INSIGHT

YEAR 12 *Trial Exam Paper*

2012

MATHEMATICAL METHODS (CAS)

Written examination 1

STUDENT NAME:

QUESTION AND ANSWER BOOK

Reading time: 15 minutes

Writing time: 1 hour

Structure of book

Number of questions	Number of questions to be answered	Number of marks
11	11	40

- Students are permitted to bring the following items into the examination: pens, pencils, highlighters, erasers, sharpeners and rulers.

- Students are NOT permitted to bring notes of any kind, sheets of paper, white out liquid/tape or a calculator into the examination.

Materials provided

- The question and answer book of 11 pages, with a separate sheet of miscellaneous formulas.

- Working space is provided throughout the question book.

Instructions

- Write your **name** in the box provided.

- Remove the formula sheet during reading time.

- You must answer the questions in English.

Students are NOT permitted to bring mobile phones or any other electronic devices into the examination.

Instructions

Answer **all** questions in the spaces provided.

A decimal approximation will not be accepted if an **exact** answer is required to a question.

In questions where more than one mark is available, appropriate working must be shown.

Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Question 1

Let $f(x) = \sqrt{3 - 2x}$. Write down the rule for $f(\cos x)$.

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_____

1 mark

Question 2

For the function $f : (-1, \infty) \rightarrow R, \ f(x) = \dfrac{1}{3}\log_e(\dfrac{x+1}{2})$

a. Find the rule for the inverse function, f^{-1}.
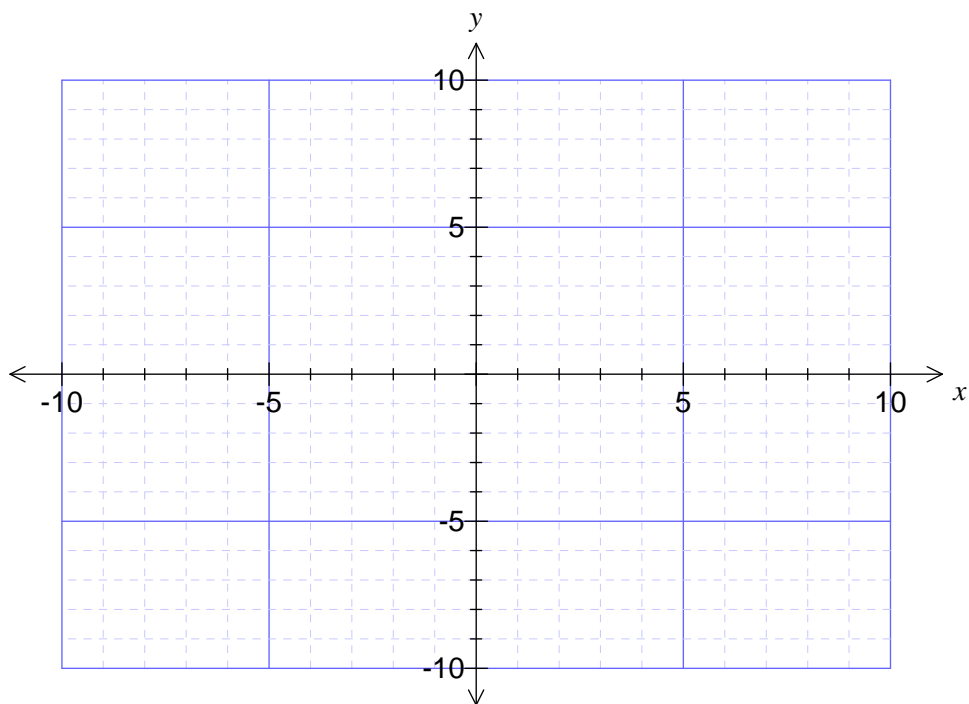
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2 marks

b. Sketch the graph of $y = f^{-1}(f(x))$ on the axes below.

1 mark

c. The function $f(x)$ undergoes a transformation as defined by the matrix $\begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$.

State the new equation.

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_____

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1 mark

End of Question 2
TURN OVER

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Question 3

a. Let $y = \dfrac{e^{2x}}{x}$. Find $\dfrac{dy}{dx}$.

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2 marks

b. Let $f(x) = \sqrt{\sin(2x)}$. Find $f'\left(\dfrac{\pi}{4}\right)$.

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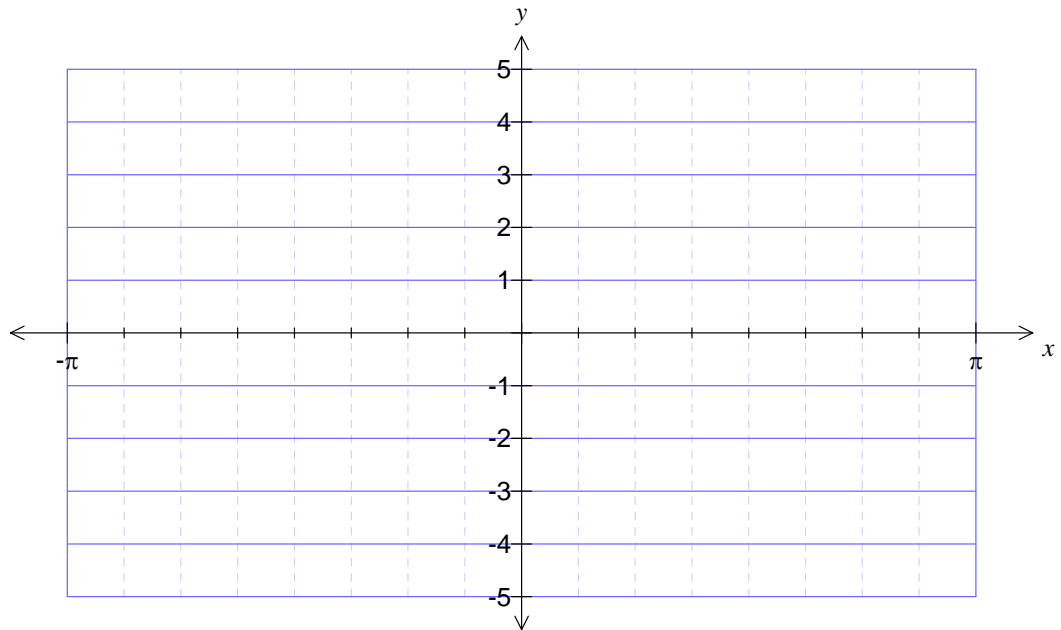
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3 marks

End of Question 3
TURN OVER

Question 4

The graph of $y = \cos(x)$ undergoes the following transformations:

- a dilation of factor $\dfrac{1}{2}$ from the y-axis
- a translation of $+3$ units up.

a. Sketch the transformed graph over the domain $[-\pi, \pi]$ on the axes below.
Label all intercepts and endpoints as co-ordinates.

2 marks

b. State the equation of the normal to the graph in part **a** at $x = \dfrac{\pi}{2}$ and sketch the normal

on the axes above.

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2 marks

End of Question 4

Question 5

Voicefone, a telephone company, currently has 30% of the market for a new type of home phone system. There are no contracts and customers simply take out a plan for a month at a time. Of the current customers, 80% will still be customers in the next month.

Assume that, for this type of system, a% of the rest of the market switches to this telephone company from one month to the next.

a. **i.** Write a transition matrix for this situation.

_____

_____

_____

     ii. Find the value of a needed in order for Voicefone to maintain its market share.

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3 marks

Voicefone runs an advertising campaign and hopes to eventually hold 60% of the total market, for this type of system.

b. Find the value of a for this situation.
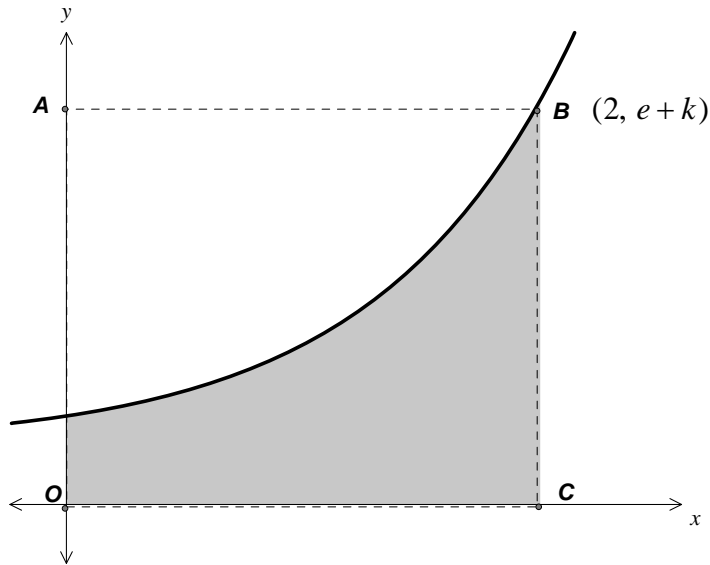
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1 mark

End of Question 5
TURN OVER

Question 6

Consider the graph of $y = e^{\frac{x}{2}} + k$. *OABC* is a rectangle, as shown in the diagram below. If the shaded and unshaded regions are equal in area, find *k*.

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<div align="right">3 marks</div>

<div align="right">**End of Question 6**</div>

Question 7

a. Find the general solution of $\sqrt{2}\cos(3x) = -1$.

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3 marks

b. Find the average value of the function $y = 2\cos(2x)$ over the interval $\left[0, \dfrac{\pi}{12}\right]$.

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3 marks

Question 8

A spherical balloon is being inflated. Its volume is increasing at the rate of 4 cm^3 per second. Find the rate, in cm s^{-1}, at which the radius of the balloon is increasing when the radius is 2 cm.

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3 marks

End of Question 8
TURN OVER

Question 9

Let X be a random variable with a normal distribution with mean 6 and variance 4, and let Z be a random variable with the standard normal distribution. If $\Pr(Z > 1) = 0.16$,

a. Find $\Pr(X > 8)$.

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1 mark

b. Find $\Pr(X > 8 \mid X > 6)$.

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2 marks

c. Find a such that $\Pr(Z > a) = \Pr(X < 5)$.

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2 marks

End of Question 9

Question 10

a. Using the linear approximation $f(x+h) \approx f(x) + hf'(x)$, where h is 0.03, and $f(x) = \sqrt{x}$, find an approximate value of $\sqrt{16.03}$.

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<div align="right">2 marks</div>

b. Explain why your answer to part **a** overestimates the value of $\sqrt{16.03}$.

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<div align="right">1 mark</div>

Question 11

If $f(x) = 3x^2$, show that $f(u+v) + f(u-v) = 2(f(u) + f(v))$.

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<div align="right">2 marks</div>

<div align="center">**END OF QUESTION AND ANSWER BOOK**</div>