, \frac{\pi}{2}\right] \cup \left[\frac{3\pi}{2}, 2\pi\right] \\ \sin(x), & x \in \left(-\frac{3\pi}{2}, -\frac{\pi}{2}\right) \cup \left(\frac{\pi}{2}, \frac{3\pi}{2}\right) \end{cases}$

B. $f'(x) = \begin{cases} \sin(x), & x \in \left(-2\pi, -\frac{3\pi}{2}\right) \cup \left(-\frac{\pi}{2}, \frac{\pi}{2}\right) \cup \left(\frac{3\pi}{2}, 2\pi\right) \\ -\sin(x), & x \in \left(-\frac{3\pi}{2}, -\frac{\pi}{2}\right) \cup \left(\frac{\pi}{2}, \frac{3\pi}{2}\right) \end{cases}$

C. $f'(x) = \begin{cases} -\sin(x), & x \in \left(-2\pi, -\frac{3\pi}{2}\right) \cup \left(-\frac{\pi}{2}, \frac{\pi}{2}\right) \cup \left(\frac{3\pi}{2}, 2\pi\right) \\ \sin(x), & x \in \left(-\frac{3\pi}{2}, -\frac{\pi}{2}\right) \cup \left(\frac{\pi}{2}, \frac{3\pi}{2}\right) \end{cases}$

D. $f'(x) = \{-\sin(x), \quad x \in (-2\pi, 2\pi)$

E. $f'(x) = \{\sin(x), \quad x \in (-2\pi, 2\pi)$

Question 14

The maximal domain D of the function $f : D \to R$ with rule $f(x) = \log_e(9 - x^2)$ is

A. [0, 3]

B. (0, 3)

C. [−3, 3]

D. (−3, 3)

E. R

Question 15

Let $f'(x) = g'(x) - 5$, where $f(0) = 3$ and $g(0) = 1$.
Hence, $f(x)$ is given by

A. $f(x) = g(x) - 5x + 7$

B. $f(x) = g(x) - 5x + 2$

C. $f(x) = g(x) + 2$

D. $f(x) = g(x) - 5x - 2$

E. $f'(x) = g'(x) + 2$

Question 16

The function f satisfies the functional equation $f\left(\frac{x-y}{2}\right) = \frac{1}{2}(f(x) - f(y))$, where x and y are non-zero real numbers. A possible rule for the function is

A. $f(x) = \sin(x)$

B. $f(x) = x^2 - 4x$

C. $f(x) = e^x$

D. $f(x) = \log_e(x)$

E. $f(x) = 5x$

Question 17

The discrete random variable X has a probability distribution as given in the table below.
The mean of X is 4.

x	0	2	4	6	8
$\Pr(X = x)$	0.2	a	0.1	0.4	b

The values of a and b are

A. $a = 0.25,\ b = 0.25$

B. $a = 0.15,\ b = 0.15$

C. $a = 0.1,\ b = 0.2$

D. $a = 0.2,\ b = 0.1$

E. $a = 0.2,\ b = 0.3$

Question 18

The heights of the teenage girls in a queue for ‘Australia’s Next Top Model’ are normally distributed with mean 180 cm and standard deviation 9.2 cm.
A total of 35% of the girls are not allowed to audition because they are considered too short.
Therefore, the minimum acceptable height, correct to the nearest centimetre, is

A. 176

B. 177

C. 175

D. 183

E. 184

Question 19

There are 2000 apples in storage at a fruit shop. Of these, it is found that 250 of them have a weight greater than 125 grams. The weights are normally distributed with a mean of μ grams and a standard deviation of 3.8 grams.
The value of μ is closest to

A. 120.6

B. 120.7

C. 128.8

D. 122.4

E. 122.5

Question 20

The equation $(x+3)^2(3-x)^3 - w = 0$ has only **one** solution for x when

A. $w > 0$

B. $w < 0$

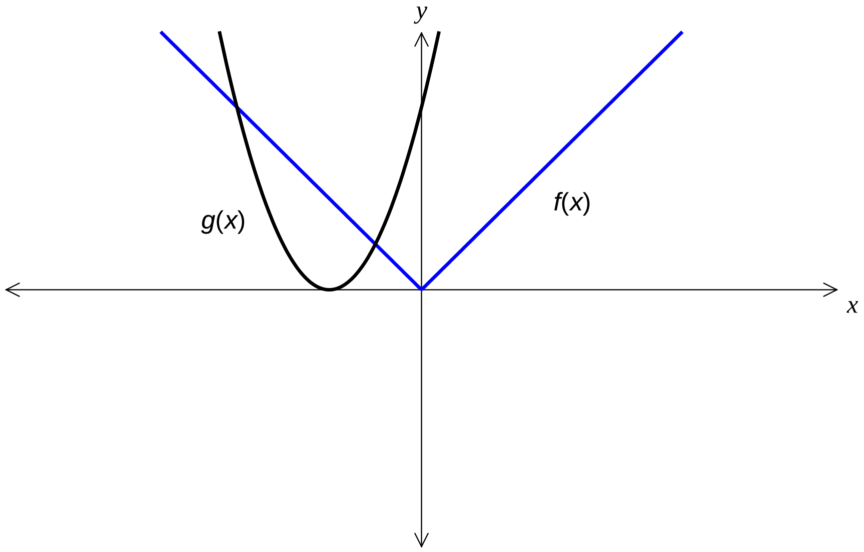
C. $w \geq 269$ or $w < 0$

D. $0 < w \leq 268$

E. $w \geq -269$

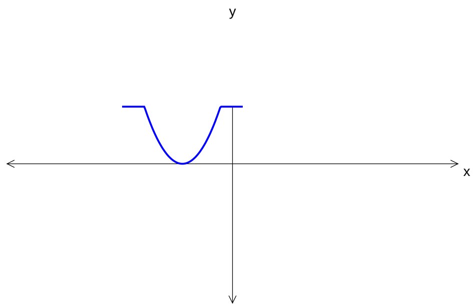
Question 21

The graphs of $y = f(x)$ and $y = g(x)$ are shown below.

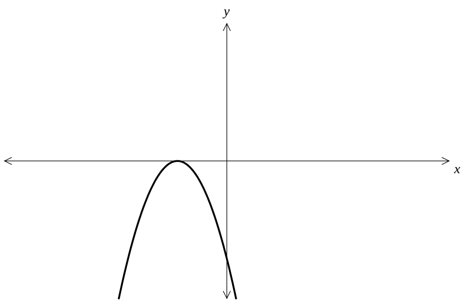


The graph of $y = f(g(x))$ is best represented by

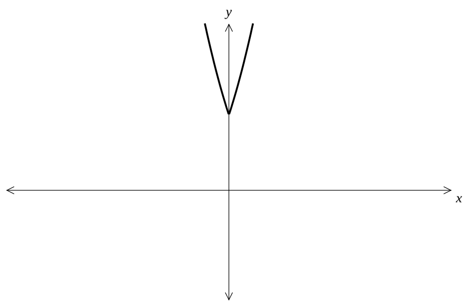
A.



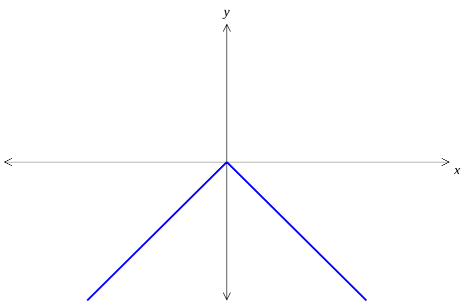
B.



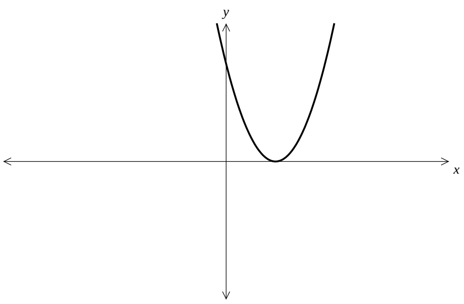
C.



D.

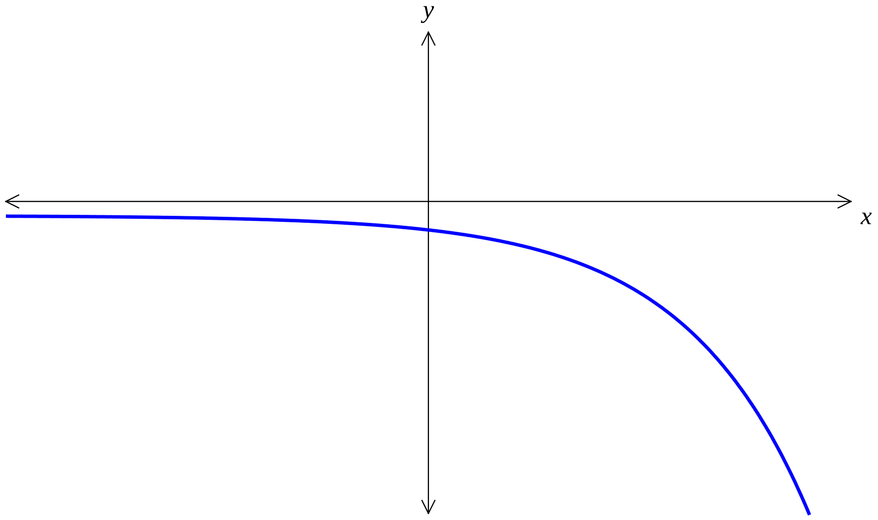


E.

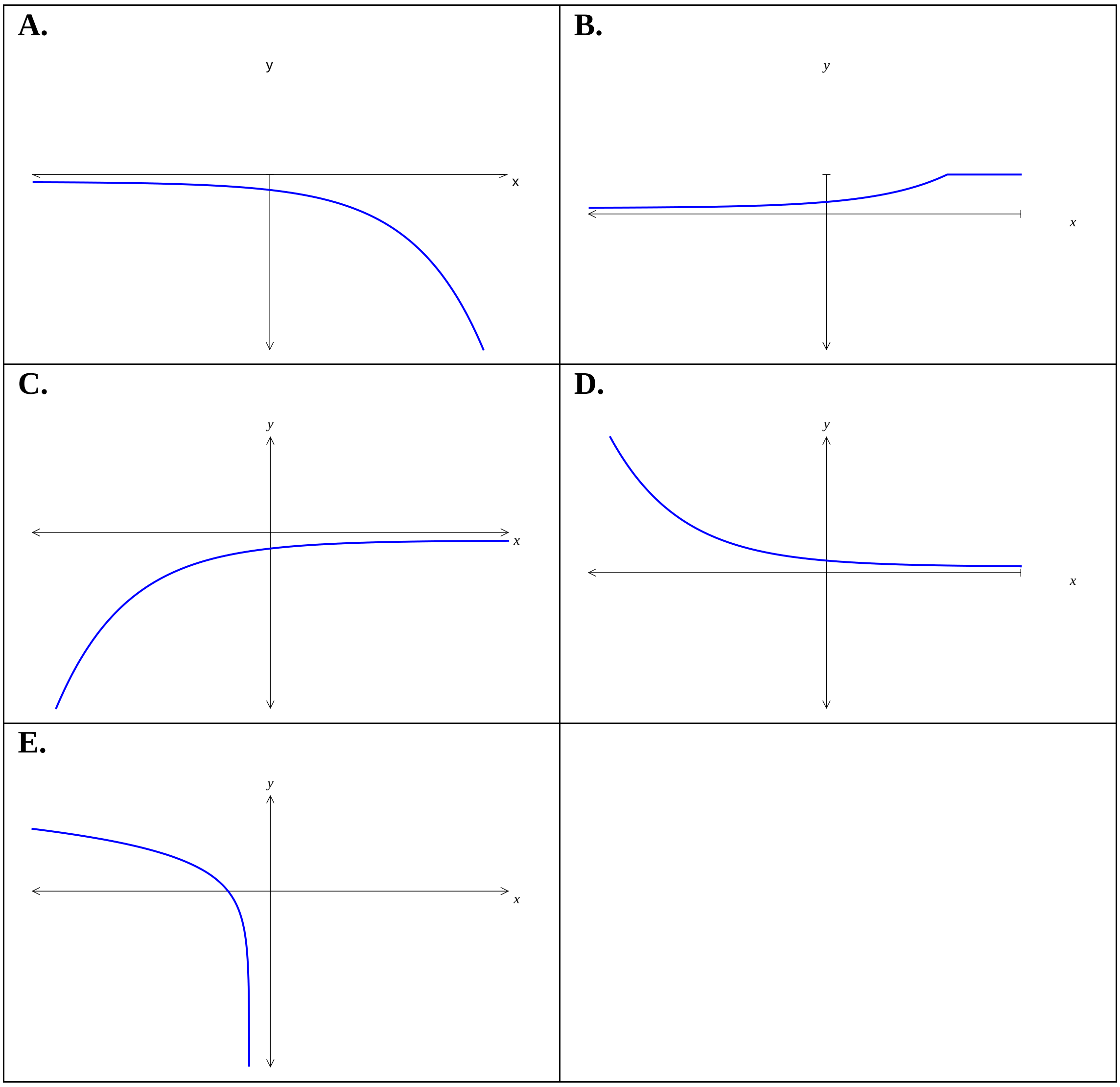


Question 22

The graph of the function f is shown below.



The graph of an antiderivative of f could be



END OF SECTION 1

Section 2

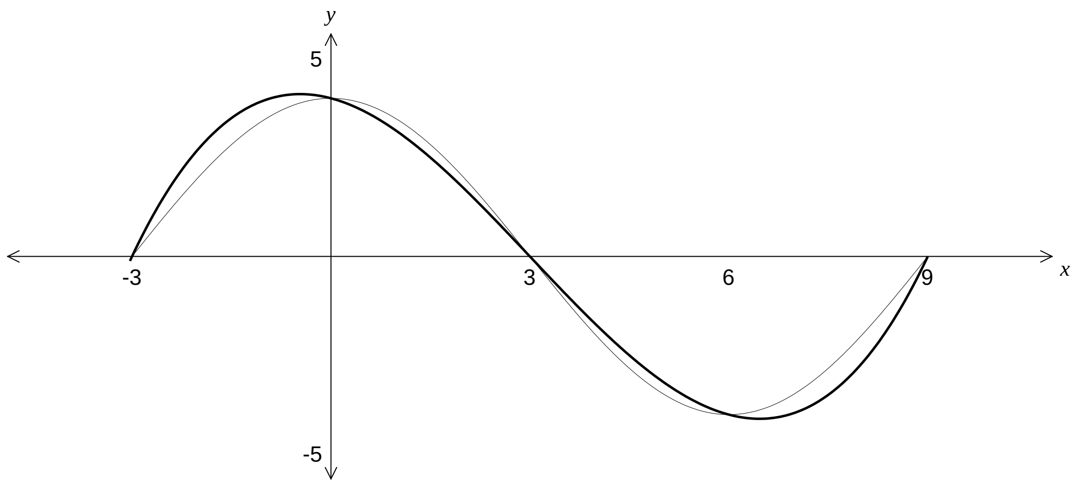
Instructions for Section 2
Answer **all** questions in the spaces provided.
Exact answers are required unless otherwise stated.
In questions where more than one mark is available, appropriate working **must** be shown.
Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Question 1

Shown below are the graphs of the two functions

$f:[-3, 9] \to R,\ f(x) = 4\cos\left(\frac{\pi x}{6}\right)$ and

$g:[-3, 9] \to R,\ g(x) = a(x+3)(x-3)(x-9)$



The point (0, 4) lies on both curves.

a. Show that $a = \frac{4}{81}$.

1 mark

b. State the period of the graph of $f(x)$.

1 mark

c. Find the exact value of x such that $g(x)$ is a maximum.

2 marks

d. **i.** Write down an integral expression that when evaluated gives the area enclosed between the curves $f(x)$ and $g(x)$ for $-3 \le x \le 9$.

2 marks

ii. Find the area between the curves $f(x)$ and $g(x)$ for $-3 \le x \le 9$.

1 mark
2 + 1 = 3 marks

e. **i.** Find the values of x, correct to 2 decimal places, that give the maximum value of $|f(x) - g(x)|$ for $-3 \le x \le 9$.

2 marks

ii. State, correct to 2 decimal places, the maximum value of $|f(x) - g(x)|$ for $-3 \le x \le 9$.

1 mark
2 + 1 = 3 marks

For a different value of a the point (0, 4) no longer lies on the graph of $g(x)$.

f. Find the exact value of a, such that the maximum value of $|f(x) - g(x)|$ occurs when $x = 1$.

3 marks

g. Find the smallest value of a, such that there are no local maximum values of $|f(x) - g(x)|$ in the interval $0 \le x \le 6$.

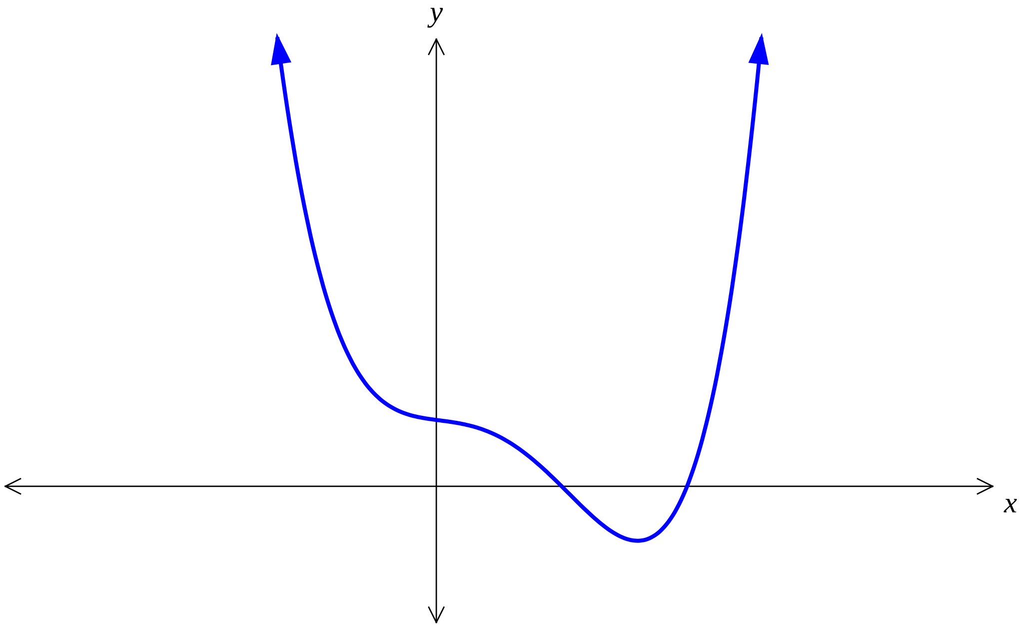
3 marks

Total 16 marks

SECTION 2 – continued
TURN OVER

Question 2

The graph of $f: R \to R,\ f(x) = (x-2)(x-4)(3x^2+4x+8)$ is shown below.



a. State the co-ordinates of the x-intercepts.

1 mark

b. State the number of stationary points.

1 mark

The quartic function g is defined by $g: R \to R,\ g(x) = (x-2)(x-4)(3x^2+ax+8)$, where a is a real number.

c. If g has m x-intercepts, what possible values can m take?

1 mark

d. If g has p stationary points, what possible values can p take?

1 mark

e. Find the values of a such that the graph of $g(x)$ has exactly **three** x-intercepts.

3 marks

f. For $a = 5$, there is only one stationary point. Write down an equation, the solution of which gives the x value of the stationary point. State the nature and co-ordinates of the stationary point, correct to 3 decimal places.

3 marks

The graph of $g(x)$ with $a = 5$ undergoes the following transformations.

- A dilation of factor 2 parallel to the x-axis
- A reflection in the x-axis
- A shift of 1 unit in the negative direction of the x-axis.

g. **i.** State the co-ordinates of the x-intercepts and the stationary point.

3 marks

ii. The new equation can be written in the form $y = -\frac{1}{16}(x-3)(x-7)(ax^2+bx+c)$.

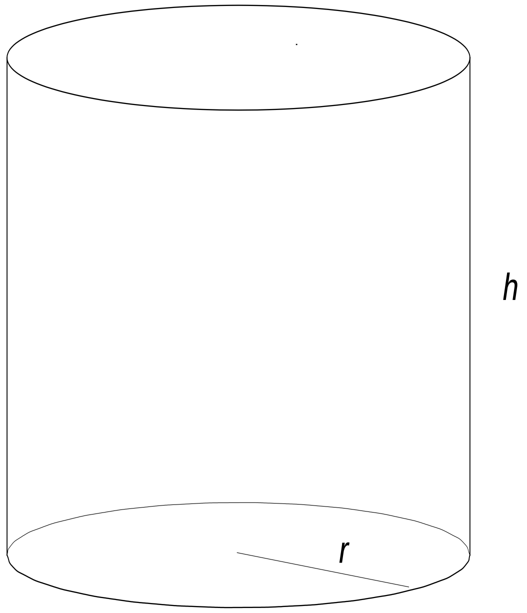
Find the values of a, b and c.

3 marks
3 + 3 = 6 marks

Total 16 marks

Question 3

Canino, a dog food company, makes dog food in cylindrical cans.
The cans have a radius of r cm and a height of h cm, as shown below.



a. If the volume of the can must be 600 cm^3, show that the surface area of the can is given by $S = 2\pi r^2 + \dfrac{1200}{r}$.

2 marks

b. **i.** Find $\frac{dS}{dr}$ and, **hence**, state the dimensions of the can that give the minimum surface area.

2 marks

ii. Find the minimum surface area, correct to 2 decimal places.

1 mark
2 + 1 = 3 marks

The lids of the cans are specially designed so that they can be opened without the need for a can opener. This adds to the cost of the cans and means that the cans are made from two different metal materials. One type of metal is used for the curved surface and the base, also called the body of the can, and the other type of metal is used for the lid.

The cost of the metal material for the body of the can is 1 cent per m^2. There is a range of metal materials available for the lid. The cost of the metal material for the lid varies depending on the quality of the metal used and is proportional to the cost of the metal used for the body of the can.

c. Show that the cost of the can is given by $C = \pi r^2 + \frac{1200}{r} + k\pi r^2$, where k is a positive constant of proportionality.

2 marks

d. **i.** When $k = 5$, find $\dfrac{dC}{dr}$.

1 mark

ii. Hence, find the dimensions of the can, correct to 2 decimal places, that will give the minimum cost.

2 marks
1 + 2 = 3 marks

e. For a particular type of metal, the value of k is such that a radius of 3.5 cm will give the minimum cost. Find this value of k.

2 marks

f. It is decided that the radius of the can must not exceed 3.5 cm. Find the values of k that give the minimum cost at $r = 3.5$.

2 marks

Total 14 marks

Question 4

Bikes are manufactured in a bicycle factory and the time, X hours, to produce a bike has the following probability density function.

$$f(x) = \begin{cases} 0 & x < 0 \\ \dfrac{1}{8} & 0 \le x \le 2 \\ 0.6e^{-0.8(x-2)} & x > 2 \end{cases}$$

a. Find, correct to 3 decimal places, $\Pr(1 \le X \le 4)$.

2 marks

b. Find the mean.

2 marks

c. Find the median, correct to 3 decimal places.

3 marks

d. It can be shown that $\Pr(X > 2) = 0.75$. A random sample of 10 bikes is chosen. Find the probability, correct to 3 decimal places, that exactly six of these 10 bikes took more than 2 hours to produce.

2 marks

SECTION 2 – Question 4 – continued

e. Again, a random sample of 10 bikes is chosen. If it is known that $\Pr(X > b) = a$, where $a, b \in R^+$, and that the probability that no more than one of these 10 bikes took more than *b* hours to produce was 0.9, find the values of *a* and *b*. Give your answer correct to 4 decimal places.

3 marks

Total 12 marks

END OF SECTION 2
END OF QUESTION AND ANSWER BOOK