MATHEMATICAL METHODS (CAS)

Units 3 & 4 – Written examination 1

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2011 Trial Examination

SOLUTIONS

Question 1

a. $f'(x) = \frac{-4(2-x)^3}{(2-x)^4} = \frac{4}{x-2}$

M1+A1
2 marks

b. $x = \log_e(2-y)^4$
$x = 4\log_e(2-y)$
$\frac{x}{4} = \log_e(2-y)$
$e^{\frac{x}{4}} = 2-y$
$y = 2 - e^{\frac{x}{4}}$
$f^{-1}(0) = 1$

M2+A1
3 marks

c. $g'(x) = 2xe^{\sin(x)} + x^2\cos(x)e^{\sin(x)}$
$g'(x) = 2 \times \frac{\pi}{2} \times e^{\sin\left(\frac{\pi}{2}\right)} + \left(\frac{\pi}{2}\right)^2 \cos\left(\frac{\pi}{2}\right) e^{\sin\left(\frac{\pi}{2}\right)} = \pi e$

M1+A1
2 marks

Question 2

a. $\int \sqrt{x} - 2\sin\left(\frac{\pi x}{4}\right) dx = \frac{2x^{\frac{3}{2}}}{3} + \frac{8}{\pi}\cos\left(\frac{\pi x}{4}\right) + c$ where c is a real constant (but not necessary)

M1+A1
2 marks

b. $\left[\frac{(2x-1)^4}{2\times 4}\right]_0^2 = e^{\log_e m}$

$\frac{81}{8} - \frac{1}{8} = m$

$m = 10$

M1+A1

2 marks

Question 3

a. Amplitude = 3

Period $= \frac{2\pi}{n} = 2\pi \times 5 = 10\pi$

A2

2 marks

b. ***Error*! *Bookmark not defined.*** $\frac{-\cos(2x)}{cos(2x)} = \frac{\sin(2x)}{\cos(2x)}$

$\tan(2x) = -1,\ -\pi \leq x \leq \pi$

Reference $\angle$: $\frac{\pi}{4}$

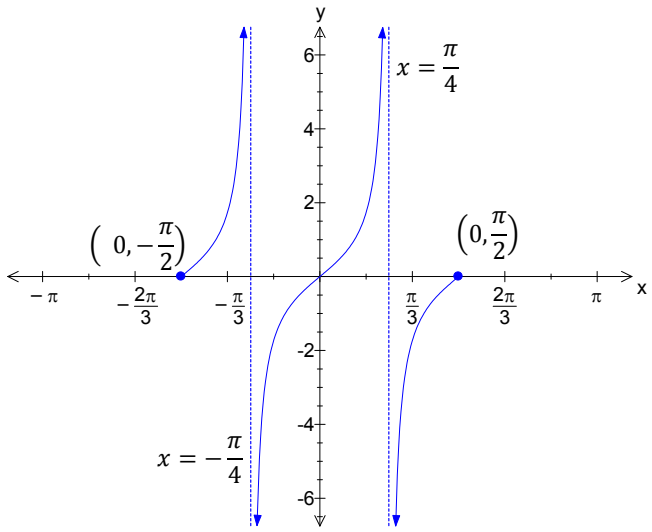
$2x = \pi - \frac{\pi}{4} = \frac{3\pi}{4}$ or $2x = \frac{3\pi}{4} - \pi = -\frac{\pi}{4}$

$x = \frac{3\pi}{8}$ $\qquad$ $x = \frac{-\pi}{8}$

M2+A1

3 marks

c.



A2

2 marks

Question 4

$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} -4x+3 \\ 2y-1 \end{bmatrix}$

$x' = -4x + 3$ and $y' = 2y - 1$

$x = \frac{3-x'}{4}$ and $y = \frac{y'+1}{2}$

$\frac{y'+1}{2} = 2\log_e(3-x') + 1$

$y' = 4\log_e(3-x') + 1$

$a = 4, b = -1, c = 3$ and $d = 1$

M2+A2

4 marks

Question 5

$\frac{dV}{dt} = 12$ mm^3/minute, $h = \frac{r}{6}$ and $\frac{dV}{dr} = \frac{\pi r^2}{2}$

$\frac{dV}{dt} = \frac{dV}{dr} \times \frac{dr}{dt}$

$12 = \frac{\pi r^2}{2} \times \frac{dr}{dt}$

$\frac{dr}{dt} = \frac{12 \times 2}{400\pi} = \frac{3}{50\pi}$ mm/min

M2+A1

3 marks

Question 6

$f(x) = \sqrt{x} = x^{\frac{1}{2}}, \quad f'(x) = \frac{1}{2}x^{\frac{-1}{2}}, \quad x = 9, \quad h = 0.01$

$f(x+h) \approx f(x) + hf'(x)$

$\sqrt{9.01} \approx \sqrt{9} + 0.01 \times \frac{1}{2}(9)^{\frac{-1}{2}}$

$\sqrt{9.01} \approx 3 + 0.01 \times \frac{1}{6}$

$\sqrt{9.01} \approx 3 + \frac{1}{600} \approx 3\frac{1}{600}$

M2+A2

4 marks

Question 7

a. $\int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} m\cos(x)dx = 1$

$m[\sin(x)]_{\frac{\pi}{2}}^{\frac{3\pi}{2}} = 1$

$m(-1-1) = 1$

$m = \frac{-1}{2}$

M2+A1

3 marks

b. $Pr(X > \pi) = \int_{\pi}^{\frac{3\pi}{2}} -0.5\cos(x)dx$

$-0.5[\sin(x)]_{\frac{\pi}{2}}^{\frac{3\pi}{2}} = -0.5(-1) = 0.5$

M1+A1

2 marks

Question 8

a. $Pr(X < 7) = 0.025$

$z = \frac{x-\mu}{\sigma} = \frac{7-11}{2} = -2 \Longrightarrow$ 2 standard deviations below the mean

$Pr(Z < -2) = Pr(X < 7) = 0.025$

A1

1 mark

b. $Pr(X < 8) = Pr(X > 14) = Pr\left(Z > \frac{14-11}{2}\right) = Pr\left(Z > \frac{3}{2}\right),\ m = \frac{3}{2}$

M1+A1

2 marks

Question 9

a.

$$\begin{array}{r} -2 \\ x+2\overline{)\,-2x+3} \\ \underline{-2x-4} \\ 7 \end{array}$$

$\frac{3-2x}{x+2} = \frac{7}{x+2} - 2$

M1+A1

2 marks

b. $\int \frac{3-2x}{x+2}dx = \int \frac{7}{x+2} - 2\ dx = 7\log_e|x+2| - 2x + c$

A1

1 mark