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Units 3 and 4 Maths Methods (CAS): Exam 1

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Marks allocated are indicated by a number in square brackets, for example, [1] indicates that the line is worth one mark.

Question 1a

$\frac{d}{dx}\left(xlog_e(x)\right) = \frac{d}{dx}x \times log_e(x) + x \times \frac{d}{dx}log_e(x)$ [1]

$= log_e(x) + x\frac{1}{x} = 1 + log_e(x)$ [1]

Question 1b

$\int_1^2 log_e(x)\,dx = \int_1^2 [1 + log_e(x)]dx - \int_1^2 (x)dx$ [1]

$=[xlog_e(x)]_1^2 - [x]_1^2 = 2log_e(2) - 1$ [1]

Question 2

$\mu = np = 5, \sigma^2 = np(1-p) = 4$ [1]

$\frac{\sigma^2}{\mu} = 1 - p = \frac{4}{5} \Rightarrow p = \frac{1}{5}$ [1]

$n = \frac{\mu}{p} = 25$ [1]

Question 3

$f(x) = |x^2(x^2-1)| = \begin{cases} x^4 - x^2, & x < -1 \; or \; x > 1 \\ x^2 - x^4, & -1 \le x \le 1 \end{cases}$

$f'(x) = \begin{cases} 4x^3 - 2x, & x < -1 \; or \; x > 1 \\ 2x - 4x^3, & -1 \le x \le 1 \end{cases} = 0$ at stationary points [1]

for $x < -1$ or $x > 1$, $(4x^2 - 2) = 0 \Rightarrow x = 0, \pm\frac{1}{\sqrt{2}}$ none of which lie within $x < -1 \; or \; x > 1$

for $-1 \le x \le 1$, $x(2 - 4x^2) = 0 \Rightarrow x = 0, \pm\frac{1}{\sqrt{2}}$ [1]

$\therefore$ we have stationary points at (0,0), $\left(\frac{1}{\sqrt{2}}, \frac{1}{4}\right), \left(-\frac{1}{\sqrt{2}}, \frac{1}{4}\right)$ [1]

sign diagrams may be used to show that $(0,0)$ is a local minimum, others are local maxima [1]

Question 4a

$log_e 2 = log_2 x = \frac{log_e x}{log_e 2}$ [1]

$\Rightarrow log_e x = [log_e 2]^2$

$\Rightarrow x = e^{[log_e 2]^2}$ [1]

Question 4b

$25^x - 5^{x+1} + 6 = 0$

$\Rightarrow (5^x)^2 - 5(5^x) + 6 = 0$ [1]

$\Rightarrow (5^x - 2)(5^x - 3) = 0$

$\Rightarrow 5^x = 2 \; or \; 3$ [1]

$\therefore x = log_5 2 \; or \; log_5 3$ [1]

Question 5

For infinite solutions, the lines described by the equations must be parallel [1]

So: $\frac{3k}{3} = \frac{k}{1} = \frac{6}{k-1}$ [1]

$\Rightarrow k(k-1) = 6 \Rightarrow k^2 - k - 6 = 0 \Rightarrow (k-3)(k+2) = 0$

$\therefore k = -2$ or 3 [2]

Question 6a

$2\sin\left(x+\frac{\pi}{2}\right)+1=0$

$\sin\left(x+\frac{\pi}{2}\right)=-\frac{1}{2}$ [1]

$x+\frac{\pi}{2}=\frac{7\pi}{6}\ or\ \frac{11\pi}{6}$

$x=\frac{2\pi}{3}\ or\ \frac{4\pi}{3}$ [2]

Question 6b

$x=\frac{2\pi}{3}-\frac{\pi}{3}\ or\ \frac{4\pi}{3}-\frac{\pi}{3}=\frac{\pi}{3}\ or\ \pi$ [1]

Question 7

Curves intersect where $ax=x^2 \Rightarrow x(x-a)=0 \Rightarrow x=0, a$ [1]

$\int_0^a ax-x^2dx=\left[\frac{1}{2}ax^2-\frac{1}{3}x^3\right]_0^a=\frac{1}{2}a^3-\frac{1}{3}a^3=\frac{1}{6}a^3=\frac{9}{2}$ [2]

$\Rightarrow a^3=27$

$\Rightarrow a=3$ [1]

Question 8

$\frac{dV}{dt}=10$ [1]

$V=\frac{4}{3}\pi r^3 \Rightarrow \frac{dV}{dr}=4\pi r^2=100\pi\ \mathrm{cm}^3/\mathrm{cm}$ when $r=5$ cm [1]

$\frac{dr}{dt}=\frac{dr}{dV}\frac{dV}{dt}$ [1]

$=\frac{1}{100\pi}\frac{10}{1}=\frac{1}{10\pi}\ cm/s$ [1]

Question 9

Shape of f and g correct (g is given by f reflected in both axes) [1]

Shape of $f+g$ similar to $y=x^3$ [1]

with intercept at $(0,0)$ [1]

and no stationary points [1]

Question 10a

area $\approx \frac{1}{2}\frac{1}{2^2}+\frac{1}{2}\frac{1}{2.5^2}$ [1]

$=\frac{1}{2}\left(\frac{1}{4}+\frac{4}{25}\right)=\frac{1}{2}\left(\frac{25}{100}+\frac{16}{100}\right)=\frac{1}{2}\frac{41}{100}=0.205$ [1]

Question 10b

$\int_2^3 \frac{1}{x^2}dx=\left[-\frac{1}{x}\right]_2^3=-\frac{1}{3}+\frac{1}{2}=\frac{1}{6}$ [1]

Question 10b

Larger: $\frac{1}{x^2}$ is a decreasing function of x over $[2,3]$ hence left rectangles will overestimate the area under the curve on this interval. [1]